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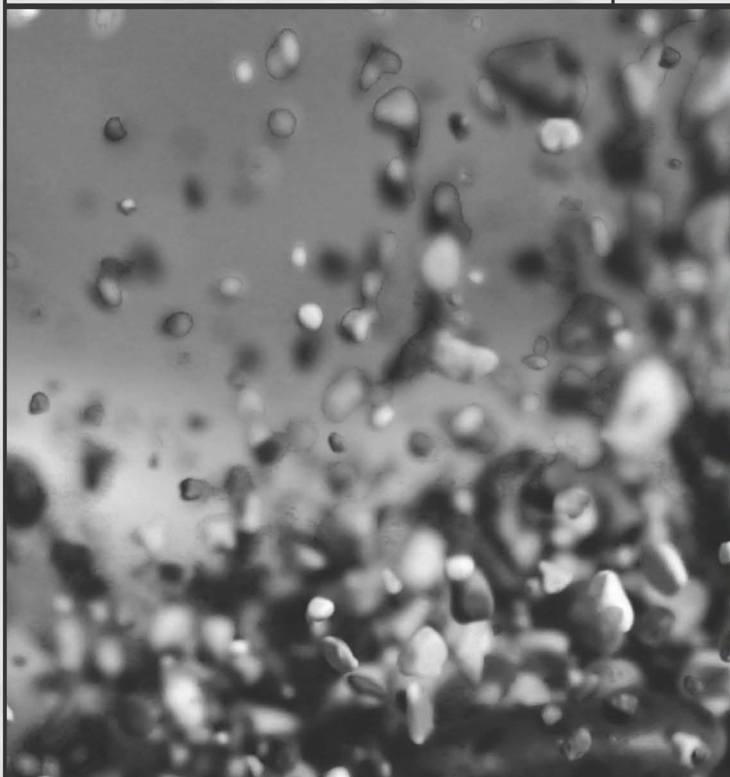

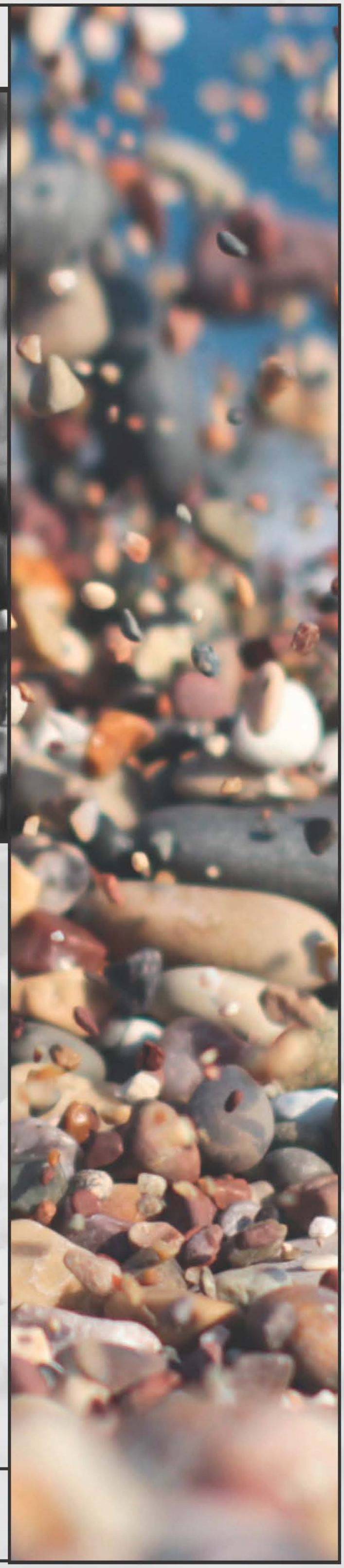

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Oxford Calculators and the new interpretation of Aristotle

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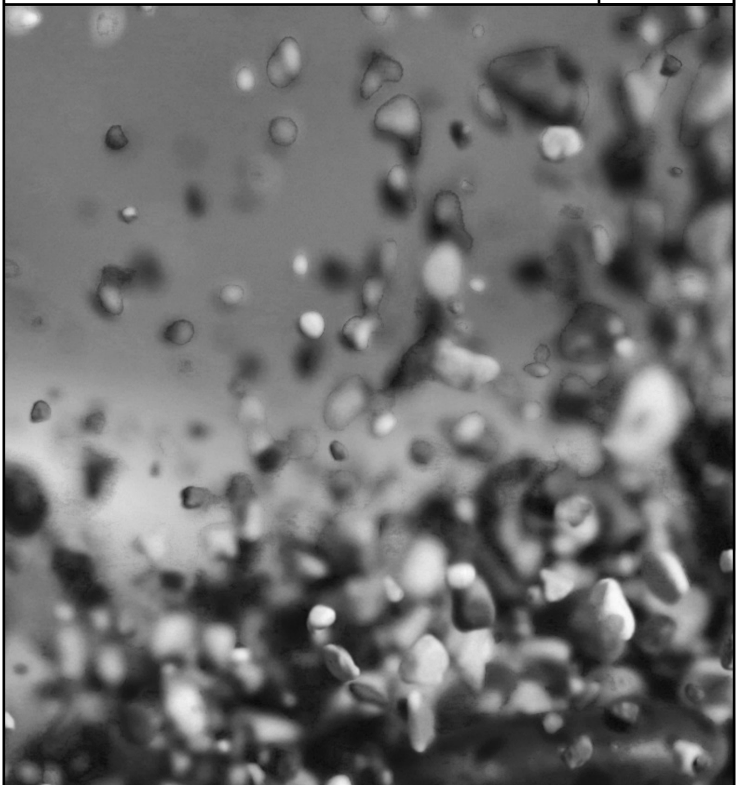

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Elżbieta Jung, Robert Podkoński – University of Lodz
Faculty of Philosophy and History, 90-131 Lodz, 3/5 Lindleya St., Poland

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Preface

Chris Schabel in his excellent book: "Theology at Paris 1316–1345. Peter Auriol and the problem of divine foreknowledge and future contingents" opens his Preface with a statement which suitably reflects the context of our research into the Oxford Calculators' 14th-century philosophy of nature. We repeat after Schabel: "The path that this present study has taken has been as roundabout as the historiographical path [...] that led to the serious lacuna that this book attempts to fill."

Studies into the Oxford Calculators tradition had their beginnings with Pierre Duhem's research published at the start of the 20th century. The discovery of mathematical physics, which, in accordance to the common opinion of historians of medieval science, was "introduced" by Thomas Bradwardine, initiated intensive research in the field. Konstanty Michalski, Marshall Clagett, Annelise Maier, Lamar Crosby, Curtis Wilson, John Murdoch, Ernest Moody, George Molland, John Longway, Stephen Read, Fabienne Pironet, Sabine Rommevaux, and Edith Sylla, to mention only a few names, devoted their studies either to preparing critical editions of the Oxford Calculators' texts or to presenting the main ideas of the Calculators themselves. The primary and secondary literature, as our Bibliography shows, is extensive.

The predominant belief, expressed by Edith Sylla, and commonly accepted, is that: "The Calculators carried their analyses and calculations a bit too far for it to be plausible that their main goal was discoveries in natural philosophy". In her opinion the works of such personalities of fourteenth-century Oxford philosophy as Richard Kilvington, Thomas Bradwardine, William Heytesbury, John Dumbleton and Richard Swineshead, albeit full of discussion of problems from natural philosophy, were intended from the outset to be first of all, more or less advanced, logical exercises, meant primarily for advanced undergraduates. We, however, made an afford to prove that the Oxford Calculators works were aimed not at formulating increasingly complicated logical riddles, but rather at developing the natural science, with a specially attention put on "science of motion' within the typically Aristotelian scheme of theoretical science.

Taking into account how much has been discovered, edited and written on the Oxford Calculators, we decided to revise and compare the results of our and other historians' studies on the intellectual heritage of these fourteenth-century English thinkers in order to provide those interested with an updated and well supplemented account on the Oxford Calculators natural philosophy in perhaps its most fundamental aspect – at least from the point of view of Aristotelian philosophy – namely on the "science of local motion". The first conclusion that must be form here, at the very beginning, is that the term "the Oxford Calculators' school" is perfectly adequate and well-grounded as a general notion with respect to the thinkers we refer to below. As will be shown, the concepts and solutions these thinkers included in their philosophical works were developed within the context of the ideas presented by the other group members – sometimes as simply borrowed ideas, sometimes as ones deemed dubious, and sometimes as mere impulses for further discussions and solutions. The other conclusion, perhaps far more subversive, is that it was not Thomas Bradwardine who introduced mathematics in the form of the new 'calculus of ratios' to the Aristotelian "science of local motion", but his contemporary, one of the most ingenious and unorthodox personalities of those times – Richard Kilvington. Only because there remained so few manuscript copies of Kilvington's works on natural philosophy, ones hitherto poorly scrutinized, did historians of medieval philosophy and science better know Thomas Bradwardine and his handbook "On the proportions of speeds in motions".

To achieve our main goal, i.e., to answer questions about continuity or discontinuity in the development of science from the Medieval period up to the Scientific Revolution we offer detailed analyses based on the first published critical editions of Latin-manuscript texts by Richard Kilvington, William Heytesbury, the anonymous author of the treatise *De sex inconevnientibus* and a part of Dumbleton's *Summa logiacae and philosophia narturalis* (Part III: *De motu locali*).

Our research confirms our belief that scientific truths in general, and even historical facts in particular, are never established once and forever, thus, through the present book we intend to revise the story of the Oxford Calculators' school.

In the course of the research and preparing critical editions, we have greatly benefited from the financial support provided by National Science Centre, Poland (UMO-2015/17/B/HS1/02376).

We would like to express our gratitude to Edith Sylla, André Goddu, Daniel Di Liscia and Chris Schabel, who have helped us in our work, both through constructive criticism and support. We thank Barbara Bartocci, Rodney Thompson and David Rundle for their help with the manuscripts of Dumbleton's *Summa*. Research for this book has been based mainly on manuscripts. The following libraries provided us with direct access to manuscripts: the Biblioteca Apostolica Vaticana, Gonville & Caius College Library, Cambridge, Peterhouse College Library, Cambridge, the Biblioteca Malatestiana in Cesena, the Wellcome Historical Medical Library, London, Lambeth Palace Library, London, the Biblioteca Nazionale Marciana in Venezia. Elżbieta Jung expresses special thanks to Verity Parkinson – the Resource Service and Support Librarian at Merton College Library Archive, University of Oxford and Anne Chaster – Deputy Librarian at Magdalen College Library and Archives, University of Oxford, for their kind assistance and help.

Chapter I

Lives and Works of Oxford Calculators

The fourteenth-century English thinkers active in Oxford formed the School, these being the so-called Oxford Calculators, a gathering previously known as the Merton School, since – as the precedent historians of science thought – its members were affiliated with Merton College.[1] Twentieth century scholars were sure that the founder of the School of Oxford Calculators was Thomas Bradwardine, who in 1328 had composed his famous *Treatise on Ratios of Speeds in Motions* (*Tractatus de proportionibus velocitatum in motibus*). In this work he offered the so-called New Rule of Motion, later known and discussed by the next generation of Oxford Calculators as well as by continental thinkers right up to the sixteenth century.[2] Elżbieta Jung's long lasting research, however, has revealed that already before 1328 there were intense, fruitful discussions on this issue between the members of Baliol College, Oxford. The anonymous author of *De sex inconvenientibus* written after 1335 mentions two names: Thomas Bradwardine and Adam of Pipewelle.[3] Bradwardine was already famous during his lifetime, while the second thinker is almost unknown – we only know that he was the member of Baliol College in 1326.[4] But yet we have a perfect witness of those discussions,

1 See for example J.A. Weisheiple, *Ockham and some Mertonians*, "Medieval Studies" 30 (1968), pp. 163–213; Idem, *Ockham and the Mertonians*, [in:] "The History of the University of Oxford", T.H. Aston (ed.), Oxford 1984, pp. 608–658; M. Clagett, "The Science of Mechanics in the Middle Ages", Wisconsin 1959.

2 The secondary literature on this subject is so extensive that it is difficult to mention even the most important works. In the footnotes below there are references to relevant works.

3 See infra, Anonimus, *De sex inconvenientibus*, q. *Utrum in omni motu sit certa servanda velocitas*, (Editions), §. 95, p. 334.

4 See, G.C. Brodrick, "Memorials of Merton College with biographical notices of the wardens and fellows", Oxford 1884, p. 195; A.B. Emden, "A Biographical Register of the University of Oxford to A.D. 1500", vol. III, P to Z, Oxford 1959, p. 1484; S. Rommevaux-Tani, *The study of local motion in the "Tracta-*

that is Richard Kilvington's commentaries on Aristotle's *On generation and corruption* and on the *Physics*. Kilvington's works were written in 1326 at the latest, and – as it appears – they were the source for the new theory of motion presented by Bradwardine in 1328.[5] Thus, the founders of the School, about whom we can be sure, are Richard Kilvington and Thomas Bradwardine. The next generation of Oxford Calculators are formed by William Heytesbury, John Dumbleton, with the last, well-known Calculator who "gave the name to this group of thinkers", being Richard Swineshead. It seems that to this group also belongs the anonymous author of the treatise *De sex inconvenientibus*, written after Hetesbury's *Regulae solvendi sophismata* (1335) and before Nicolas of Autreourt's question *Utrum visio craturae rationalis beatificalis per verbum possit intendi naturaliter* (1339).[6]

This chapter briefly presents the biographies and works of the *dramatis personae* of this book as well as short descriptions of their works devoted to local motion, which is the main subject of the book itself.[7]

tus de sex inconvenientibus": an example of inheritance form the Oxford Calculators, [in:] "Quantifying Aristotle. The Rise and Decline of the Oxford Calculators", D. Di Liscia, E. Sylla (eds), (forthcoming); E. Jung, "Zmiany ilościowe i ich miara w traktacie *O sześciu niedorzecznościach*, (Research on Science & Natural Philosophy, vol. III), Łódź 2020, pp. 15, 19; Eadem, *The New Interpretation of Aristotle. Richard Kilvington, Thomas Bradwardine and the New Rule of Motion*, [in:] "Quantifying Aristotle…", (forthcoming).

5 See, E. Jung, *The New Interpretation of Aristotle*…, (forthcoming); Ricardus Kilvington, *Quaestiones super libros Physicorum*, q. *Utrum in omni motu potentia motoris excedit potentiam rei motae*, (Eiditons), pp. 215–266.

6 See below, p. 22–28.

7 With the Oxford Calculators was associated also Roger Swineshead (fl. 1330, d. ca. 1365) a Master of Sacred Theology and a Benedict monk of Glastonbury. His work, variously entitled as *Descriptiones motuum*, *De Primo Motore* or, *De motibus naturalibus* was written after Bradwardine's *Tractatus de proportionibus velocitatum in motibus*, i.e., after 1328 but before 1337, when it was copied in Erfurt Amplonian Ms F 135, the only complete extant copy. Roger Swyneshead is also the author of the logical works: *Obligationes* and *Insolubilia* edited and commented by Paul Spade ("Roger Swineshead's *Obligationes*: Edition and Comments", "Archives d'histoire doctrinale et littéraire du moyen âge" (AHDLMA), 44 (1977), pp. 243–85; "Roger Swineshead's *Insolubilia*: Edition and Comments", AHDLMA 46 (1979), pp. 177–220). Since Edith Sylla described this work *On natural motion* in detail and, in fact, there is nothing specially interesting with regard to the theory of local motion we shall pass over this work here. See, E. Sylla, "The Oxford Calculators and the Mathematics of Motion 1320–1350. Physics and Measurement

1. Richard Kilvington

Richard Kilvington (we know almost seventy different spellings of his name) was born at the beginning of the fourteenth century in the village of Kilvington, Yorkshire in 1302/03. He was the son of a priest of the diocese of York. During his study at arts, he could have been first in Baliol, where he most likely met Bradwardine.[8] In Oxford he became Master of Arts (1325/26) then a Doctor of Theology (ca. 1335). Most likely, Kilvington was a fellow of Oriel College, Oxford.[9] Richard Kilvington's activities after Oxford are better known than his academic career. Having finished his studies, he joined the household of Richard of Bury, whose patronage helped some bachelors and doctors in their ecclesiastical careers and royal service. Between 1334–1345 Bury's household included Thomas Bradwardine, Walter Burley, Richard Bentworth, Richard FitzRalph, Robert Holcot, Richard Kilvington, Walter Segrave, John Maudith and John Acton.[10] Even after Bury's death, in 1345, Kilv-

by Latitudes", New York 1991, pp. 111–128; Eadem, *Mathematical physics in the work of the Oxford Calculators Roger Swineshead's On Natural Motion*, [in:] "Mathematics and Its Applications to Science and Natural Philosophy in the Middle Ages. Essays in Honor of Marchall Clagett" E. Grant, J.E. Murdoch (eds), Cambridge 1987, pp. 69–102; Spade, Paul Vincent and Read, Stephen, "Insolubles", *The Stanford Encyclopedia of Philosophy* (Fall 2018 Edition), Edward N. Zalta (ed.), URL = <https://plato.stanford.edu/archives/fall2018/entries/insolubles/>.

8 See, N. Kretzmann, B.E. Kretzmann, "The 'Sophismata' of Richard Kilvington. Introduction, Translation and Commentary", p. XXIV. Annelise Maier described Kilvington as Bradwardine's student. She, however, did not offer any specific evidence for this claim, and hence the Kretzmanns see this as unlikely (see, N. Kretzmann, B.E. Kretzmann, "The 'Sophismata'…", p. XX, n. 9).

9 There is documentary evidence that Kilvington was a fellow of Oriel College, Oxford University. In 1333 he was mentioned as a "provisor" of Oriel; in 1331 he donated a substantial number of books to Oriel's library (see N. Kretzmann, B.E. Kretzmann., "The 'Sophismata'…", p. XXV, n. 28–29).

10 W. Chambre, "Continuatio Historiae Dunelmensis", Newcastle 1839, p. 128: "Multum <enim> delectabatur de <comitiva> clericorum; et plures semper clericos habuit in sua familia. De quibus fuit Thomas Bradwardyn, postea Cantuariensis Archiepiscopus, et Ricardus Fyzt Rauf, postmodum Archiepiscopus Arnmanachae, Walter Burley, Johannes Maudit, Robert Holcot, Ricardus de Kylwyngton, omnes doctores in theologia: Ricardus Benworth, postea Episcopus Londoniensis et Walterus Segraffe, postea Episcopus Cicestrensis".

ington was still a "king's clerk" going abroad "on necessary business" with royal "protection and safe conduct". In 1350 Kilvington was the Archdeacon of London. In 1354 he was appointed Dean of Saint Paul's cathedral in London. Along with Richard FitzRalph, Kilvington was involved in the battle against mendicant friars. The struggle began in London and in 1357 it moved to Avignon, where FitzRalph appeared to defend his views before Innocent VI. Kilvington was active in the support of FitzRalph in his treatise: *In causa domini Armachani allegationes magistri Ricardi devoti viri contra Fratres.* It seems certain that FitzRalph's and Kilvington's argument with the mendicants continued almost until the ends of their lives. Kilvington was probably a victim of the Black Death and died in 1361, two years after the papal bull reconfirmed the mendicant privileges. Richard Kilvington was buried in Saint Paul's cathedral in London.[11]

We do not know any of Kilvington's philosophical or theological works, which might have been written after his transition from the university to a public career. His diplomatic and ecclesiastical career did not stimulate his further scholarship, nor did his being a member in Richard of Bury's household. Apart from a few sermons, all of Kilvington's known works stem from his teaching at Oxford, and they often reflect the lively class discussions.[12] None of his works is written in the usual commentary format, following the order of books in the respective works of Aristotle. In accordance with the fourteenth-century Oxford practice, Kilvington reduced the number of topics discussed to certain central issues, which were fully developed with no more than ten questions constituting a commentary.[13] The reduction in the range of

11 For more details see E. Jung, "Works by Richard Kilvington", AHDLMA 67 (2000), pp. 184–225; Eadem, "Między filozofią przyrody a nowożytnym przyrodoznawstwem. Ryszard Kilvington i fizyka matematyczna w średniowieczu", Łódź 2002; Eadem, "Arystoteles na nowo odczytany. Kwestie o ruchu Ryszarda Kilvingtona", Łódź 2014; Jung, Elzbieta, "Richard Kilvington", *The Stanford Encyclopedia of Philosophy* (Winter 2016 Edition), Edward N. Zalta (ed.), URL = <https://plato.stanford.edu/archives/win2016/entries/kilvington/>. In these works there is also an extensive bibliography.

12 See Ryszard Kilvington "Kwestie o ruchu" (Ricardus Kilvington, *Quaestiones de motu*), Polish translation by E. Jung, [in:] "Arystoteles na nowo odczytany…", Łódź 2014, pp. 107–316.

13 See, for example, Richard Kilvington, *Quaestiones super libros Ethicorum*", [in:] "Richard Kilvington's *Quaestiones super libros Ethicorum.* A Critical Edition with an Introduction" by Monika Michałowska, Leiden 2016, pp. 63–336.

topics is counterbalanced by deeper analysis of the questions chosen for treatment. Some of Kilvington's questions cover twenty folios, which in a modern edition yield about 180 pages. Only his logical treatise was not written as a commentary, but rather as "a guide" for students showing how to solve sophisms. In the preface to his *Sophismata* Kilvington says:

> When we are able to call both sides into question, we will readily discern what is true and what is false, as Aristotle says in Book One of his *Topics*. Therefore, in order that we may more readily discern what is true and what is false, in the present work, which consists of *sophismata* to be thoroughly investigated, I intend, to the best of my ability, both to demolish the two sides of the contradiction and also to support them by means of clear reasoning. I am led to do this by the request of certain young men who have been pressing their case very hard. And so, wishing to give them something I have often heard them ask for, I have undertaken an attempt in that direction.[14]

Richard Kilvington's philosophical works, the *Sophismata* and *Quaestiones super De generatione et corruptione*, composed before 1324, came from his lectures as a bachelor of arts; the *Quaestiones super Physicam* composed at the latest in 1326 and *Quaestiones super Libros Ethicorum* (before 1333) date from his time as an arts master; after he advanced to the Faculty of Theology, he produced eight questions on Peter Lombard's *Sentences (*1333 or 1334).[15] Most of Kilvington's works are still to be found in manuscripts, only his commentary to the *Ethics*, which consists of eight questions,[16] and 48 sophisms, which form his *Sophismata*, are critically edited, *Sophismata* have also been translated into English by Kretzmanns. His commentary on the *On generation and corruption* form a set of

14 N. Kretzmann, B.E. Kretzmann, "The ‚*Sophismata*'…, (*Introduction*), p. 1.

15 See E. Jung, *The New Interpretation of Aristotle*…., (forthcoming).

16 See M. Michałowska, "Richard Kilvington's *Quaestiones super libros Ethicorum*…", (Introduction, pp. 11–26).

nine questions,[17] his commentary on the *Sentences* is formed in a set of eight questions.[18]

From the perspective of the present book the most important is a set of eight questions which belong to his commentary on Aristotle's *Physics*. Recently Jung has proven that Kilvington's questions on the *Physics* perfectly testify to the dispersed tradition of this commentary. The whole set consists of eight questions: one question with an exposition of the *Physics*, to be found in a Vatican manuscript (Vat. Lat. 4353),[19] a set of

17 The questions are as follows: *Utrum augmentatio sit motus ad quantitatem*; *Utrum numerus elementorum sit aequalis numero qualitatum primarium*; *Utrum ex omnibus duobus elementis possit tertium generari*; *Utrum continuum sit divisibile in infinitum*; *Utrum omnis actio sit ratione contrarietatis*; *Utrum omnia elementa sint adinvicem transmutabilia*; *Utrum mixtio sit miscibilium alteratorum unio*; *Utrum omnia contraria sint activa et passiva adinvicem*; *Utrum generatio sit transmutatio distincta ab alteratione*. They are to be found, as a complete or incomplete set, in the following mss.: Brugia, Stedelijke Openbare Bibl. 503; Cambridge, Peterhouse 195; Erfurt, Wissenschaftliche Allgemeinbibliothek, Amploniana Cms 8º 74; Kraków, BJ 648; Paris, BnF lat. 6559; Sevilla, Bibl. Colombina 7–7–13.

18 The commentary on the *Sentences* is to be found in the following libraries: Bologna, Bibl. Comunale dell'Archiginnasio A. 985; Brugge, Stedelijke Openbare Bibliotheek, Hs. 188, Hs 503; Erfurt, CA. 2° 105; London Harley, British Library, 3243; Paris, BnF lat. 14576, 15561; Praha, Národní Knihovna České Republiky, Cod. III B. 10; Wrocław, Bibl. Uniw., IV F 198; Vatican, Vat. lat. 4353; Firenze, Bibl. Nationale Centrale Cod. II. II 281 ; Tortosa, Bibl. de la Catedral y del Cabildo de la Sanctísima Iglesia Catedral, Cod. 186. The eight questions, from ms. Bologna are titled as follows : 1) *Utrum Deus sit super omnia diligendus*; 2) *Utrum per opera meritoria augeatur habitus caritatis quo Deus est super omnia diligendus* ; 3) *Utrum omnis creatura sit suae naturae certis limitibus circumscripta* ; 4) *Utrum quilibet actus voluntatis per se malus sit per se aliquid* ; 5) *Utrum peccans mortaliter per instans solum mereatur puniri per infinita instantia interpolata*; 6) *Utrum aliquis nisi forte in poena peccati possit esse perplexus in his quae pertinent ad salutem*; 7) *Utrum omnis actus factus extra gratiam sit peccatum*; 8) *Utrum aliquis possit simul peccare venialiter et mereri vitam aeternam*. For a description of the manuscripts see M. Michałowska, "Richard Kilvington on the capacity of created beings, infinity, and being simultaneously in Rome and Paris. Critical edition of question 3 *Utrum omnis creatura sit suae naturae certis limitibus circumscripta* from *Quaestiones super libros Sententiarum* with an Introduction" (forthcoming).

19 In my paper *Works by Richard Kilvington* (p. 203, n. 102) I claimed that only four questions on motion from the Marciana library form Kilvington's commentary to the *Physics*. Detailed study, however, revealed that *expositio* of the *Physics* as well as one question not two, as I had claimed before, were also composed by Richard Kilvington.

three questions in a Seville manuscript (Biblioteca Colombina 7–7–13), a set of four questions on motion, to be found in Venice library (Venezia, Bibl. Naz. Marciana, lat. VI, 72), single questions are also to be found in other manuscripts.

The questions are as follows:

Expositio super primum librum Physicorum (Ms. Vatican, Vat. lat. 4353).

1. *Utrum omne scitum sciatur per causam* (Ms. Vatican, Vat. lat. 4353).
2. *Utrum omne quod generetur ex contrariis generetur* (Ms. Vatican, Vat. lat. 4353; Seville Colomb. 7–7–13).
3. *Utrum in omni generatione tria principia requirantur* (Ms. Seville Colomb. 7–7–13; Paris BnF lat. 6559; Bruges, Stedelijke Openbare Bibliotheek 503).
4. *Utrum omnis natura sit principium motus et quietis* (Ms. Seville Colomb. 7–7–13).
5. *Utrum potentia motoris excedit potentiam rei mote* (Ms. Venezia, Bibl. Naz. Marciana lat. VI, 72 (2810); Vat. lat. 2148).
6. *Utrum qualitas suscipit magis et minus* (Ms Venezia, Bibl. Naz. Marciana, lat. VI, 72 (2810); Paris, BnF lat. 16401; Vatican, Vat. lat. 2148; Vat. lat. 4429; Paris, BnF lat. 6559; Oxford, Bodl., Canon Misc. 226; Praha, Narodni Knihovna III. B; Cambridge, Peterhouse 195).
7. *Utrum aliquod motus simplex possit moveri aeque velociter in vacuo et in pleno* (Venezia, Bibl. Naz. Marciana, lat. VI, 72 (2810)).
8. *Utrum omne transmutatum in transmutationis initio sit in eo ad quod primitus transmutatur* (Venezia, Bibl. Naz. Marciana, lat. VI, 72 (2810)).[20]

From the point of view of the main problem of this book, the fifth question devoted to the problem of local motion is the most interesting. This question is – as Kilvington says – divided into four articles, where he firstly presents and discusses different opinions describing the way of "measuring" the primary conditions necessary for motion to occur, such as an excess of an acting power over the passive one; the possible limit of an acting power causing the motion; the possible limit of a passive power to be overtaken; and the result of their actions i.e., the speed of motion as well as possible rule describing it. The issues raised here will be discussed in Chapter III.

20 On details see E. Jung, *The New Interpretation of Aristotle…*., (forthcoming).

2. Thomas Bradwardine

Thomas Bradwardine (ca 1295–1349) in 1321 was a Bachelor of Arts at Balliol College, in 1323 he became a fellow of Merton College, Oxford where he probably remained for the next twelve years. In the same year he became Master of Arts, in 1333 a bachelor and in 1340 a Doctor in theology. Like Kilvington, Bradwardine belonged to the circle of friends and courtiers of Richard de Bury who introduced him to the royal court of Edward III. Bradwardine actively participated in the life of the Church and the royal court. His career as an ecclesiastic began in 1333 when he was made Canon at Lincoln Cathedral and was to be crowned with his election in 1349 as Archbishop of Canterbury. As the chancellor of St. Paul's Cathedral in London, Bradwardine was appointed royal chaplain in 1337 and, probably, the king's confessor. He accompanied Edward in his travels to Flanders and France during the campaign of 1346. Immediately after his episcopal consecration, which was held in Avignon, Bradwardine returned to England to assume his position, yet he died a month later, on the 26th of August 1349, as a victim of the first wave of the Black Death.[21]

Thomas Bradwardine authored many significant works, which cover a number of scholarly domains. His insight and intellectual inquisitiveness earned him the title of *Doctor profundus* and a mention in Chaucer's *Canterbury Tales*. The philosophical works of his that have been preserved to our time are the following: two treatises in mathematics: *Speculative Arithmetic* (*Arithmetica speculativa*) and *Speculative Geometry* (*Geometria speculativa*),[22] a number of logical treatises (all written before 1328), a famous work on the theory of motion *Treatise on Ratios of Velocities in Motions* (*Tractatus de proportionibus velocitatum in motibus*), written in 1328,[23]

21 On Bradwardine see, for example, http://www-history.mcs.st-andrews.ac.uk/Biographies/Bradwardine.html, the article by J.J. O'Connor and E.F. Robertson.

22 A critical edition in G. Molland, "*Geometria speculativa* of Thomas Bradwardine. Text with critical Discussion" (unpublished Ph.D. dissertation), Cambridge 1967.

23 A critical edition in: "Thomas of Bradwardine. His *Tractatus de Proportionibus*. Its Significance for the Development of Mathematical Physics". Edited and translated by H. Lamar Crosby, Jr., Madison 1955, pp. 64–140. In the colophon of Bradwardine's treatise one reads: "Explicit tractatus de proportionibus editus a magistro Thoma de Bradelbardin. Anno Domini MCCC28."

Treatise on the Continuum (*Tractatus de continuo*).[24] The theological works are the commentary to the *Sentences*, some questions of which are edited by Kathrine Tachau and Jean-Francois Jenest;[25] to this commentary also belongs a question on future contingents, which is edited as a separate work: *On Future Contingents* (*Utrum Deus habeat praescientiam futurorum contingentium ad utrumlibet*).[26] The most famous of Bradwradine's theological works, printed in 1618, is: *In Defense of God Against the Pelagians and On the Power of Causes, to his Fellow Mertonians* (*De causa Dei contra Pelagium et de virtute causarum ad suos Mertonenses*).[27] He – as he says – started to elaborate this work when he was a philosophy student,[28] but the final version was composed in 1344. Bradwardine is also an author of the treatise *De memoria artificiali adquirenda* (*On Acquiring a Trained Memory*).[29]

It seems that most of Bradwardine's philosophical treatises were composed as "a guide" or a textbook for students. Beyond any doubts

24 A critical edition in: J.E. Murdoch, "Geometry and the Continuum in the Fourteenth Century: A Philosophical Analyses of Thomas Bradwardine's *Tractatus de continuo*" (unpublished Ph.D. dissertation), Microfilm Ann Harbor, Harvard University, 1957.

25 See J.-F. Genest and K. Tachau, *La lecture de Thomas Bradwardine sur les Sentences*, AHDLMA, 57 (1990), 301–306.

26 A critical edition by Jean-Francois Genest, *Le De futuris contingentibus de Thomas Bradwardine*, "Recherches Augustiniennes et Patristiques", 14 (1979), pp. 249–336.

27 "Thomae Bradwardini Archiepiscopi Olim Cantuariensis De causa Dei contra Pelagium et de virtute causarum ad suos Mertonenses, libri tres", Opera et studio Henrici Savilli (...) Londini 1618.

28 Thomas Bradwardine, *De causa Dei…* [in:] E. Jung, *Determinism and Freedom in Thomas Bradwardine's View*, [in:] "If God exists… Human freedom and theistic thesis", A. Stefańczyk, R. Majeran (eds), Lublin 2019, p. 247: " Later, yet before I had begun my study of theology, provided with these words as with a ray of grace and in possession of some representation of truth, it appeared to me that I saw from afar God's grace preceding as to timing and nature all good meritorious works, namely the desired will of God who, prior as to time and nature, wills the salvation of a deserving human being and produces his deserts in himself before that man does it himself. Just as God is Prime Mover with respect to all motions, so I was provided with God's grace before any effort of mine, for which I render Him my thanks."

29 For *De memoria* see M. Carruthers (ed.), "Journal of Medieval Latin" 2, (1992), 25–43; translation in M. Carruthers, "The Book of Memory: A Study of Memory in Medieval Culture", New York 1990, pp. 228–281; see also M. Carruthers, J. Ziolkowski, *The Medieval Craft of Memory*, [in:] "An Antology of Texts and Pictures", M. Carruthers, J. Ziolkowski (eds), Philadelphia 2002, pp. 205–214.

such a role was played by his famous *Tractatus de proportionibus*, in which he made an extensive use of Kilvington's question on local motion. Bradwardine's treatise is divided into four chapters. The first one recapitulates the knowledge about proportionality to be found in Boethius' *Arithmetic* and Campanus de Novara's *Commentarium super quantum librum "Elementorum" Euclidis*; in the second chapter, Bradwardine criticizes four theories interpreting Aristotle's statement that speed is proportional to the acting and passive powers involved; in Chapter III Bradwardine introduces his own solution of the problem and "he commences his exegesis by quoting Aristotle and Averroes in general support of his view, after which he launches directly into his twelve theorems concerning velocity";[30] chapter IV deals with circular motions. Bradwardine's theory is a subject of detailed study in Chapter III below.

3. William Heytesbury

William Heytesbury was born sometime before 1313 in Wiltshire in the Salisbury Diocese. He is first mentioned as a fellow at Merton College in Oxford in 1330. He held the administrative position of a bursar (i.e., the recipient of a scholarship) of Merton in 1338–1339, responsible for determining dues, auditing accounts, and collecting revenues. By 1340 he had completed his regency in arts at Merton and, together with John Dumbleton, had been named a foundation fellow at the new Queen's College in 1340, but soon he returned to Merton College. He was a Doctor of Theology by July 1348, chancellor of the University in 1371–72, and may have been chancellor also in 1353–1354. He died between December 1372 and January 1373.

Heytesbury obtained his fame thanks to his logical works, none of his theological works is known. Heytesbury's extant writings, which are tentatively dated to the period 1331–1339 are (with one exception) concerned with the analysis of fallacies and sophisms. *Sophismata* is a collection of sophisms for advanced students working on natural philosophy ("sophisms – as Paul Spade describes it – are problematic sentences about which one can give plausible arguments both that they are true

30 L. Crosby Jr, Thomas of Bradwardine His *Tractatus de proportionibus*…., p. 38.

and also that they are false").[31] *Sophismata asinina* is a collection of sophistical proofs that the reader is a donkey. *Iuxta hunc textum*, also known as *Consequentiae Heytesbury*, is a collection of sophisms designed for testing formal inference rules. *Casus obligationis* is a collection of epistemic sophisms. *De sensu composito et diviso* is a manual on the logical analysis of the *de re/de dicto* ambiguity. *Termini naturales* is a vocabulary of basic physical concepts. Most of these have not been critically edited, but early prints, recent editions, and several modern translations are available.[32]

His most important and influential work, written in 1335, is, beyond any doubt, *Rules for Solving Sophisms* (*Regulae solvendi sophismata* or *Logica*).[33] The *Rules* are divided into six chapters. The first three chapters are principally logical in character and they respectively discuss: 1) the rules for

31 William Heytesbury, "On *Insoluble* Sentences. Chapter One of His *Rules for Solving Sophisms*", translated with an Introduction and Study by Paul Vincent Spade, Toronto 1979, p. 2.

32 For details see Hanke, Miroslav and Jung, Elzbieta, "William Heytesbury", *The Stanford Encyclopedia of Philosophy* (Spring 2018 Edition), Edward N. Zalta (ed.), URL = <https://plato.stanford.edu/archives/spr2018/entries/heytesbury/>.

33 This work (complete or incomplete) is to be found in the following mss: Bergamo, Bibl. Civica "Angelo Mai", MA 481; Brugge, Hoofdbibliotheek Biekorf (Stadsbibliotheek), 497; Brugge, Hoofdbibliotheek Biekorf (Stadsbibliotheek), 500; Cesena (Forlì-Cesena), Bibl. Comunale Malatestiana, S.X.5; Vatican, Chig. E.V.161; Vatican Chig. E.VI.193; Vatican, Ottob. lat. 662; Vatican, lat. 2136; Vatican, lat. 2138; Vatican, lat. 3144; Crema (Cremona), Bibl. Comunale, 190; Erfurt, Amplon. 2° 135; Erfurt, Amplon. 2° 313; Erfurt, Amplon. 4° 270; Firenze, Biblioteca Riccardiana, 790; Firenze, Bibl. Riccardiana, 821; Kraków, BJ 621; Kraków, BJ 704; Leipzig, Universitätsbibliothek, 529; Leipzig, Universitätsbibliothek, 1360; Leipzig, Universitätsbibliothek, 1370; London, Wellcome Library 350; München, Bayerische Staatsbibliothek, Clm 23530; Oxford, Bodl., Canon. misc. 221; Oxford, Bodl., Canon. misc. 409; Oxford, Bodl. Canon. misc. 456; Padova, Bibl. Antoniana, Manoscritti 407; Padova, Bibl. Universitaria, 1123; Padova, Bibl. Universitaria, 1434; Padova, Bibl. Universitaria, 1570; Praha, Národní Knihovna Ceské Republiky, III.A.11 (396); San Gimignano (Siena), Bibl. e Archivio Comunale, 25; Sarnano (Macerata), Biblioteca Comunale, E. 15; Venezia, Bibl. Naz. Marciana, lat. VIII. 38(3383); Verona, Bibl. Civica, 2881; Warszawa, BN III. 8058. For a detailed description on codices see. P.V. Spade, *The Manuscripts of William Heytersbury's* Regulae solvendi sophismata. *Conclusions, Notes and Descritptions*, "Philosophical Quarterly" 31 (1981), pp. 271–313; see also http://www.mirabilweb.it/title/regulae-solvendi-sophismata-guillelmus-hentisberus-title/3600.

handling so-called "insoluble" sentences in disputations, i.e., paradoxes; 2) the sophisms involving the words "know" and "doubt"; 3) the logical problems arising from the use of "relative" terms. The next three chapters are concerned with the philosophy of nature and they respectively deal with: 4) the problem of the beginning and ceasing of continuous processes; 5) the limit decision problem on maxima and minima of the physical factors of the different type of changes. In the sixth chapter – *On the three categories*, Heytesbury sets out rules for speed: in accelerated local motion, with regard to place; in quantitative changes, with regard to acquired quantity; in qualitative changes with regard to intensity of quality.[34] Given the main topic of this book, we are interested in debates about local movement, which we write about in the Chapter III.

4. The Anonymus Author of the *De sex inconvenientibus*

A good testimony as to the very quick assimilation of the works of Richard Kilvington, Thomas Bradwardine and William Heytesbury is an anonymous treatise entitled *De sex inconevnientibus*. The question on local motion: *Utrum in motu locali sit certa servanda velocitas* is the fourth and the last question of the anonymous treatise *De sex inconvenientibus* written by a thinker who also was associated with the Oxford Calculators.[35] Al-

34 To date, the best and only such comprehensive study of the problems presented in *Regulae* is the book by C. Wilson, "William Heytesbury. Medieval Logic and the Rise of Mathematical Physics", Madison 1960.

35 On this text (in general or on its selected fragments) see P. Duhem, "Études sur Léonard de Vinci", vol. 3, Paris 1913, pp. 420–424, 471–474; Idem, *La dialectique du Oxford et la Scolastique italienne*, "Bulletin Italien", vol. 12 (1912), pp. 22–26, 101–103, 289–292; M. Clagett, "The Science of Mechanics in the Middle Ages", Madison 1959, pp. 263–265; S. Caroti, *Da Walter Burley al 'Tractatus de sex inconvenientibus'. La tradizione inglese della discussione medievale 'De reactione'*, "Medioevo. Rivista di Storia della Filosofia Medievale", vol. 21 (1995), pp. 257–374; G. Fernández Walker, *A New Source of Nicholas of Autrecourt's 'Quaestio': The Anonymous 'Tractatus de sex inconvenientibus'*, "Bulletin de Philosophie Médiévale", vol. 55 (2013), pp. 57–69; S. Rommevaux, *Six inconvénients découlant de la règle du mouvement de Thomas Bradwardine dans un texte anonyme du XIV^e^ siècle*, [in:] "L'homme au risque de l'infini: Mélanges d'histoire et de philosophie des sciences offerts à Michel Blay", M. Malpangotto, V. Jullien, E. Nicolaïdis (eds), Turnhout, 2013, pp. 35–47; Eadem, *Un auteur anonyme du XIV^e^ siècle,* à Oxford, lecteur de Pierre de Maricourt,

though the exact date of its composition is unknown, it can be narrowed down considerably, because it refers to *Tractatus de proportibus* (1328) and Heytesbury's *Regulae* (1335). Marshal Clagett and Jean Celeyrette[36] claim that *De sex* is quoted by John Dumbleton in his *Summa logicae et philosophiae naturalis*, which was written before 1349.[37] Recently Gustavo Fernandez Walker has indicated the influence of this treatise over Nicholas of Autrecourt's second version of his theological *Quaestio*.[38] Autrecourt's question was most likely written before 1339, when he was summoned to Avignon to face allegations of false teaching.[39] Thus the treatise *De*

"Revue d'Histoire des Sciences", vol. 61/1 (2014), pp. 5–33; Eadem, *La détermination de la rapidité d'augmentation dans le 'De sex inconvenientibus': comparaison avec les développements sur le même sujet de William Heytesbury*, [in :] "Miroir de l'amitié: mélanges offerts à Joël Biard", Ch. Grellard (ed.), Paris 2017; Eadem, *The study of local motion…*, (forthcoming); J. Papiernik, *Metody matematyczne w badaniach z zakresu filozofii przyrody. Problem szybkości powstawania form w XIV-wiecznym traktacie 'De sex inconvenientibus'*, "Przegląd Tomistyczny", vol. 23 (2017), pp. 95–145; Eadem, *How to measure different movements? The 14th-century treatise 'De sex inconvenientibus'*, "Przegląd Tomistyczny", vol. 25 (2019), pp. 445–460 (in this paper, she proves that the four *quaestiones* make up the whole anonymous treatise. Some, like e.g., Duhem were convinced that the whole treatise consists of eleven questions). For the Polish translation of questions I and II of *De sex inconvenientibus*, see J. Papiernik, "Zmiany jakościowe i ich miara w traktacie *O sześciu niedorzecznościach* (Research on Science & Natural Philosophy, vol. I), Łódź 2019, pp. 91–216; Polish translation of questions III and IV of *De sex inconvenientibus*, see E. Jung, "Zmiany ilościowe i ich miara w traktacie *O sześciu niedorzecznościach* (Research on Science & Natural Philosophy, vol. III), Łódź 2019, pp. 75–192.

36 M. Clagett, "Nicole Oresme and the medieval geometry of qualities and motions: A treatise on the uniformity and difformity of intensities known as *Tractatus de configurationibus qualitatum et motuum*", Madison 1968, p. 619; J. Celeyrette, *Bradwardine's Rule: A Mathematical Law?*, [in:] "Mechanics and Natural Philosophy before the Scientific Revolution", W.R. Laird, S. Roux (eds), (Boston Studies in the Philosophy and History of Science, 254), Dordrecht 2008, p. 58. See also S. Rommevaux-Tani, *The study of local motion…*., (forthcoming).

37 1348 is the last year in which the Merton College Record mentions John Dumbleton as its Fellow, and there is no evidence of his activity after this year. Researchers assume that he died of the plague ca. 1349. See e.g., J.A. Weisheipl, *The place of John Dumbleton in the Merton School*, "Isis" 50 (1959), p. 450; E. Sylla, *John Dumbleton*, [in:] "Encyclopedia of Medieval Philosophy: Philosophy Between 500 and 1500", H. Lagerlund (ed.), Dordrecht 2011, p. 608.

38 G. Fernandez Walker, *A New Source of Nicholas of Autrecourt's 'Quaestio'…*.

39 See S. Rommevaux-Tani, *The study of local motion…*, (forthcoming).

sex inconvenientibus must have been written in between 1335 and 1339, or even earlier, since already in 1339 Autrecourt had made use of it in his question. The anonymous treatise is to be found in the same manuscript in Paris, BnF lat. 6559, as Autresourt's work.[40] The work is quite extensive (in a modern edition it would give ca. 350 pages), so it likely was composed between 1335–1338.

Although the author's name is unknown, his affiliation to the Oxford Calculators School is highly probable for several reasons. Firstly, he discusses and accepts some of Bradwardine's and Heytesbury's solutions.[41] Secondly, he refers twice to Adam of Pipewell, a fellow of Baliol College in 1321, then a fellow at Merton College by 1325, and still present there in 1327.[42] Regrettably, none of Pipewell's works have survived to our time. Finally, although Richard Kilvington's name does not appear in *De sex*, there are numerous examples that show a clear dependence on Kilvington's commentaries on the *De generatione et corruptione* (1324) and on the *Physics* (1326); some of those references are quoted in the footnotes to the critical edition of question IV from *De sex*.[43]

40 What should be explained here is that this is considered to be the earlier version of the Autrecourt's *Quaestio* and is held in Paris, Bibliothèque Nationale de France, Ms. lat. 6559, ff. 191r-193v, so in the same manuscript, which includes the text of the *De sex inconvenientibus* – the basis for the presented critical edition. The second version of the *Quaestio* is in Ms. Paris, BnF lat. 14576, ff. 212r-214r. On this see: Z. Kaluza, "Nicolas d'Autrécourt. Ami de la vérité" (Histoire Littéraire de la France 42.1), Paris 1995, pp. 195–204; C. Grellard, *L'usage des nouveaux langages d'analyse dans la quaestio de Nicolas d'Autrécourt. Contribution à la théorie autrécourienne de la connaissance*, [in:] "Quia inter doctores est magna dissensio. Les débats de philosophie naturelle à Paris au XIVe siècle", S. Caroti, J. Celeyrette (eds), Firenze 2004, pp. 69–95; G. Fernandez Walker, *A New Source of Nicholas of Autrecourt's 'Quaestio'*….

41 See S. Rommevaux, *Six inconvénients découlant de la règle du mouvement...* ; Eadem, *La détermination de la rapidité...* . For references to Bradwardine see E. Jung, "Zmiany ilościowe i ich miara...", p. 119 ; p. 151, n. 25 ; 152, n. 26 ; for references to Heytesbury see ibidem, pp. 78, 79, 81.

42 See, G.C. Brodrick, "Memorials of Merton College…", p. 195; A.B. Emden, "A Biographical Register…", vol. III, P to Z, p. 1484.

43 For the references in the fourth question to Richard Kilvington's Commentary to *Physics*, see below, Anonimus, q. *Utrum in motu locali sit certa servanda velocitas* (Editions), §§ pp. 308, 311, 322–323. It is also worth noticing that in the discussion of a possible cause for accelerated motion, the anonymous author explicitly says that his opinion is in line with Adam of Pipewell's view, but while we can

In the opinion of James A. Weisheipl and Curtis Wilson,[44] since the anonymous author while presenting different opinions uses the expression "*ut tenet/dicit/ponit scola Oxoniensis*",[45] which suggests some distance to the Oxford's group, he may have been a continental thinker. This hypothesis may confirm the information given in the *explicit* of the Prague manuscript (Národní knihovna České republiky, VIII. G.19) which ends on the second article of the fourth question, but in the *explicit* we read: *expliciunt quaestiones de motu Parisius disputatae*.[46] The questions

find very similar debates in Kilvington's question *Utrum in omni motu potentia motoris excedit potentiam rei motae*, we do not know of any such treatise with analyses resembling these fragments, which is attributed to this master Adam. For this see E. Jung, *The New Interpretation of Aristotle…*, (forthcoming). For some examples of the references to Kilvington's to be found in *De sex* see: Ryszard Kilvington, *Kwestie o ruchu* (Ricardus Kilvington, *Quaestiones de motu*), p. 113, n. 150; 161, n. 283. An article on the influence of Kilvington's commentaries on the treatise *De sex inconvenientibus* is being prepared by Elżbieta Jung and Joanna Papiernik.

44 J.H. Weisheipl, *The place of John Dumbleton*, p. 440; see also C. Wilson, *William Heytesbury. Medieval Logic and the Rise of Mathematical Physics*, Madison, 1956, p. 7.

45 The wording 'scola Oxniensis' is used three times in the anonymous treatise: 1) In the first question, in the third main answer to the question, Ms. BnF 6559, f. 3ra: "Tertio ad principale arguitur sic: si in generatione formarum sit certa ponenda velocitas, igitur talis velocitas attenderetur solum penes latitudinem formae acquirendae, sicut ponit tertia positio et tenet tota scola Oxoniensis…"; 2) in the part *Ad oppositum* in the first question, Ms. BnF 6559, f. 4ra-b: "Ad oppositum argumentorum et pro in titulo quaestionis sunt positiones iam dictae, sed praecipue tertia, quam tenet et sustinet tota scola Oxoniensis, magis valens et ceteri quamquam tenent scholars"; 3) in the third main answer to the second question, Ms. BnF 6559, 14rb: "Tertio et ultimo ad principale arguo sic. Si in motu alterationis velocitas sit signanda etc., igitur talis velocitas attenditur penes proportionem latitudinum intensarum, sicut ponit tota scola Oxoniensis…".

46 The Prague codex, Národní knihovna České republiky, VIII. G.19, from the 14th century consists of several treaties and questions, including fragments of: Thomas Bradwardine's *Tractatus de proportionibus*, Richard Kilvington's *Sophismata*, Roger Bacon's *Perspectiva*, Jacobus de Sancto Martino's *Tractatus de latitudinibus formarum*, William Heytesbury's *De sensu composito et diviso*. There are also fragments of logical works (sophisms), as well as fragments of texts concerning uniform and non-uniform motions, the proportion theory, geometric issues, and issues in the field of natural philosophy. *De sex inconvenientbus* is contained on ff. 25r-46v, it is incomplete and finishes at the end of the second article of the fourth question. "Inc.: Utrum in omni generatione formarum sit ponenda velocitas. Circa propositam quaestionem ac circa dubia disputanda de proportionibus velocitatum in motibus..." Expl.: "...et totum pertransitum ab a ante

which form the treatise, however, are devoted to the problem previously pondered by Kilvington, Bradwardine, Heytesbury, i.e., how to set out rules for the speed of accelerated motion, quantitative and qualitative changes, as well as to the question, discussed only by Kilvington: how to set down rules for the speed of changes in the qualities of the elementary bodies, such as hotness, coldness, dryness or wetness.[47] Also the frequent presentation of arguments discussed in the form of sophisms is characteristic for the Calculators.[48] It is also worth mentioning that the list of references, to be found in *De sex inconvenientibus* testifies rather to the English than to the Continental provenance of this text. The anonymous author quotes and refers to: Aristotle's *Physica*, *De generatione et corruptione*, *De caelo*, *Metaphysica*, *Meteorologica*, *De anima* and *Parva naturalia*, as well as Averroes' commentaries to these works; Pseudo-Aristotle *De secretis* (known also as *Secretum Secretorum*); Euclid's *Elementa*; Boethius's *De Institutione Arithmetica*; Albumasar's *Introductio in Astronomiam*; Alhazen's *Perspectiva*; Zahel's *De iudiciis astrorum*; Jordanus de Nemore's *De ponderibus*; Petrus Peregrinus's *De magnete.*

Undoubtedly, if the *explicit* of the Prague manuscript gives veritable information, we still would hypothesize that, due to the arguments presented above, the anonymous treatise in the form of four questions was composed in Oxford, and then it may have been discussed in Paris.

The anonymous treatise consists of four main question, which contain three articles also in the form of question.[49] The structure of the *De sex inconventibus* is as follows:

finem horae et sic non sequitur inconveniens adductum et probatio claret. Patet quia in eodem casu ad alia sic dicendum. Expliciunt quaestiones de motu Parisius disputatae". A comprehensive description of the Prague manuscript is given by D. Di Liscia and S. Rommevaux-Tani, [in:] "Quantifying Aristotle…", (forthcoming). For the previous description see, J. Truhlář, "Catalogus codicum manu scriptorum latinorum, qui in c. r. bibliotheca publica atque Universitatis Pragensis asservantur", t. I., Pragae 1905. It is also available online: https://archive.org/details/cataloguscodicu03truhgoog/page/n625/mode/2up.

47 See J. Papiernik, "Zmiany jakościowe i ich miara…", pp. 31, 117, n. 92; pp. 120–124.

48 See S. Rommevaux-Tani, *The study of local motion…*., (forthcoming); E. Jung, "Zmiany ilościowe i ich miara…", pp. 33–39.

49 Some researchers state that *De sex inconvenientibus* originally contained eleven questions. This conclusion is drawn from the reading of the content of the list of questions to be found on f. 194v in ms. 6559, which gives eleven titles: the titles

Question I on generation: Should the specific rule for measuring the speed of the generation of forms be set up? (*Utrum in generatione formarum sit certa ponenda velocitas*). To this question belong the following articles: 1) Does the generating factor give as much the place as the form? (*Utrum generans tantum loci contribuat quantum formae*); 2) Are the intermediate colors generated from the extreme colors? (*Utrum ex coloribus extremis intermedii generentur colores*); 3) Do the celestial bodies generate primary qualities through light? (*Utrum caelestia corpora generent qualitates primarias lumine mediante*).

Question II on the motion of alteration: Should acceleration and slowness be measured in the motion of alteration? (*Utrum in motu alterationis velocitas sit signanda vel tarditas*). To this question belong the following articles: 1) Is a magnet able to change a piece of iron placed next to it? (*Utrum magnes suppositum sibi ferrum sufficiat alterare*); 2) Is a change of a luminous medium instantaneous in an instant? (*Utrum alteratio medii luminosi sit subita in instanti*); 3) Is the factor producing a change the subject of action? (*Utrum quodlibet alterans in agendo repatiatur*).

Question III on the motion of augmentation*:* Does a subject of augmentation continuously accelerate its motion in the process of augmentation? (*Utrum augmentum continuum in augendo velocitet motum suum*). To this question belong the following articles: 1) Is rarefaction possible? (*Utrum rarefactio sit possiblis*); 2) Is rarefaction a motion to some quantity? (*Utrum rarefactio sit motus ad aliquam quantitatem*); 3) Does rarefaction occur in something rare or dense? (*Utrum rarefactio sit per rarum et densum*).

of four main questions of the *De sex* and seven others. Of the remaining titles, a part of the alleged fifth question (*Utrum caelum possit suo motu et lumine inferiora corpora transmutare*) is also copied in the manuscript BnF 6559. The structure of this question, however, is different than the four others, and it clearly shows that it does not belong to the same work. The list can be misleading, as on the top of folio f. 194v there is written: *prima quaestio primo libro*, and right below is the title of the first question of the anonymous treatise. What is more important, and, in fact this declaration is conclusive, at the beginning of his work the author himself announces that he is going to analyze four kinds of motion, Ms. BnF 6559, f. 1ra: "Utrum in generatione formarum sit certa ponenda velocitas. Circa propositam quaestionem et cetera dubia disputanda de proportionibus velocitatum in motibus generationis, alterationis, augmentationis ac motu locali presentem servabo processum. In primis, ut potero, disputabo materias antedictas, deinde materias illas tradam per modum tractatus." For a detailed discussion of this issue see J. Papiernik, "How to measure…", pp. 449–450.

Question IV on local motion: Can local motion be measured through a certain speed? (*Utrum in motu locali sit certa servanda velocitas*). To this question belong the following articles: 1) Is the acceleration of a heavy body caused by a specific factor? (*Utrum velocitatio motus gravis sit ab aliqua causa certa*); 2) Is the speed of a sphere moving in time measured by a point only or by a space? (*Utrum velocitas motus sphaerae cuiuslibet penes punctum tantum vel spacium aliquod attendatur*); 3) Is acceleration of any uniformly difform local motion, starting from non-degree, equal to its middle degree? (*Utrum velocitas omnis motus localis uniformiter difformis incipiens a non gradu sit aequalis suo medio gradui*).

The first question deals with the motion of generation. Its author is of the opinion that the speed of motion in the process of generation – understood as the process of transition, for example, from coldness to hotness or from a less intense hotness to a more intense one, or the process of the generation of one element from another, when two qualities are changed simultaneously, e.g., hotness and dryness into coldness and wetness – should be measured by the latitude of the acquired form, that is, in modern terms, by the certain amount of the quality gained. In the second main question dealing with the speed of the motion of alteration, the author also concludes that the speed is proportional to the ratio of the latitude of quality gained to the previous latitude of quality. As for the third main question on augmentation, the author claims that the speed of such a change is rightly measured by the ratio of the latitudes of rarefaction and it is determined by the ratio of the lengths traversed by the fastest moving point or points.[50]

The rule for local motion will be discussed in detail in Chapter III below. The complete text of question IV on local motion is be found in the following manuscripts: Paris, BnF lat., 6559; Paris, BnF lat. 6527; Venezia, Biblioteca Marciana, lat. VIII. 19; Oxford, Bodleian Library, Canon. Misc. 177, and in an old print Bonetus Locatellus and printed in Venice in 1505.[51]

50 See S. Rommevaux-Tani, *La détermination de la rapidité d'augmentation...*, pp. 153–162; E. Jung, "Zmiany ilościowe i ich miara…", p. 39.

51 The old print is full of errors and omissions. Besides the four questions of *De sex inconvenientibus* the collection contains: *Quaestio de modalibus* Bassani Politi; *Tractatus proportionum introductorius ad calculationes Suisset*; *Tractatus proportionum* Thomae Braduardini; *Tractatus proportionum* Nicholai Orem; *Tractatus de latitudinibus formarum* eiusdem Nicholai; *Tractatus de latitudinibus formarum Blasii de Parma*; *Quaestio*

5. John Dumbleton

John Dumbleton was born ca. 1310, he was a native of the Dumbleton village community in Gloucestershire, within the diocese of Worcester. He is mentioned as a fellow of Merton College, Oxford in 1338. Along with William Heytesbury, he was named as one of the original fellow of Queen's College, Oxford in 1340. "This means – as Weisheipl points out – that Dumbleton had completed his regency in arts by that date, and that he at least intended to study theology and to take Holy Orders, for these were required by the statutes of Queen's."[52] Wiesheipl says that Dumbleton returned to Merton, where he is again mentioned in the records for 1344–45.[53] "A late fourteenth or early fifteenth-century MS of the *Summa* at Padua contains a note at the end of the first part describing Master John of Dumbleton as an Englishman and a Bachelor in Sacred Theology: *Explicit prima pars Summe magistri Johannis de dulmenton anglici baccularii sacre theologie*."[54] Edith Sylla, on the other hand, claims that "Dumbleton was a fellow of the Sorbonne at Paris probably between 1344 and 1347" and in Merton again in 1347–1348. As she says: "one manuscript mentions "Master John Dumbleton, one-time fellow of the Sorbonne, in his *Summa*…". Regrettably, she does not give any specific references.[55] Since we do not have any clear information about his academic career, and the only sure information is the Merton's record[56] we can rather propose the hypothesized claim that he left Oxford for theological study at the Sorbonne in 1345 and stayed there until his death. Most likely, Dumbleton succumbed to the Black Death in 1348 or 1349.

John Dumbleton is the author of: *Compendium sex conclusionum*, *Expositio capituli quarti Bradwardinis de proportionibus*, written in 1332 and *Summa logicae et philosophiae naturalis*, most likely written just before his death, i.e.,

subtilis doctoris Johannis de Casali De velocitate motus alterationis and *Quaestio Blasii de Parma de tactu corporum durorum*.

52 J.A. Weisheipl, O.P., *The Place of John Dumbleton…*, p. 450.

53 See ibidem, n. 60.

54 Ibidem, n. 61.

55 See E. Sylla, "John Dumbleton" [in:] "Ecyclopedia of Medieval Philosophy", Springer 2011, p. 608.

56 See Merton College Rec. 3676–7, [in:] A.B. Emden, *Biographical Register*, I, p. 608.

in 1348/49. There are at least twenty extant manuscripts of this work.[57] All copies are incomplete with the missing part Ten on Platonic forms that Dumbleton refers to, but it probably never was completed. Doubleton's *opus magnum* runs to some 400,000 words.[58] The whole work was transcribed by James Weisheipl from a single manuscript (Vat. lat. 6750) in the early 1950s, but that transcription was never published, with it being deposited at the Library of the Pontifical Institute for Mediaeval Studies in Toronto.[59]

Dumbleton's *Summa*, in the form it has survived to our time has nine parts, which present extant discussion: I) On logic beginning with an extended discussion of the signification which is important for understanding Dumbleton's solution to the insoluble, this part is followed by two chapters on knowledge and doubt;[60] II) On first principles, matter and form, intension and remission of qualities; III) On local motion, alteration and augmentation, on the causes of motion, on the rules for the speed of different types of motion, on the definition of time and motion; IV) On the nature of the elements and their qualities, on the relations of elemental and qualitative forms, on density and rarity, on the relative weights of pure and mixed bodies; V) On spiritual action and light, on the nature of the medium receiving light; VI) On the limits of active and passive powers, on the motion of the heavens and their

57 The Mss are: Cambridge, Gonville & Caius, 499/268; Cambridge, Peterhouse, 272; Dubrovnik-Ragusa, Dominikanerbibliothek 32; Klosterneuberg SB 670; London, B.L. Royal 10. B. XIV; London, Lambeth Palace 79; Oxford, Magdalen 32; Oxford, Magdalen 195; Oxford, Merton 279; Oxford, Merton 306; Padua, Anton. XVII, 375; Paris, BnF lat. 16146; Paris, BnF lat. 16621; Paris, Universitaire 599; Prague, Capit. Metropol. 1291 (L. XLVII); Vatican, Pal. lat. 1056; Vatican, Vat. Lat. 954 (lacks Part I); Vatican, Vat. Lat. 6750; Venezia, Bibl. Naz. Marciana VI, 79 (2552); Worcester, Bibl. Cathed., F. 6; Worcester, Bibl. Cathed., F. 23 (Part I, II and incomplete Part III).

58 See P. V. Spade, "The Medieval Liar", Toronto 1975, pp. 63–65.

59 Weisheipl's Ph. D. dissertation is titled: *Early fourteenth century physics of the Merton 'school': with special reference to Dumbleton and Heytesbury*, 2 vols. Fragments of Part II – V are to be found in E. D. Sylla, Ph. D. at Harvard, published as "The Oxford Calculators and the Mathematics of Motion 1320–1350…, pp. 565–625. There is the project running at St. Andrews University and Barbara Bartocci is responsible for the critical edition of Part I and II of the *Summa*, see https://www.st-andrews.ac.uk/~slr/paradox.html.

60 See https://www.st-andrews.ac.uk/~slr/paradox.html; E. Sylla, "John Dumbleton", p. 608.

movers, on the limits of the size of natural bodies; VII) On the Prime Mover and eternity of the world; VIII) On the generation of substances, on the numerical unity of the soul; IX) On the five senses.[61]

The problems pondered in Part III of the *Summa* will be discussed in Chapter III of this book,[62] a working edition of chapters 5 to 11 of Part III is also offered.[63]

6. Richard Swineshead

The name Richard Swineshead (*fl* ca. 1340–1354) is now decisively associated with the most monumental and sophisticated treatise originating from the Oxford Calculators school commonly known as the "Book of calculations" (*Liber calculationum*). In fact, we know nothing about the mentioned author, save that besides this treatise there are preserved also two short texts on local motion and the fragment of a commentary on *De coelo* attributed to him, too.[64] Quite paradoxically, the "Book of

61 See E. Sylla, "John Dumbleton", pp. 608–609. The secondary literature on John Dumbleton is not as extensive in comparison to e.g., Thomas Bradwardine. See, G. Molland (1973), E. Sylla (2008), "John of Dumbleton", [in:] "Complete dictionary of scientific biography", vol. 7, 20; G. Molland, *John Dumbleton and the status of geometrical optics*, [in:] "Mathematics and the medieval ancestry of physics. Collected papers", London 1995; E. Sylla, *Medieval concepts of the latitude of forms: The Oxford calculators*, AHDLMA 40 (1973), pp. 223–283; Eadem, *The Oxford calculators and mathematical physics: John Dumbleton's "Summa logicae et philosophiae naturalis", Parts II and III*, [in:] "Physics, cosmology and astronomy, 1300–1700: tension and accommodation", S. Unguru (ed.), (Boston studies in the philosophy of science, vol. 126), Dordrecht 1991, pp. 129–161; Eadem, *The Oxford Calculator's Middle Degree Theorem in Context*, "Early Science and Medicine" vol. 15, no. 4/5 (2010), pp. 353–357; J. Weisheipl, *The place of John Dumbleton in the Merton school*, "Isis" 50 (1959), pp. 439–454.

62 See chapter III, pp. 112–125.

63 See Editions, pp. 393–425.

64 See J.E. Murdoch, E.D. Sylla, "Swineshead", in: *Dictionary of Scientific Biography*, Vol. 13, H. Staudinger, G. Veronese (eds), New York 1976, 184–185. The form of the surname is quite certain, if we consider the versions appearing in manuscripts and other historical sources, like *Suiseth*, *Suinshead*, *Swyinshede* or *Swynyshed*, as those reflecting the presently accepted one. With respect to the name of this author, besides Richard (or Ricardus), also John (Johannes), Roger, and even Raymundus can be found in these sources. See R. Podkoński, *Richard*

calculations" was the best known Calculatory work throughout the next centuries. Inferring from the number of manuscript copies of this work preserved in Italian libraries, as well as the fact that there were at least three printed editions of it in Italy in the late fifteenth and early sixteenth century, one can safely assume that *Liber calculationum* enjoyed a considerable interest among Italian scholars of the time.[65] At the beginning of the sixteenth century a group of Spanish and Portuguese thinkers active at the Paris University extensively used the conclusions drawn from Swineshead's treatise in their own works. One of these philosophers, Alvaro Thomaz in his *Liber de triplici motu* (published 1509) effectively employed and developed Richard Swineshead's ideas concerning local motion.[66] The "Book of calculations" was also highly appraised by such personalities of sixteenth-century natural science as Julius Scaliger and Girolamo Cardano. In the seventeenth century Richard Swineshead's main work was appreciated by no one else than Gottfried Wilhelm Leibniz, who explicitly declared the need for it to be in print again and even ordered a transcription of the last printed edition of this work for his own use.[67] From Leibniz's correspondence with John Wallis, one of the leading mathematicians of these times, we can safely assume that the

Swineshead's Liber calculationum *in Italy: Some Remarks on Manuscripts, Editions and Dissemination*, "Recherches de Théologie et Philosophie médiévales" LXXX,2 (2013), 308–309.

65 It does not mean, of course, that these scholars appreciated the treatise as a whole. Fifteenth-century Italian thinkers were mostly interested in the issues relative to intension and remission, and especially reaction omitting generally in their analyses and commentaries the problem of local motion (see R. Podkoński, *Richard Swineshead's* Liber calculationum *in Italy…*, pp. 313–337).

66 See J.E. Murdoch, E.D. Sylla, "Swineshead", p. 210. On Alvaro Thomaz's account on local motion, see: E.D. Sylla, *Alvarus Thomas and the Role of Logic and Calculations in Sixteenth-Century Natural Philosophy*, [in:] "Studies in Medieval Natural Philosophy", S. Caroti (ed.), Firenze 1989, pp. 257–298. On the dependence of Alvaro Thomaz's solutions on Richard Swineshead's concepts, see: R. Podkoński, "*Suisetica inania*, Ryszarda Swinesheada spekulatywna nauka o ruchu lokalnym", Łódź 2017, pp. 216–220.

67 The handwritten copy included in the codex Hannover, Niedersächssische Landesbibl. MS 615 that belonged to G.W. Leibniz had been copied from the Venice 1520 edition of Richard Swineshead's "Book of calculations" prepared by Victor Trincavellus (see J.E. Murdoch, E.D. Sylla, "Swineshead", p. 210); G.W. Leibniz, "Sämtliche Schriften und Briefe", Bd. 13, Berlin 1987, p. 513: (318. Leibniz an Thomas Smith, Hannover, 29. Januar 1697): "Vellem etiam

latter was at least perfunctorily acquainted with the reasonings included in Swineshead's treatise.[68] In the late eighteenth century Jacob Brucker in his *Historia critica philosophiae* quoted a short passage from the "Book of calculations" as well as numerous fragments from other authors referring to Swineshead's conclusions.[69] Cantor's late nineteenth century "History of mathematics" only mentions the thinker's name, but thanks to Pierre Duhem and Lynn Thorndike's pioneering investigation on late medieval science Richard Swineshead has gradually, from the beginning of the twentieth century, regained his rightful place in the history of medieval natural philosophy.[70] It is worth mentioning here that it was Richard Swineshead's traditional sobriquet: the Calculator, that served Edith Sylla to establish the name "Oxford Calculators school" for these fourteenth-century Oxford thinkers who introduced the calculus of ratios into their philosophical analyses.[71]

The "Book of calculations" concerns only natural philosophical issues limited, in fact, to the problems of the 'measurement' of alteration, augmentation and local motion on the general background of the Aristotelian laws relating to the sublunar world. The range of topics discussed is best reflected by the titles of chapters (or treatises) included in this work:

Treatise I: *De intensione et remissione* (On intension and remission).
Tr. II: *De difformibus* (On difformly qualified subjects).
Tr. III: *De intensione elementi* (On the intension of an element).
Tr. IV: *De intensione et remissione mixtorum* (On the intension and remission of mixed subjects).
Tr. V: *De raritate et densitate* (On rarity and density).
Tr. VI: *De velocitate motus augmentationis* (On the velocity of augmentation).

edi scripta Joh. Suiseth, vulgo dicti Calculatoris, qui Mathesin in philosophiam scholasticam introduxit".

68 See "Leibnizens Gessamelte Werke, aus den Handschriften der Königlichen Bibliothek zu Hannover", G.H. Pertz (ed.), B. IV. Halle 1859, pp. 18–38.

69 See J. Brucker, "Historia critica philosophiae", III, Leipzig 1766, pp. 849–853.

70 See L. Thorndike, *Calculator*, "Speculum" VII 2 (1932), pp. 221–224.

71 This informal group was known earlier as the "Merton school" or simply the "Mertonians" after the title of Thomas Bradwardine's gargantuan theological treatise *De causa Dei contra Pelagianum…ad suos Mertonenses*. Sylla introduced this new name as being more adequate, since not all of these thinkers were fellows of Merton College, Oxford (see E.D. Sylla, "The Oxford Calculators…", pp. 540–541).

Tr. VII: *De reactione* (On reaction).
Tr. VIII: *De potentia rei* (On the power of a thing).
Tr. IX: *De difficultate actionis* (On the difficulty of action).
Tr. X: *De maximo et minimo* (On maximum and minimum).
Tr. XI: *De loco elementi* (On the place of an element).
Tr. XII: *De luminosis* (On luminous bodies).
Tr. XIII: *De actione luminosi* (On the action of a luminous body).
Tr. XIV: *De motu locali* (On local motion).
Tr. XV: *De medio non resistente* (On non-resisting medium).
Tr. XVI: *De inductione gradus summi* (On induction of the highest degree).

Presented here are the order, titles and division of the "Book of calculations" according to the printed versions of this text. Only one manuscript copy reflects such an order (with one exception), however. Nevertheless, since John Murdoch and Edith Sylla in their concise summary of Richard Swineshead treatise accepted the above-presented order, to avoid possible confusion we choose not to change it here. Closer analysis of these chapters, as well as the evidence of the preserved manuscripts, suggest that the order and the division of these parts, not mentioning the titles, as intended by the author were different. It is enough to note here that, e.g., the chapter *De actione luminosi* should be taken as an integral part of the treatise *De luminosis*, and the chapter *De potentia rei*, similarly, as an integral part of the treatise *De reaction*.[72] The treatise XI *De loco elementi* includes a famous discussion on the imaginary case of a long, heavy rod traversing the centre of the Earth.[73] The hypothesis, formulated by Murdoch and Sylla on the basis of the fact that chapters XIV and XVI are absent in the oldest known handwritten copies of the *Liber calculationum*, that these were written later than the remaining ones simply has to be rejected, however. The number and strictness of internal quotations within the "Book of calculations", especially relating to the "rules" included in the chapter XIV "On local motion", assures us

72 See. R. Podkoński, *Richard Swineshead's* Liber calculationum *in Italy*…, pp. 311–318, 338–343; Idem, *Richard Swineshead's* De luminosis: *natural philosophy from an Oxford Calculator*, "Recherches de Théologie et Philosophie médiévales", LXXXII 2 (2015), pp. 365–366.

73 See, M.A. Hoskin, A.G. Molland, *Swineshead on Falling Bodies: An Example of Fourteenth-Century Physics*, "British Journal for the History of Science", 3 (1966), pp. 150–182.

that the whole *Liber calculationum* must had been intended from the very beginning as a consistent and complete compendium on basic natural philosophical issues.[74]

The first impression one gets after a closer look at the reasonings included in any part of the "Book of calculations" is their logico-mathematical complexity on the one hand, and no obvious nor direct relation to the common experience, on the other. In general, only the techniques for calculating the changes of physical variables or for unravelling the sophisms about such changes are provided, with no explanation as to how, and for what purpose these could be used in the description of real natural phenomena. Richard Swineshead rarely, if ever, took the trouble of explaining the basics or sources of theories underlying the methods he used. Within the whole text of *Liber calculationum* he explicitly referred only twice to Euclid's and once to Boethius' and Averroes' works, respectively. When compared with John Dumbleton's *Summa logicae et philosophiae naturalis*, described above, Richard Swineshead's "Book of calculations" gives the impression of an academic handbook intended for advanced students or readers already well acquainted with the theories and solutions proposed, for example, in Dumbleton's treatise.[75] Most probably the above-mentioned features of *Liber calculationum* caused such an aversion in humanist thinkers like Ermolao Barbaro, Leonardo Bruni or Pietro Pomponazzi for the treatise to be recognized as *sophisticas quisquilias et suisetica inania*.[76] The very same features of Richard Swineshead's main work were highly appraised by Cardano and Leibniz, and that – in our opinion – clearly delineates the narrow circle of its intended recipients.

In the third chapter of the present tome the "science of local motion" developed by Richard Swineshead will be presented in detail on the basis of the contents of chapter XIV "On local motion", chapter XV "On non-resisting medium", and chapter X "On maximum and minimum" of the "Book of calculations". Also we will refer to the above-men-

74 See R. Podkoński, *Suisetica inania...*, pp. 133–140.

75 At least with respect to Dumbleton's statements concerning the propagation of light and the proper 'measurement' of its intensity it seems certain that Richard Swineshead intended to correct and supplement his theory within the "Book of calculations" (see R. Podkoński, *Richard Swineshead's* De luminosis: *natural philosophy…*, pp. 366–375).

76 See J.E. Murdoch, E.D. Sylla, "Swineshead", p. 209.

tioned short treatises (*opuscula*) on local motion ascribed to Swineshead, as these clearly show the continuity of the development of the Oxford Calculators science of motion.[77] The reasonings and conclusions included in these *opuscula* let us see these texts as the intermediate "steps" between William Heytesbury's statements from the section *De motu locali* of his "Rules for solving sophisms" and the treatise *De motu locali* of Richard Swineshead's "Book of calculations". Certain solutions and rules we encounter in these short treatises help us also to understand some specific features of Richard Swineshead's accomplishment.

77 The critical edition of these short treatises accompanied with a detailed presentation has been published recently (see R. Podkoński, *Richard Swineshead's science of motion*, Łódź 2019).

Chapter II

Theories of Local Motion before the Oxford Calculators

As for the story this book tells, the most important theories of local motion were the ideas put forward by Aristotle, Averroes, Robert Grosseteste, and William of Ockham. While for all the Oxford Calculators, though Aristotle's theory and the interpretation of his works by Averroes were the starting point for their own concepts, it was the methodology of Robert Grossesteste and William of Ockham, regarding mathematics as the actual language for describing physical phenomena, that constituted a turning point in the history of the theory of motion as presented by the Oxford Calculators. Therefore, the present chapter offers short descriptions of these theories.[1]

1. Aristotle's "Mathematical Physics"

As Edward Hussey notices:

> Aristotle's quest for mathematical laws in the physical world is, like everything else in his physics, closely grounded on the experience of ordinary life. The central "laws of proportionality" must be so understood. But the governing assumption that the mathematical relationships are there to be discovered must be due to the realization that in basic physical processes it is only the quantities and their relationships that hold out some prospect of a reasonable explanation; a purely qualitative law would inevitably be full of unexplained exceptions. But a law relating quantities is necessarily a mathematical one. […] For Aristotle, mathematics was a study of certain particularly basic properties of physical bodies.[2]

1 Part of this chapter was presented in R. Podkoński, "Richard Swineshead's Theory of Motion", Łódź 2019, pp. 15–22.

2 E. Hussey, *Aristotle's Mathematical Physics: A Reconstruction*, [in:] "Aristotle's *Physics*: A Collection of Essays" L. Judson (ed.), Oxford 1991, p. 241. For Aristotle's

In the conclusion of his papers he says: "The aim has been only to show that Aristotle did deliberately formulate mathematical "laws" of physics which were based firmly on observed facts."[3]

Aristotle devotes mainly two Books of his *Physics* to the problem of local motion, namely Book IV and VII. In the final part of Book VII he provides us with the famous "laws of motion" relating the factors of motion as follows:

> If, then, A the movent have moved B a distance C in a time D, then in the same time the same force A will move ½ B twice the distance C, and in ½ D it will move ½ B the whole distance C: for thus the rules of proportion will be observed. Again, if a given force moves a given weight a certain distance in a certain time and half the distance in half the time, half the motive power will move half the weight the same distance in the same time. Let E represent half the motive power A and F half the weight B: then the ratio between the motive power and the weight in the one case is similar and proportionate to the ratio in the other, so that each force will cause the same distance to be traversed in the same time.[4]

At first glance, these laws describing the relationship between the factors causing motion, i.e., its causes of motion (acting force and resistance) and distance traversed and time consumed in its effects (speed) seem to be correct. Observation and "common sense" convince us that if an acting force A can move some weight B to distance C, the same force at the same time will move half of the weight B to twice the distance C, or to the same distance, but in twice shorter time. In both cases B would be moved twice as fast. Similarly, the force that is half the force A will move half the weight B for the same distance C in the same time, since the ratio of force to weight is the same in this case. However, a further statement shows that the relationships between these factors are more complicated because, as Aristotle says:

"laws of proportionality" see e.g., A. Gregory, *Aristotle, Dynamics and Proportionality*, "Early Science and Medicine" 6 1 (2001), pp. 1–20.

3 E. Hussey, *Aristotle's Mathematical Physics*, p. 242.

4 Aristotle, *Physics*, 250a, 2–9, Bk VII, R.P. Hardie, R.K. Gaye (transl.), [in:] "The Basic Works of Aristotle", R. Mc Keaon (ed.), New York 2001, p. 353.

> But if E moves F a distance C in a time D, it does not necessarily follow that E can move twice F half the distance C in the same time. If, then, A moves B a distance C in a time D, it does not follow that E, being a half of A, will in the time D or in any fraction of it cause B to traverse a part of C the ratio between which and the whole of C is proportionate to that between A and E (whatever fraction of A E may be): in fact it might well be that it will cause no motion at all; for it does not follow that, if a given motive power causes a certain amount of motion, half that power will cause motion either of any particular amount or in any length of time [...]. If on the other hand we have two forces each of which separately moves one of two weights a given distance in a given time, then the forces in combination will move the combined weights an equal distance in an equal time: for in this case the rules of proportion apply.[5]

From the above statement it appears that the same force would not necessarily move the doubled weight to half the distance than previously or indeed in twice longer time. This argument is used by Aristotle to refute Zeno's of Elea reasoning, according to which a single falling grain as well as the entire bag makes a noise, which presupposes that the part has the same capacity to act as the whole. Accordingly to Aristotle an acting force can only act as a whole overcoming the whole resistance.

However arbitrarily formulated, the condition was in fact interpreted by medieval thinkers in the sense that local movement can occur if and only if the force of the agent is greater than resistance, whether in the form of the weight or resistance of a medium, or a force acting in the opposite direction. The same condition Aristotle puts more straightforwardly in his *On the Heavens*, in the context of the explanation of the phenomenon of the flotation of heavy bodies on the surface of water. Aristotle says:

> since there are two factors, the force responsible for the downward motion of the heavy body and the disruption-resisting force of the continuous surface, [thus] there must be some ratio between the two. For in proportion as the force applied by the heavy thing

5 Aristotle, *Physics*, 250a10–28, pp. 352–353.

> towards disruption and division exceeds that which resides in the continuum, the quicker will it force its way down; only if the force of the heavy thing is the weaker, will it ride upon the surface.[6]

Given the above condition, it is easy to understand why Aristotle made a reservation about the occurrence of motion in cases where the weight of the moved object increases. If a given weight, here presumably understood as the resistive factor, were doubled, in result it can simply exceed the motive force. Consequently, the acting force would not be able to overcome the resistance and the condition that the force must exceed the resistance for motion to occur will not be fulfilled. It is worth noting that in the context of the discussion on the void Aristotle specifically introduced the resistance of the medium, rather than the weight of the moved body, as the factor opposing the motive force in local motion. Therefore, his famous "equations" regarding this kind of motion appeared first in the *Physics* in the following formulas:

> [The body] A, then, will move through B in time C, and through D, which is thinner, in time E (if the length of B is equal to D) in proportion to the density of the hindering body. For let B be water and D air; then by as much as air is thinner and more incorporeal than water, A will move through D faster than through B. Let the speed have the same ratio to the speed, then, that air has to water. Then if air is twice as thin, the body will traverse B in twice the time that it does D, and the time C will be twice the time E.[7]

John Murdoch and Edith Sylla, in one of their articles on the topic, have suggested that Aristotle's purpose was not to establish the strict, mathematically consistent relations between factors correlated with local motion and to formulate "laws of motion". In their opinion

6 Aristotle, *On the Heavens*, IV, 313b18–23, J.L. Stocks (transl.), [in:] "The Basic Works…", p. 466.

7 Aristotle, *Physics*, 215a, 1–9, Bk. IV, p. 284. It is worth noting here that the ratio of the density of air and water is given here on purely speculative grounds. In fact the average density of air equals 0.001225 g/cm^3, while the density of water is simply 1g/cm^3, therefore water is *ca.* 816(!) times more dense than air. See URL=<https://en.wikipedia.org/wiki/Density_of_air>; URL=<https://en.wikipedia.org/wiki/Properties_of_water#Density_of_water_and_ice>.

the whole of Book VII should be rather seen as a draft, an unfinished version of the planned introduction to the concept of the Prime Mover presented in the treatise we know now as Book VIII of the *Physics*.[8]

Surely, Aristotle was not very strict there, not determining exactly the nature of the relation between weight and resistance, even though he obviously assumed such a relation.[9] Nevertheless, the Aristotelian "laws of motion" introduced in Book VII of the *Physics*, as well as the "rules" from Book IV referred to the mathematical description of the relations between forces, resistances or weights, and speeds in local motion, explicitly alluding to the equal ratios between these factors and bringing up the "law of proportion." What is more important in the context that interests us most here, is that successive generations of natural philosophers, either ancient or medieval, had no doubt that these "rules" were intended by Aristotle from the outset, and should be seen, as mathematically sound and consistent equations.

Aristotle recognized mathematics as one of the theoretical science that "are more to be desired than the other sciences."[10] He even sought for expressions of goodness and beauty in mathematical procedures:

> Those who assert that the mathematical sciences say nothing of the beautiful and the good are in error. For these sciences say and prove a great deal about them; if they do not expressly mention them, but prove attributes which are their results or their definitions, it is not true to say that they tell us nothing about them. The chief forms of beauty are order and symmetry and definiteness, which the mathematical sciences demonstrate in a special degree. And since these (e.g. order and definiteness) are obviously causes of many things, evidently these sciences must treat this sort of

8 J.E. Murdoch, E.D. Sylla, *The Science of Motion…*, p. 224.

9 Establishing such a relation would be even more difficult taking into account the fact that Aristotle considered also the shape and dimensions of a moving body to be equally significant factors. See Aristotle, *On the Heavens*, 313b, 2–16, Bk. IV, pp. 465–466.

10 Aristotle, *Metaphysics*, 1025b1–1026a, 32, Bk. E(VI), W.D. Ross (transl.), [in:] "The Basic Works…", pp. 778–779.

> causative principle also (i.e., the beautiful) as being in some sense a cause.[11]

Aristotle did not write any specific book on mathematics and, as it seems, he was not well acquainted and fluent in mathematical methods and procedures.[12] What is more important here, when introducing the division of the sciences, with regard to their proper subject in the *Posterior Analytics* Aristotle unambiguously and authoritatively stated that in the course of demonstration it is forbidden to use arguments derived from the scope of one science to prove conclusions in another. He says:

> It follows that we cannot in demonstrating pass from one genus to another. We cannot, for instance, prove geometrical truths by arithmetic. (…) Arithmetical demonstration and the other sciences likewise possess, each of them, their own genera; so that if the demonstration is to pass from one sphere to another, the genus must be either absolutely or to some extent the same. If this is not so, transference is clearly impossible, because the extreme and the middle terms must be drawn from the same genus (…). Nor can the theorem of any one science be demonstrated by means of another science.[13]

11 Ibidem, 1078a32–b5, Bk. M(XIII), pp. 893–894.

12 Diogenes Laertius mentioned, among Aristotle's works, in fact, the short treatise *On mathematics*, but this is now lost (see: Diogenes Laertius, "Lives of the Philosophers", V, as quoted in: J. Barnes, *Life and works*, [in:] "The Cambridge Companion to Aristotle", J. Barnes (ed.), Cambridge 1995, p. 8. With regard to two other short treatises, *Mechanics* and *On Indivisible Lines*, that include advanced mathematical argumentations it is established beyond any doubt that these were not written by Aristotle (see ibidem, pp. xxiii–xxiv).

13 Aristotle, *Posterior Analytics*, 75a37–b15, Bk. I, G.R.G. Mure (transl.), [in:] "The Basic Works…", pp. 121–122. Since in Aristotle's day optics, harmonics and mechanics were well developed sciences, ones already employing mathematical arguments and proofs, he made a reservation that these are a few "middle" or "subalternated" sciences that do not obey the above prohibition (see ibidem, 76a9–10, Bk. I, p. 123: "The only exceptions to this rule are such cases as theorems in harmonics which are demonstrable by arithmetic."; Ibidem, 76a 23–25, pp. 123–124: "But, as things are, demonstration is not transferable to another genus, with such exceptions as (…) the application of geometrical demonstration to theorems in mechanics or optics, or of arithmetical demonstrations to those of harmonics."

This rule, described in the secondary subject literature as the prohibition of *metabasis*, undoubtedly impacted substantially, and strictly speaking adversely, on the development of natural philosophy in subsequent centuries. The prohibition of *metabasis* scrupulously observed by medieval philosophers forbid them effectively from introducing any mathematical tools and explanations into natural philosophy itself. Since, if Aristotle had clearly considered the use of arithmetic to prove geometrical truths wrong, thus forbidding the passing from one branch of mathematics to another, then *a fortiori* he would surely have found as fallacious the introduction of mathematical arguments to support proofs in physics.[14] This was in fact the commonly accepted interpretation of his prohibition of *metabasis* that prevailed among medieval natural philosophers, at least from the moment the text of the *Posterior Analytics* had been translated into Latin in the 12th century and had become available to these thinkers.[15] The first who reinterpreted this prohibition in such a way whereby the introduction of mathematical arguments into scholastic natural philosophical issues was acceptable, or even preferable, was actually William of Ockham, possibly the most innovatory thinker of his times.

2. Theories of Motion in Arabic Medieval Philosophy

With Averroes' commentaries on Aristotle's works the Middle Ages was to become acquainted with the tradition of Arabic science. In his commentaries on the *libri naturales*, Averroes presented and discussed arguments advanced by John Philoponus, Al-Kindi, Al- Farabi, Avempace and Avicenna; the last two Arabic philosophers having presented an innovative and entirely different concept of motion than the Aristotelian one.

As Marwan Rashed notes:

> Combining some arguments in Aristotle (*Physics*, IV.8, VII.5, and *De caelo*, I.6 in particular) – which originally have very different

14 With the exception of the "middle" or "subalternated' sciences, see e.g., S.J. Livesey, *The Oxford Calculators, Quantification of Qualities, and Aristotle's prohibition of metabasis*, "Vivarium", 24 (1986), pp. 51–56.

15 See B.G. Dod, *Aristoteles Latinus*, [in:] "The Cambridge History of Later Medieval Philosophy" N. Kretzmann, A. Kenny, J. Pinborg (eds), Cambridge 1982, p. 75.

purposes – Philoponus and his followers constructed an Aristotelian "law" of motion. It expresses mean speed (*S*) as a function of force (*F*, which is weight in the case of free fall and the resistance of the medium (*R*):

$$S = F/R$$

For ontological and empirical reasons, many physicists of antiquity and the Middle Ages reformulated the relation of the force and the resistance. Philoponus, in particular, replaces this "law" he attributes to Aristotle with another one, which does not divide the force by the resistance but postulates that the time *t* required for an object to fall a certain distance through the medium will be inversely proportional to its weight (*W*), plus a certain time (*x*):

$$t = I/\ W + x$$

It is only the determination of *x* that the density of the medium plays a role. Thus, the mean speed of a free fall is directly proportional to the weight of the body, but is also partially influenced by the density of the medium.[16]

Philoponus was not the only one who tried to reformulate the Aristotelian "law". The other theories, known to Avicenna, suggest that two bodies, such as a feather and a stone would fall in a void with the same speed. This conclusion follows from the belief that a body composes of atoms, whose weight is the same, and the weight of a separate atom – the primary cause of its motion in free fall – in a void would cause the same speed. The difference in the speeds of different bodies results from a lesser or greater density of atoms. Less densely packed atoms have less intense a force to cut through the medium, and, thus the body moves slower. The following "law" is obtained:

16 See M. Rashed, *Natural philosohy*, [in:] "The Cambridge Companion to Arabic Philosophy", P. Adamson, R.C. Taylor (eds), Cambridge 2006, pp. 295–296. See also M. Wolff, "Fallgesetz und Massbegriff", Berlin 1971; Idem, *Philoponus and the Rise of Preclassical Dynamics*, [in:] "Philoponus and the Rejection of Aristotelian Science", R. Sorabji (ed.), London 1987, pp. 125–160.

$$S = c(W - R)/W,$$

"where c is a constant, S the speed, W the weight of the body, and R the resistance of the medium. In a void (where R = 0), S = c, whatever the value of W, yielding the above-mentioned result that any two bodies fall through a void at equal speed, whatever their weight."[17]

Avicenna rejects Philoponus' interpretation of Aristotle because of a precise reason explained in detail by Rashed, who, after a detailed study of Avicenna's *Physics* of the *Shifāʾ* (Bk. II, Ch. I), concludes as follows:

> Avicenna stresses that we can mean two things when we speak of "motion": motion as a trajectory, which pertains to our imaginative faculty and is conceived of only as linking a starting point to an end; and motion as an intermediate state, which must be attributed to each moment of the trajectory. Motion in the second sense characterizes as an infinitesimal moment, and nothing else. (…) Each substance spatially or qualitatively removed from its natural state (e.g., a stone thrown up away from its natural resisting place) returns to it, passing through *all* intermediary states. Each of these intermediary states, because it is not the end point of the process, produce a new *mayl*, which adds itself to the impulsion produced by the others. Every moment is thus characterized by its own kinetic intensity. (…) Avicenna thus seems to stand the crossroads of two traditions. With the mathematicians, he recognizes that every one of the infinite points on a special interval AB, without perhaps being perfectly real, is however notionally and qualitatively distinct from every other point. But with the *mutakallimūn*, he sees in a dynamic of impetus the efficient principle of such distinction. Thus, starting from a classificatory project of the different types of impetus, Avicenna arrives at a complex – because partially "ontological" – doctrine of instantaneous motion. This combination of the kinematics of the geometers and the dynamics of *mutakallimūn*

17 See M. Rashed, *Natural philosophy*, pp. 296–297. It is possible that Bradwardine gets use of this theory, but he interprets it all way around while stating that s = F – R/R to prove that the speed of motion is not caused by the proportion of equality, when F = R, then s = R – R/R = 0/R = 0. This peculiar theory is to be found only in Bradwardine's *Tractatus de proportionibus*, pp. 92–94.

> deeply influenced Avicenna's successors in the East and West. It is probably Avicenna's main achievement in natural philosophy that after him, for every lucid reader, the discussion of motion must focus on what happens at an infinitesimal level.[18]

It is worth noting that John Dumbleton, in presenting the new proof of "the mean speed theorem", refers to Avicenna' s *Physics* of the *Shifā'*.[19]

The next Arab philosopher with whom Averroes argues is Avempace. Avempace is of the opinion that natural science is a theoretical one and it should utilize the results of other science. It is not as demonstrative as geometry, because its objects are material things in motion, however, not each of its statements are inductive. Although Avempace accepts the Aristotelian basic condition of motion that: "everything that is moved, is moved by something", but he interprets it in his own way. Obviously a motion can occur only if an active power acts causing the motion, but a resistance is not a necessary condition for motion. While discussing the issue of two different types of motion "violent" and "natural", as introduced by Aristotle, Avempace states that, even though "violent" motion is opposed to the "natural" one, in this sense that one is directed in an opposing direction than its "natural place", and the other goes towards its natural place, both motions should be described in the same way. He accepts John Philoponus' concept and claims that there is a minimum amount of moving power for each moveable, but the powers can be added, i.e., the moving and resistive ones. When they are equal, motion does not occur. In "violent" motion while the acting power overcomes the passive one it is constantly exhausted by it, since the initial impetus is diminishing by the resistance during the motion. In the case of "natural" motion in a medium, a body has to overcome the resistance of the medium when cutting it out. Natural motion in a void, however, would occur with the speed directly proportional to the weight of the body.

> As a proof that motion without any medium, namely, through a void, is possible, Avempace adduces the movement of the spheres:

18 See M. Rashed, *Natural philosohy*, pp. 301–302.

19 See Johannes Dumbleton, *Summa logicae et philosophiae* naturalis, Part III: De *motu locali* (Editions), § 5, p. 395.

> [In the heavens] there are no elements of violent motion, because nothing bends their movement, the place of the sphere remains the same and no new place is taken by it. Therefore, circular movement should be instantaneous, but we observe that some spheres move slowly—such as the sphere of fixed stars—and others fast—the daily movement—and that there is neither violence nor resistance among them. The cause for the different velocities is the difference in nobility (*sharf*) between mover and movable.[20]

Averroes' explanation of Avempace, as presented in commentary 72 to Book IV of the *Physics* gives the following "law": s = F – R. This rule was quoted and criticized as the wrong theory by all the Oxford Calculators, who believed, after Aristotle, that motion cannot occur without two powers (moving and resistive ones) being involved in it.[21]

Ruth Glasner has examined most carefully Averroes' commentary to the Aristotelian *Physics*. The results of this detailed study are presented in her excellent book entitled: "Averroes' Physics. A Turning Point in Medieval Natural Philosophy".[22] From the perspective of our book, the most important findings concern Averroes' concept of the relation between mathematics and natural philosophy, as well as his original interpretation of the "laws of motion" leading to nominalism.

As Glasner asserts:

> Averroes' interpretation openly conflicts with many statements of Aristotle, who often refers to physical entities as *continua*. Averroes' strategy, when dealing with such statements, was to translate 'continuous' to 'continuous qua continuous' or 'continuous qua quantity', namely to translate the physical term into a mathematical one.[23]

20 See Montada, Josép Puig, "Ibn Bâjja [Avempace]", *The Stanford Encyclopedia of Philosophy* (Spring 2018 Edition), Edward N. Zalta (ed.), URL = <https://plato.stanford.edu/archives/spr2018/entries/ibn-bajja/>.

21 See Chapter III, pp. 68–71.

22 See R, Glasner, "Averroes' Physics. A Turning Point in Medieval Natural Philosophy", Oxford 2009.

23 See ibidem, pp. 174–175.

Averroes, like his Arab predecessors, claims that the moving body is composed of minimal particles which are not divisible, while time and distance are mental constructs and as such they are infinitely divisible. The minimal particle is identified as the First-Moved, because natural bodies are not divisible *in actu qua* natural bodies. Thus the First-Moved moves essentially, as a whole and not as a part. With the notion of the First-Moved part, Averroes breaks with Aristotle's notion of homoeomerity "and denies continuity in the physical world" – as Glasner notes – since for him "natural *minima* mean much more than a theoretical limit of divisibility."[24]

Averroes distinguishes between two types of motion: the continuous celestial motion and the motion of the sublunary region, which consists of multiple motions that succeed one another. Any material body, such as elements, has its specific natural motion, but such a body moves as a whole, i.e., as the aggregate of the motion of its particles, thus, the sum of their motions results in the speed of the whole. Each First-Moved unit is composed of its matter and form. The matter is the subject of being moved and being a divisible quantity; the form is the faculty of moving and is indivisible.

> The form of earth is associated with each minimal part of earth, which is indivisible and an ontologically stable entity. The First-Moved part is thus a holomorphic unit: it "carries" the specific form of the body and dictates its specific motion. In conclusion, Averroes' theory of *minima naturalia* is a theory of actual and essential parts and, as such, it bridges the gap between the two opposing systems, the Aristotelian and the atomistic. It is atomistic in the sense that a physical body is made of actual minimal building stones, and that no physical magnitude is infinitely divisible. It is deeply Aristotelian as the minima are essential parts, that is, units having matter, form and specific natural motion.[25]

The second significant, from our point of view, of Averroes's concepts concerns the relationship between mathematics and physics. On the one hand, Averroes accepts Aristotle's' belief that mathematical beings do not exist in separation from matter, and only intellect regards

24 See ibidem, p. 168.

25 See ibidem, p. 172.

them as such. Averroes considers mathematical objects to be separated in the process of abstraction by noting the similarities and dissimilarities until the nature of the components become intelligible. The geometer can always extend or divide his object, the natural philosopher cannot do this freely. Continuum is thus a concept characteristic for geometry. Physical entities do not form a continuum which may be infinitely divided. As Glasner notes: "Averroes's strategy, when dealing with such statements, was to translate 'continuous' to 'continuous qua continuous' or 'continuous qua quantity', namely to translate the physical term into a mathematical one."[26]

As Glasner points out:

> For Aristotle time, motion, magnitude (the distance traversed), and the moved body are all equally divisible. For Averroes they are not. The concrete physical entity, the moved body, is composed of minimal units, while time and distance are mental constructs and as such infinitely divisible. Commenting on the *Metaphysics,* Averroes inquiries into the mode of existence of the objects of mathematics: 'Is it the existence [1] of the substance or [2] of the accidents or [3] of the things that the soul makes out of that which exists in reality, like many of the relations and combinations?' The answer is the third: 'Unless there was soul, there would be no number, as there would be no time; so too for magnitudes.'[27]

We conclude this section with the brilliant observation made by Glasner:

> While time is an entity that has only mental existence, motion has two modes of existence. Aristotle distinguished two kinds of divisibility of motion: 'In the first it is divisible in virtue of the time that it occupies. In the second it is divisible according to the motions of the several parts of that which is in motion.' I shall refer to the first as divisibility 'in length' and to the second as divisibility 'in mass'. In the long commentary Averroes claims that the division in mass is prior to the division in length, because the former is 'outside the soul', whereas the latter 'does not have a [natural]

26 See ibidem, p. 174.

27 See ibidem, p. 176.

cause' and is 'in the soul'. Averroes may have been acquainted with Avicenna's admirable observation that motion 'leaves an impression on the imagination only because its form subsists in the mind by reason of the relation between that which is moved and the two places: the place that it leaves and the place that it reaches.' For both Avicenna and Averroes motion qua continuous entity exists only in the mind.

Of the isomorphic continuous entities that Aristotle lists in *Physics* VI.4 – time, motion, magnitude, and body – Averroes regards time, distance, and motion 'in length' as entities that exist only in the soul. All continuous entities are 'expelled' from the real world and become 'explanatory devices'. What exists in reality is the moved body which is composed of particles and in them parts of motion 'in mass'. Real motion is an entity that resides in a body. This is Averroes' axiom of inherence—motion is in a body. It is the last step in the development of his interpretation of premise VIII. Let me conclude the discussion on Averroes' nominalism with a quotation from the *Tahafut̄*: "Motion has existence only in the intellect, since outside the soul there exists only the thing moved and in it a part of the motion (جزء من الحركة) without any lasting existence." In summary, for Averroes the *continuum* is a mathematical structure, not a physical one.[28]

It seems that, at least Richard Kilvington followed Averroes' concept in his last question from the *Physics* commentary, entitled: *Utrum omne transmutatum in transmutationis initio sit in eo ad quod primitus transmutatur.*[29]

3. The English Tradition in Mathematical Natural Science

In regarding the presentation of any of the issues discussed by Oxford medieval natural philosophers one should always refer first to the theo-

28 See ibidem, p. 177.

29 Richard Kilvington's questions forming his commentary of the *Physics* are to be edited next year by Elżbieta Jung.

ries developed by Robert Grosseteste (1168—1253), bishop of Lincoln and the University's first chancellor, "the real founder of the tradition of scientific thought in medieval Oxford" – as he has been described by one of the most influential historians of science, Alistair Crombie.[30] Robert Grosseteste was highly regarded by the Oxford Calculators and, along with Aristotle and Averroes, was considered the greatest authority. His commentary to the *Posterior Analytics* was frequently used in their methodological discussions on the status of natural philosophy.

Grosseteste is perhaps better known in the history of philosophy for his original cosmological and cosmogonic speculations, aptly described in the secondary literature as the "metaphysics of light." In short, Grosseteste advanced the idea that the whole universe in its very beginning, that is at the moment of Creation, emanated from the first indivisible and infinitely small point of light (*lux*) that multiplicated itself *in infinitum* in every direction, thus constituting spherical, finite cosmic space. This primordial light was defined by Grosseteste as the first corporeal form (*forma prima corporalis*). The resulting, secondary light (*lumen*) emanating from the cosmic sphere towards the centre of the universe was the factor that produced the elementary matter. This process, described quite perfunctorily and not clearly by Grosseteste, was to occur as a result of the condensation and rarefaction of the *lumen*.[31] His most influential statement given the context of the later development of Oxford medieval natural philosophy was, however, that this process took its course according to the geometrical laws of optics and catoptrics. Consequently, the structure of the created, physical world conformed necessarily to the laws of geometry.[32]

30 A.C. Crombie, "Medieval and Early Modern Science", London 1952, pp. 11–12. See also, G. Beaujouan, *Medieval Science in the Christian West*, [in:] "Ancient and Medieval Science", R. Taton (ed.), London 1963, p. 491. On the biography, works and philosophy of Robert Grosseteste, see: Lewis, Neil, "Robert Grosseteste", *The Stanford Encyclopedia of Philosophy* (Summer 2019 Edition), Edward N. Zalta (ed.), URL = <https://plato.stanford.edu/archives/sum2019/entries/grosseteste/>.

31 <Robertus Grosseteste,> *Tractatus de luce secundum Lincolniensem*, [in:] "Robert Grosseteste and His Intellectual Millieu. New Editions and Studies", J. Flood, J.R. Ginther, J.W. Goering (eds), Toronto 2013, pp. 226–238.

32 Robertus Grosseteste, *De lineis, angulis et figuris*, [in:] "Die Philosophischen Werke des Robert Grosseteste, Bischofs von Lincoln", L. Baur (ed.), Münster 1912.

The idea that the laws of nature are mathematical in their essence was the main concept of Robert Grosseteste's natural philosophy, something inherited by subsequent generations of medieval English scholars. His detailed description of the natural world was soon superseded by the Aristotelian worldview, which was an inevitable result for the Latin-speaking world of the rediscovery of Aristotle's natural philosophical works.[33] Still, "the special importance [given] to mathematics in attempting to provide a scientific explanation of the physical world"[34] was the ever present distinguishing feature of Oxford medieval natural science. No wonder then that the application of *calculationes* to Aristotle's own assumptions concerning the relations between forces, resistances and speeds in local motions was first undertaken by English thinkers. It is worth noting here that the form these relations were formulated in his *Physics* actually could also suggest reference to the calculus of ratios in order to explain or interpret them consistently.

William of Ockham (*ca.* 1288—1347) is recognized as one of the three most influential thinkers of the High Middle Ages, the remaining two being Thomas Aquinas (1224/5—1274) and John Duns Scotus (*ca.* 1266—1308).[35] Such a reputation is founded on two aspects of his philosophy; namely on Ockham's "reduction of metaphysics," commonly seen as the direct effect of the introduction of the principle of parsimony – better known in the history of science as "Ockham's Razor," – and his "political" writings directed against the popes John XXII, Benedict XII and Clement VI. In fact, for almost twenty years, from 1328 till his death, William of Ockham devoted all his energy and wit to advocate the deposition of the successively reigning popes as mentioned.[36] In the context that interests us most here the first issue is more important, however, for Ockham's "reduction of metaphysics" influenced his con-

33 See B.G. Dod, *Aristoteles Latinus…*, pp. 69–74.

34 A.C. Crombie, *Grosseteste's Position in the History of Science*, [in:] "Robert Grosseteste: Scholar and Bishop", D.A. Callus (ed.), Oxford 1955, p. 111.

35 P.V. Spade, *Introduction*, [in:] "The Cambridge Companion to Ockham", P.V. Spade (ed.), Cambridge 1999, p. 1. On William of Ockham's biography and works see ibidem, pp. 2–11, and: W.J. Courtenay, *The Academic and Intellectual Worlds of Ockham*, [in:] "The Cambridge Companion to Ockham", pp. 17–27.

36 J. Kilcullen, *The Political Writings*, [in:] "The Cambridge Companion to Ockham", p. 302.

cept of theoretical sciences in general, and allowed in particular for the introduction of mathematical arguments into natural philosophy.

Generally speaking, by employing his "Razor" Ockham reduced the catalogue of real ontological categories to only two, namely: substance and quality.[37] Consequently, all the remaining categories enumerated by Aristotle and his medieval followers, like quantity, relation, action, passion, place, motion, etc. were not considered to be separately existing entities in Ockham's worldview. This does not mean that they are not real. He did not reject abstract terms, like 'whiteness' or 'humanity.' Moreover, he allowed there to exist distinct real things that can be described as distinct 'whitenesses' or 'humanities,' as long as these are distinct substances: white colored things or humans respectively. There is just no distinct thing, or a really existing ontological component of any substance that can be properly described as 'whiteness' or 'humanity.' As Paul Vincent Spade puts it:

> The main vehicle Ockham uses to show we do not need distinct entities for all these abstract nominalizations is his semantic theory of connotation together with the related theory of "exposition" and "exponibiles". With these tools, Ockham argues that true statements containing words appearing to signify such entities are in fact equivalent, in some fairly strong meaning-based way, to statements that do not contain such words; hence, we can say all the true things we want without committing ourselves to such entities.[38]

With regard to the notion of 'local motion' specifically, William Ockham explained that local motion is nothing more than a distance

37 The most commonly quoted formula of "Ockham's Razor" is: "Beings are not to be multiplied beyond necessity." Still, as Paul Vincent Spade states, Ockham never actually put it this way. The formulas we find in his works are: "Plurality is not to be posited without necessity"; "what can happen through fewer [principles] happens in vain through more"; or "when a proposition is verified of things, more [assumptions] are superfluous if fewer suffice." It is worth noting here that the principle of parsimony is not Ockham's discovery, versions of it can be found in Aristotle's works as well as in those of many medieval thinkers. See P.V. Spade, *Ockham's Nominalist Metaphysics: Some Main Themes*, [in:] "The Cambridge Companion to Ockham", pp. 100–102.

38 Ibidem, p. 104.

continually traversed by a moving body.[39] Accepting such a definition led his followers eventually to distinguish the "kinematical" and the "dynamical" descriptions of local motion. If the motion is in fact determined by the distance traversed by a moving body in a given time, then the description "with regard to effect" (*tamquam penes effectum*), to put it in medieval terms, concerning the relation between speeds, times and distances, seems to be the most proper one. On the other hand, the Aristotelian account on motion, "with regard to cause" (*tamquam penes causam*), determining the intensity of motion, i.e., speed, related to the intensity of its factors, namely motive force and resistance, was still considered adequate. Consequently, in the works of the Oxford Calculators when taken as a whole, both these attitudes are encountered.

The more important, though yet a side effect of Ockham's "reduction of metaphysics" was his reformulation of the concept of theoretical science. All Aristotelian categories, save substance and quality, as well as other abstractive notions in Ockham's view, are only connotative terms and exist as a part of our mental language. Their special feature is, however, that they can be properly defined, while absolute terms, those concerning individuals, cannot.[40] Consequently, every theoretical science is developed on the level of true propositions and statements, deduced logically from the principles and premises that are necessarily true, independently from the existence of physical reality.[41] According to Ockham such premises can be derived only from logic, mathematics and theology.[42] What is more, while defining 'science' in the *Prologue* to his *Commentary on "Physics,"* Ockham pointed out that 'science' (*scientia*, knowledge) can be understood either as a singular sentence, statement or proof, or as the "collection of many conditions ordered in a determined and fixed way."[43] He repeated these explanations in

39 Gulielmus Ockham, *Quaestiones physicae*, qu. 22, [in:] A. Goddu, "The Physics of William of Ockham", Leiden-Köln 1984, p. 202, n. 134: "Motus localis est spatium particulariter et continue acquisitum mobili." See also: A. Goddu, *Ockham's Philosophy of Nature*, [in:] "The Cambridge Companion to Ockham", p. 156.

40 See C. Panaccio, *Semantics and Mental Language*, [in:] "The Cambridge Companion to Ockham", p. 57.

41 See A. Goddu, *Ockham's Philosophy of Nature*, pp. 144–146.

42 See A.J. Freddoso, *Ockham on Faith and Reason*, [in:] "The Cambridge Companion to Ockham", pp. 331–335.

43 See Gulielmus Ockham, *De scientia in generali et de scientia naturali in speciali. Prologus in Expositionem super VIII libros Physicorum*, [in:] "William of Ockham, Philosoph-

the *Prologue* to his *Ordinatio* with the complementary statement that the Aristotelian prohibition of *metabasis* applied only to science taken in the former meaning, that is as a singular, separately taken statement. Such a statement, moreover, when derived on the basis of its proper premises, can be transferred and used within the scope of the other science, as understood in the latter sense. In logic, for example, one can refer to mathematical proofs on the one hand, and use them as tools of analysis in natural philosophy, on the other. Consequently, exploiting mathematical argumentations in order to solve natural philosophical issues is acceptable, since mathematical proofs have all the properties required in theoretical science.[44]

ical Writings. A Selection," P. Boehner, S. Brown (eds), Indianapolis 1990, p. 3.

44 See Gulielmus Ockham, *Scriptum in librum primum Sententiarum. Ordinatio*, Prologus, qu. 1, [in:] "Opera theologica", vol. 1, G. Gál, S. Brown (eds), New York 1967, pp. 7–15. See also: S.J. Livesey, *The Oxford Calculators*, pp. 57, 63.

Chapter III

Oxford Calculators on Local Motion

1. Richard Kilvington's Theory of Local Motion

The problem of local motion is the main issue in the fifth question of Richard Kilvington's commentary on Aristotle's *Physics*, that is related to motion in a medium and entitled: Whether in every motion the power of a mover exceeds the power of a thing moved? (*Utrum in omni motu potentia motoris excedit potentiam rei motae*),[1] and also in the seventh one of the same set, which concerns the possibility of motion in a void: Whether a simple body can be moved equally fast in a medium and in a void? (*Utrum corpus simplex posset aeque moveri in pleno et in vacuo*).[2] Discussing the problem of local motion, Richard Kilvington approaches it in a twofold manner, considering the causes of motion (*tamquam penes causam*) as well as its effect (*tamquam penes effectum*).

It must be stated from the outset that Richard Kilvington, like most medieval natural philosophers, accepts the Aristotelian conditions for motion to occur: 1) everything that moves is moved by something, and there cannot be a motion when there is no resistive power (*virtus resistiva*), since if there was no resistance, the resulting change would be instantaneous; 2) an active power must be greater, or more intense, to use medieval terms, than a resistive power for motion to be initiated and, what is most important here: to be continued. With respect to the nature of motion and its factors however, Kilvington follows William of Ockham's particularist ontology and claims: "I say that all things either in reality or in the soul are only singular things, since to be universal means only to signify things universally, but it does not mean that

1 See Richard Kilvington, *Utrum in omni motu potentia motoris excedit potentiam rei motae* (Editions), pp. 215–266.

2 For description of Kilvington's commentary on the *Physics* see Chapt. I, pp. 16–17.

there are other, i.e., universal things, different form the singular ones."[3] Thus, Kilvington accepts substance and quality as the only two distinct kinds of realities[4] and consequently states that the reality of motion is limited to a thing moved, places, qualities, and quantities it successively acquires.[5] Thus, Kilvington is interested in measuring local motion in terms of the actions of the causes of motion, distances traversed and time consumed more than in determining the intensity of speed.

Richard Kilvington's questions on local motion, from his commentary on the *Physics* are perfect testimony to the debates that took place at Oxford university in the second decade of the 14th century. They should be recognized also as the "great opening" of the new discussions and solutions providing the original reinterpretation of Aristotle's theory and the new, logically and mathematically consistent, yet Aristotelian, rules of local motion.

1.1. Motion with respect to its Causes

In accordance with Averroes's interpretation of Aristotle's "rules", a motion can occur only if an active power – F exceeds a resistive power (a total resistance) – R, i.e., when a ratio of F : R is a ratio of a greater inequality (*proportio maioris inequalitatis*). This fundamental principle is the basic one in considering the problem of the reac-

3 Ricardus Kilvington, q. *Utrum omne quod scitur sicatur per causam*, Ms. Vat. Lat. 4353 f. 141r: "Dico enim quod omnes res modum sive universales sive animae sunt res singulares, quia universales non sunt res universales nisi quia significant universaliter et non quia sunt aliae res a singularibus".

4 Richard Kilvington, *Utrum qualitas suscipit magis et minus*, Ms Venezia, Bibl. Naz. Marciana, VI, 72 (2810), f. 90rb: "In intensione caliditatis praecise semper manet eadem caliditas quae praecise substantialiter intenditur, sed latitudo illius caliditatis est eadem res realiter sicut tempus et coelum; sic alia est ratio ut est latitudo, et alia ut est caliditas." Ibidem, f. 94rb: "Motus est mobile successive vel partialiter pertransiens spatium supra quod erit motus"; "Motus alterationis non est aliud quam mobile." See also E. Jung[-Palczewska], *The Concept of Time in Richard Kilvington*, [in:] "Time, Aevum, Aeternitas. La Coneceptualizatione del Tempo nel Pensiero Tardomedievale", G. Alliney, L. Cova (eds), Firenze 2000, pp. 198–202.

5 On the basic ontology of motion in the Oxford Calculators see for example: E. Sylla, "The Oxford Calculators and the Mathematics of Motion 1320–1350. Physics and Measurement by Latitudes", New York 1991, pp. 182–187.

tion between agents of two different kinds: inanimated and animated ones. Inanimated bodies, that is either pure elements or bodies that are mixed from these, act "from their nature" i.e., each element strives to reach its "natural place" in the sublunar world. Animated bodies act thanks to their intellects and/or free will, and therefore they are of a finite, debilitable power and they can act only for some time, since these, e.g., human beings, are prone to weaken when acting against any resistance, be it walking or lifting a weight. Elementary bodies, like fire, air, water and earth, are of indebilitable power, and at least theoretically they could act *in infinitum*. Active powers can cause either the natural or violent motions of inanimated bodies, such as simple element or mixed bodies. In the case of a mixed body the natural motion is caused by the element that dominates, and as such causes the motion in a direction that is "proper" to its own natural place and opposite to the natural places of other elements that make up the mixed body.[6]

In his discussion on the measurement of motion with respect to its causes, Kilvington presents two major aspects of the issue: the physical one, involving relations between active powers (forces – F) and resistive powers (resistances – R), and the mathematical one, involving the concepts of continuity, limits and a new calculus of ratios. The three initial *articula* of his fifth question on local motion concern the problem of assigning the value of an excess of an acting power over a resistance and of setting the limits of active and passive factors.[7]

1.1.1. An Excess of Acting Power over Resistance – the Condition Necessary for Motion

At the beginning of the part of his question *Utrum in omni motu…* concerning the necessary condition for motion to occur, Kilvington cites four different opinions about how an excess of the acting power over resistance may be described: 1) by a minimum excess of an acting power

6 E. Jung[-Palczewska], *Motion in a Vacuum and in a Plenum in Richard Kilvington's Question: Utrum aliquod corpus simplex posset moveri aeque velociter in vacuo et in pleno from the 'Commentary on the* Physics, "Miscellanea Mediaevalia", Bd. 25 (1998), pp. 179–193.

7 See Richardus Kilvington q. *Utrum in omni motu*...(Editions), § 1–47, pp. 215–233.

over a resistance, which is sufficient for motion (*minimum quod sic*); 2) by a maximum excess, which is not sufficient for motion (*maximum quod non*); 3) by such an excess which is sufficient to continue motion but not to initiate it; 4) by such an excess, which, when sufficient to continue motion, is sufficient to initiate it. While the first two opinions state that it is possible to assign an intrinsic or an extrinsic limit of the sequences of the F : R ratios, the last two differ in deciding whether necessary it is to initiate a motion a greater excess of active power over resistance than the one that sustains the motion.

Kilvington refutes the first three opinions on the basis of imaginable, theoretical cases and due to an observation of facts.[8] He accepts the fourth opinion, since, as he claims, Aristotle, Averroes, Euclid and Jordanus Nemorarius prove that any excess of active power over a resistance which is sufficient to sustain motion is sufficient to initiate it. The following three arguments are put forward in support of this opinion. Firstly, it is impossible to observe the exact moment when motion begins and when it lasts continually, and thus, it is impossible to assign a minimum excess *quod sic* or a maximum excess *quod non*. Secondly, an observation of different weights placed on a balance with equal arms shows that even a minimally heavier body lifts the other one. Thirdly, in accordance with Aristotle and Averroes, motion is initiated and sustained in every case when the active power is greater than the resistance.[9] Consequently, motion occurs whenever the ratio of F to R is a ratio of *maioris inequalitatis*, i. e., when F : R is greater than the ratio of equality, i.e., the ratio that equals 1 : 1. Thus Kilvington affirms that any force greater than resistance can produce motion, though it takes a longer time for a smaller force to initiate that.

Kilvington's elaborated theory of local motion, having taken into accounts the authorities of Aristotle, Averroes, Archimedes, Euclid and Jordanus Nemorarius, is based on the statement put forth in the above-presented fourth opinion, that any excess of an active power over a passive one, which is sufficient to initiate motion, is sufficient to continue it.

The theory that any excess of an active power over resistance is sufficient for motion has an unexpected consequence: that the Earth can move rectilinearly. The motion of the Earth is caused by its inclination to bring the geometric center of the universe into coincidence with its

8 Ibidem, § 1–24, pp. 215–225.

9 Ibidem, § 93–105, pp. 252–258.

center of gravity, and because the Earth as a whole is unequally heavy and dense, and geological changes perpetually alter it, its center of gravity shifts. If the world were eternal, this slight rectilinear motion would be infinite.[10]

1.1.2. Inalienable Conditions of Motion

Having established a fundamental condition for motion to occur, Kilvington concentrates on setting boundaries for the range of active and passive powers that cause motion. An active power is limited by the patient, e.g., by the weight that can be lifted or the distance that can be traversed. A passive power is limited by the agent it can be affected by, as sight is limited by the smallest object that can be seen.

The question of the limits of active and passive powers originates in Aristotle's reflection on the common observation of human capacities, and his opinion is in accord with the everyday use of language. Aristotle states that an active power is limited by a *maximum quod sic*. He says:

> We speak for instance, of a power to move or to lift a hundred talents or walk a hundred stades – though a power to effect the maximum is also a power to effect any part of part of the maximum – since we feel obliged in defining the power to give the limit or maximum. A thing, then, which is capable of a certain amount as maximum must also be capable of that which lies within it. If, for example, a man can lift a hundred talents, he can also lift two, and if he can walk a hundred stades, he can also walk two. But the power is of the maximum, and a thing said, with reference to its maximum, to be incapable of so much is also incapable of any greater amount.[11]

10 Ibidem, § 95, pp. 252–254. See also E. Jung[-Palczewska], *Works by Richard Kilvington*, pp. 216–217; E. Grant, *Cosmology*, [in:] "Science in the Middle Ages", pp. 290–291. John Buridan in his *Quaestiones super libros quattuor* «*De coelo et mundo*», also presents the same conclusions (see Johannes Buridanus, *Quaestiones super libris quattuor "De celo et mundo",* ed. by E. Moody, London 1948, p. 231.

11 Aristotle, *On the Heaven*, Bk. I, 281a8–11, [in:] "The basic work of Aristotle", p. 422.

A passive power, such as the power of vision, is to be determined by the minimum by which it may be affected, since as Aristotle states: "he who sees a stade need not see the smaller measure contained in it, while, on the contrary, he who can see a dot or hear a small sound will perceive what is greater."[12]

Averroes' commentary on Aristotle's remarks sticks closely to his text. He agrees that an active power is always limited by the maximum it can accomplish. He goes on, however, to suggest that also the incapacity of an active power (*defectus potentiae*) should be measured in this way. In other words, there is a need to establish a *minimum quod non* as a limit for the range of what an active power cannot accomplish. And with that we describe an active power by both: a *maximum quod sic* (the maximum weight Socrates can lift) and *minimum quod non* (the minimum weight Socrates cannot lift). The other difference occurs in Averroes' treatment of passive powers. The power of vision is indeed determined by the minimum that it can perceive, but this power belongs to the general class of passive powers. Thus, every power is active or passive and, in accordance with Averroes, every action should be analyzed from two points of view, that of the agent (to settle the limit for the range of what an active power can accomplish – *maximum quod sic*, or what it cannot accomplish – *minimum quod non*) and that of the patient (to settle the limit of the range of active powers, i.e., a minimum active power by which a passive one can be acted upon – *minimum quod sic*).[13] For example we

12 Ibidem, 281a21–23, p. 422.

13 Averroes, *Commentarium super libros quattuor de coelo et mundo*, I, com.116, pp. 221–222, 19–40: "…cum manisfestum est per se quod si aliquid potest aliquam actum quod potest illud quod est sub eo, manisfestum est quod potentia eius non est diffinienda nisi per ultimum illius actionis; et cum ita sit, manisfestum est quod potentie rerum non terminantur nisi per suos fines et per eos distinguitur quelibet potentiarum rerum habentium potentias diversas, verbi gratia quod cum voluerimus diffinire potentiam illius quod habet potentiam moveri per minus quam per centum; et cum ita sit, potentia igitur non diffinitur nisi per finem sui actus (…) et quod ipse intedit per hoc finem potentiae non quecumque, incepit dicere modum ex quo diffinitur debilitas potentie et suum defectum, (…) idest deffectus autem potentie terminator per minimum in posse; cum enin non poterit minus, necessarium est ut non possit maius, econtrario determination actionis potentie, scilicet quia potentia diffinitur ex fine sui actus, defectus autem eius diffinitur et primo in posse; cum enim non poterit ferre quinque libras quod est plus, dicemus quod est impotens ferre quinque libras et non plus, quoniam si quinque non potest necessario plus non potest…".

can, on the one hand, determine the upper limit of Socrates' power by the heaviest stone he can lift or by the one, heavier than the one he lifted, that he cannot lift. On the other hand, we can determine the passive power of a weight by the minimum active power that can lift it.

While commenting on Aristotle's and Averroes' remarks, Kilvington raises some important queries, which open a new perspective on the solution of the problem and introduce mathematics into physics. He is interested in answering the following questions: how is a power to be limited if it is active or passive; if it is subject to weakening or not; if it is mutable or immutable? How to assign the limits of active powers if a body moves in a medium that is not uniformly resistant? Most of the cases considered are posed *secundum imaginationem* which, however, does not make empirical verification irrelevant. Kilvington's mathematical interest is to be observed, at the outset, in his classification of all powers as active or passive ones, even in cases in which it is hardly possible to define them as such in accordance with Aristotelian terms. Kilvington, as later on Heytesbury, "employs definitions of active and passive powers which depend on purely quantitative consideration rather than on meaning."[14] The mathematical character of Kilvington's discussion is to be observed also in his use of two kinds of limits for continuous sequences: an intrinsic limit (when an element is a member of the sequence of elements it bounds: *maximum quod sic, minimum quod sic*) and an extrinsic limit (when an element which serves as a limit stands outside the range of elements which it bounds: *maximum quod non, minimum quod non*).

1.1.2a. How to "Measure" an Active Power?

At the beginning of his discussion, presented in the second article of his question on local motion in a medium, Kilvington presents four opinions characterizing the limits of the capacity of an active power by two kinds, as affirmative (*maximum quod sic, minimum quod sic*) or negative (*maximum quod non, minimum quod non*). Affirmatively, an active power can be determined: 1) strictly by a *maximum simpliciter*, and thus we speak, for instance, about a man being able to lift some weight or to traverse some distance; 2) by a *maximum in ceteris circumstantiis*, and thus we speak, for example, about a man being able to carry a huge weight traversing

14 See C. Wilson, "William Heytesbury...", p. 70.

a small distance or, conversely, being able to carry a small weight traversing a long distance; 3) by a *maximum* in which it has a sensible effect and in any greater it does not. Negatively an active power can be determined (4) by a *minimum in quod non potest*, e. g., by a minimum weight which cannot be lifted or a minimum distance which cannot be traversed.[15] We can thus assign an upper limit of an active potency either by affirmation of the maximum (*maximum quod sic*) or by the negation of the minimum (*minimum quod non*).

Kilvington discusses these opinions at length and raises objections, which are systematically answered. To show that it is impossible to assign a maximum weight that Socrates is able to lift he uses a *reductio ad absurdum* argumentation based on the assumption that an excess of acting power over a resistance is divisible, so the precise amount of such an excess cannot be established.[16] To reject the second opinion it is enough to conclude, says Kilvington, that in this case every active power would be both large and small. To disprove the third opinion Kilvington points out that in the case of the action of simple bodies, such as fire, it is impossible to determine the last instant of their action, since they can act infinitely. Such powers finish their action only because of the patients on which they act; thus if an infinite body had existed, a fire could burn on it eternally.[17]

The most interesting is the discussion of the fourth opinion. In order to repudiate it, Kilvington refers to Averroes, who claims that the incapacity of an active power is determined by the minimum it cannot accomplish. But thus, Kilvington argues, this *minimum quod non* can also be the *maximum quod sic* of the capacity of the active potency. In Kilvington's opinion Aristotle states that a capacity of an active power is determined by a *maximum in quod potest* and its incapacity is determined by a *minimum in quod non potest.* Against this, one can argue as follows: suppose that B is the *minimum quod non* of the incapacity of an active power, and A is the *maximum quod sic* of its capacity. B must be greater than A by a divisible magnitude, let C fall within this divisible magnitude. We can ask then: is the active power able to act upon C or not? If yes, then A is not the *maximum quod sic* of the power, for it can act upon something

15 See Ricardus Kilvington, *Utrum in omni motu...*, § 25, pp. 225–226.

16 See ibidem, § 26, 27, p. 226.

17 See ibidem, § 29–31, pp. 227–228.

more. If it is not, then B is not the *minimum quod non*. Three more logico-mathematical arguments are employed against this opinion.[18]

In order to solve this apparent contradiction Kilvington points out that we can determine both the capacity and incapacity of an active power by the same term, for this limit is the extrinsic upper limit for the capacity and the intrinsic lower limit for the incapacity.[19] Having solved the problem, Kilvington goes back to Aristotle, who talks about sensible or notable differences. An active power is determined by both: an upper limit, a *maximum quod sic*, and by a lower limit, a *minimum quod non*. In order to determine Socrates' power we could observe that he is able to lift 5 pounds (and not sensibly or notably more) or traverse 10 miles (and not notably more).[20] The most proper way, however, to describe a capacity of an acting power is to determine a *minimum quod non*-limit, since every excess of an acting power over resistance is sufficient for motion, and in this case the beginning of motion is not necesserily observed at the outset. With such a conclusion Kilvington can be placed among all those masters who, according to the anonymous author of the *Treatise on maxima et minima*:

> claiming that every excess does suffice to motion, must grant and would grant, that an active capacity is limited by a minimum upon which it cannot act. And this is the resistance equal to the active capacity, because the active capacity cannot act upon that resistance since action does not occur through a proportion of equality.[21]

1.1.2b. How to "Measure" a Passive Power?

Kilvington goes on to discuss and assign the method of determining the limits of passive powers. According to the definition that Heytesbury later adopts: "a passive potency is the one which, inasmuch as it is susceptible to less or can be affected by less, is susceptible to a greater, or

18 See ibidem, § 32–34, pp. 228–229.

19 See ibidem, § 106–107, p. 258.

20 See ibidem, § 108.

21 Anonymous, *Tractatus de Maximo et Minimo*, [in:] J. Longeway, "William Heyetsbury on maxima et minima," p. 78.

can be affected by a greater, and not vice versa."[22] Thus, a passive power is determined by the capacity of an active power to affect it. Again, Kilvington begins this article with a presentation of four opinions concerning the limits of passive power, which may be determined: 1) by the *minimum quod sic* that is by the minimum that can affect it (that means that inasmuch as it is susceptible to lesser, it is susceptible to greater); 2) by the minimum with respect to circumstances; 3) by the *maximum quod non*, i.e., by the greatest power by which it cannot be affected, while it can be affected by a greater one. According to opinion (4): a passive power may be determined by the minimum that is most suitable, e.g., sight is determined by the smallest body it can be seen well.[23] Thus we can assign a lower limit of a passive power either by the affirmation of the minimum or by the negation of the maximum. Kilvington accepts the *minimum quod sic*-limit with respect to the circumstances for a passive power, since – as he rightly notices – it happens that we cannot see not only a small thing, but also a big one, for instance a cathedral, while being placed too close to it.[24] When we ask, however, about Socrates' capacity of good vision, we point out the smallest thing he can scarcely see,[25] and with this Kilvington is in agreement with Aristotle.

Nevertheless, it happens that a weak acting power, such as a drop of water overcomes the resistance of the rock on which it acts. Also a small amount of fire can act through its hotness to overcome the resistance of the coldness of a large amount of water. This happens because every passive power can be affected on its part. This is the fifth opinion, which Kilvington debates at length.[26] In accordance with this opinion, any small capacity of an acting power can cause the division of any great resistance of a divisible thing, since it can divide a part of it and a part of this part and a part of this part and so on *in infinitum*. Now, if such a continuously proportional division were possible, the resistance of the divided thing would become lesser and lesser *in infinitum*. And with that the conclusion that an arbitrarily small active power can act upon a body of any resistance is affirmed. This is observed in the action of drops of water upon rock: although each drop of water divides an

22 C. Wilson, "Wiliam Heytesury…", p. 70.

23 Ricardus Kilvington, *Utrum in omni motu…*, § 36, pp. 229–230.

24 Ibidem, § 114, pp. 259–260.

25 Ibidem, § 113, p. 59.

26 Ibidem, §§ 39–47, 118, pp. 230–233, 260–261.

imperceptibly small part of the rock itself, the final result of the drops' action is a visible hole in this rock.[27]

To sum up, Kilvington indicated most of the issues concerning the problem of setting limits to the powers involved in action-passion processes. Although he did not explicitly formulate general rules concerning different types of division, his debates reveal that he approved the following conditions for the existence of limits: there must be a range in which the power can act or be acted on, and another range in which it cannot act or be acted on. The power should be capable of taking on a continuous range of its intensity between no-power and the value, which serves as a limit, and no other values.[28] Kilvington was aware of two different types of considerations: one type refers to the everyday use of language (*frequentior usus loquendi*) describing real, physical phenomena, and the other type refers to formal, i.e., logico-mathematical language (*virtus sermonis*) dealing with questions within the realm of "mathematical physics".[29] The two ways lead him to accept multiple solutions to the limit decision problem.

1.1.3. The Resut of Action of Powers – Speed of Motion

Having settled the limits for the causes of motion, Kilvington aims at elucidating the result of their action, i.e., the speed of motion. The fourth article of Kilvington's fifth question on the *Physics* contains a lengthy debate on the subject and begins with a review two of the most commonly known opinions of his time, claiming either that the speed of motion is proportional to the arithmetical difference between an acting power and resistance (v ~ F – R) or that it is proportional to the ratio of F to R (v ~ F/R). He criticizes and eliminates these theories as erroneous[30] and finally presents his own solution in a manner typical for him, i.e., confirming some of the conclusions derived from the debates he previously presented. The fourth and last article contains an extensive debate on the proper way of "measuring" the speed

27 Ibidem, § 115–117, p. 260.

28 These terms were well defined later by Heyesbury (see C. Wilson, "William Heytesbury...", pp. 70–72).

29 Ricardus Kilvington, *Utrum in omni motu…*, §§ 114, 122, pp. 259, 261.

30 Ibidem, §§ 50–80, pp. 234–248.

of motion. This is also the main subject of Thomas Bradwardine's treatise *De proportionibus velocitatis in motibus*. This time Kilvington does not mention any opinion of his contemporaries and he begins the debate with the statement that:

> if the question is true, the speed of motion varies either as the difference whereby the power of the mover exceeds the resistance offered by the thing moved or it varies with the proportion of an acting power of the mover over passive power of the thing moved.[31]

These theories state respectively that the speed of motion is proportional to the arithmetical difference between an acting power – a moving force and a passive one – resistance (v ~ F – R) or that it is proportional to the proportion of F to R (v ~ F/R).

Kilvington reviewed the two above-mentioned theories, and he presents his arguments in the light of a solution he offers as the correct one. The same strategy was to be taken up later by Bardwardine. Kilvington composed his questions on the *Physics* in 1326, at the latest, Bradwardine prodused his treatise on motion in 1328. In his question, Kilvington presents the "rule of motion" later repeated by Bradwardine, who benefits from Kilvington's arguments.[32] Therefore while presenting Kilvington's critic I will also refer to Bradwardine's treatise.

While refuting the first opinion (v ~ F – R) Kilvington begins with quotations from Averroes' commentary on the *Physics*, Book IV, com. 71 and 39: *omnis motus est secundum excessum potentiae moventis super rem motam*; *secundum excessum potentiae alterantis super potentiam alterati erit velocitas motus alterationis in quantitate temporis*.[33] Bradwardine also supplements his pres-

31 Ibidem, § 48, p. 233.

32 For detailed discussion see E. Jung, *The New Interpretation of Aristotle*..., (forthcoming).

33 Ibidem, § 49. Averroes *Com. in*, *Phys.*, IV, com., 71, f.161rb–va: "proportio tarditatis ad tarditatem est sicut proportio impedimentis ad impediens ut dicit Avempace (...) et si concesserimus quod proporio motuum, que fuerit in medio, adinvicem est sicut proportio impedimentis ad impedientem, quando idem motum movetur in vacuo, movetur motu indivisibili, et in instanti, sed sequitur necessario ut moveatur in tempore, cuius proportio ad tempus in quo movetur in medio est sicut proportio excessus potentie motoris super rem motam."; Thomas Bradwardine, *Tractatus de proportionibus*..., p. 86, 9–11. Averroes, *Com. in*, *Phys.*,

entation with a quotation from Aristotle's *De coelo et mundo*, Book I and from Book IV of the *Physics*, commentary 35.[34] Then they both say – Kilvington: "But this theory may be refuted in many ways as follows." (*Sed contra istam opinionem potest sic argui multipliciter*); Bradwardine: "The present theory, may, however, be torn down in several ways" (*Haec autem opinio destrui poterit multis modis*).[35]

Kilvington presents the following arguments against this theory:

I). Undoubtedly, the theory challenges Aristotle's and Averroes' rule of motion, which claims that two movers would move two *mobilia*, taken together with the speed equal to the speed with which one of them would move one *mobile*. Suppose that F1 = F2 = 2 and R1 = R2 = 1 then F1 + F2 = 4, R1 + R2 = 2 and, in accordance with the rule: v2 ~ (F1 + F2) – (R1 + R2) = 4 – 2 = 2 while in the case of motion caused by a separate acting mover v1 ~ F1 - R1 = 2 – 1 = 1, thus v1 ≠ v2.[36]

II). The arithmetical proportion is also contrary to the following rule: "half of a given mobile is moved by a half of a given mover in the same time and on the same distance as a whole mobile moved by a whole mover." The given numerical example is the same: suppose that F = 4, R = 2 then F – R = 2 and the whole mover exceeds the whole mobile by 2, while a half of it F = 2 exceeds half of a mobile R = 1 by 1 (½F – ½R = 2 – 1 = 1).[37]

III). Now to demonstrate the flaw in Averroes' theory Kilvington gives examples from the "common experience":

a). "If a second man joins his strength with a single man who is moving some weight that he can scarcely manage with a very slow

VII, com., 39, f. 337va: "secundum excessum potentiae alterantis supra potentiam alterati erit velocitas alterationis et quantitas temporis, in quo est alteratio"; Thomas Bradwardine, *Tractatus de proportionibus*..., p. 86, 14–15: "secundum excessum potentiae alterantis supra potentiam alterati erit velocitas alterationis et quantitas temporis."

34 See Thomas Bradwardine, *Tractatus de proportionibus*..., p. 86, 5–16.

35 Ricardus Kilvington, *Utrum in omni motu…,* § 50, p. 234. Thomas Bradwardine, *Tractatus de proportionibus*..., p. 86, 5–16.

36 Ricardus Kilvington, *Utrum in omni motu*..., § 50, pp. 234–235; Thomas Bradwardine, *Tractatus de proportionibus*, p. 86, 17.

37 Ricardus Kilvington, *Utrum in omni motu*..., § 51, p. 235. Thomas Bradwardine, *Tractatus de proportionibus*, 86–88, 30–55.

motion, the two together can move it much more than twice as fast [than before]."[38]

b). The same principle is quite manifest in the case of weight suspended from a revolving axle, which it moves insensibly during the course of its own downward motion (as is the case with clocks). "If an equal clock weight is added to the first, the whole descends and the axle, or wheel, turns much more than twice as rapidly."[39]

Bradwardine, who also mentions these last two examples employs them in order to refute the other false theory which claims: "with the moving power remaining constant, the proportion of the speeds of motions varies in accordance with the proportions of resistances. And, with the resistances remaining constant that it varies in accordance with the proportion of moving powers."[40]

c). "Suppose that a man carries a weight on some distance, running as quickly as he can. Then if the second man, who runs as fast as the first one, joins him, they both would carry the weight and move with the same speed as before." This means that, despite the acting force being doubled, the speed of motion would remain the same. Consequently, speed does not vary in accordance to the arithmetical proportion between an acting power and a mobile.[41]

d). "When three men try to pull a ship and when their power is not sufficient to move the ship forward they only can turn it over while when a fourth man joins them they could pull the ship in a straight line over a long distance."[42]

e). "A man carrying some weight moves very slowly, and even if there were added to what he carries a quantity less than what he carries, the man will carry the whole twice as slowly and he will move with a great

38 Ricardus Kilvington, *Utrum in omni motu*..., § 51, p. 235. Thomas Bradwardine, *Tractatus de proportionibus*, 98, 287–291. In the case of *verbatim* quotes from Kilvington I use the English translation by Lamar Crosby to be found in "Thomas of Bradwardine and His *Tractatus de Proportionibus*...", see ibidem, p. 99.

39 Richard Kilvington, *Utrum in omni motu*..., § 53, pp. 235–236. Thomas Bradwardine, *Tractatus de proportionibus*..., p. 98, 292–297. English transl. p. 99.

40 Thomas Bradwardine, *Tractatus de proportionibus*, p. 94, 183–186, English transl. p. 95.

41 Ricardus Kilvington, *Utrum in omni motu*..., § 54, p. 236.

42 Ricardus Kilvington, *Utrum in omni motu...,* § 55, p. 236. See also Averroes, *Com. in Phys.*, VIII, com. 23, f. 359ra; Aristoteles, *Phys.*, VIII, 253b15–254a.

difficulty."[43] Hence speed does not vary in accordance to the excess of an acting power over resistance.

IV). Finally, the presented opinion has to be rejected because a geometric proportion (that is, a similarity of proportions) of movers to the *mobilia* would not produce equal speeds, since it does not represent an equality of excesses; for, although the proportion of 4 to 2 and 2 to 1 are the same the excess of the one term over the other is 2 in the first case and 1 in the second case. The conclusion is contrary to Aristotle's and Averroes' statement that equal proportions between movers and mobiles result in equal speed.[44]

The above presented arguments, says Kilvington, convincingly prove that "speed of motion varies in accordance with the proportion of the power of the mover to the power of the thing moved and not with regard to an excess."[45] Bradwardine begins Chapter III with the following sublime words:

> Now that these fogs of ignorance, these winds of demonstration, have been put to flight, it remains for the light of knowledge and of truth to shine forth. For true knowledge proposes a fifth theory which states that the proportion of the speeds of motion varies in accordance with the proportion of the power of the mover to the power of the thing moved.[46]

Then, to support this theory each of them quotes the same paragraphs from Aristotle and Averroes.[47] "From what has been said the second theory derives" (*ex quibus concluditur haec opinio secunda*) – says Kilvington.[48] Next Kilvington presents seven conclusions against this theory, which are also to be found in Chapter II and III of Bradwardine's

43 Ricardus Kilvington, *Utrum in omni motu*..., § 56, p. 236.

44 Ricardus Kilvington, *Utrum in omni motu*..., § 57, pp. 236–237. Thomas Bradwardine, *Tractatus de proportionibus*..., p. 88, 50–58.

45 Ricardus Kilvington, *Utrum in omni motu*..., § 58, p. 237. Here Kilvington gives a supportive argument, which is to be found in Bradwardine's treatises in chapter II, see Thomas Bradwardine, *Tractatus de proportinbus*..., p. 92, 126–132.

46 Thomas Bradwardine, *Tractatus de proportionbus*..., p. 110, English transl., p. 111.

47 Ricardus Kilvington, *Utrum in omni motu*..., § 58, pp. 237–238. Thomas Bradwardine, *Tractatus de proportionibus*, p. 110, 7–26.

48 Ricardus Kilvington, *Utrum in omni motu*..., § 59, p. 238.

treatise. However, since Kilvington's discussion extends over 15 pages,[49] we will present only the main arguments which can be found in both parts (in article IV and in *responsio ad argumenta*). In the footnotes we also give references to Bradwardine's treatise. Kilvington formulates his conclusions as counterarguments against the second theory, Bradwardine presents them in different chapters of his treatise.

Conclusion I. In accordance with the above rules a body twice as heavy would not move in the same medium, with a doubled speed. Suppose that a body moves with its natural downward motion and that the resistance of a medium is exceeded by the moving power (gravity) like 3 to 1; then if we double the gravity of a body (that is, if we double the weight of a body), the power would exceed resistance as 6 to 1, though this does not mean that the speed of motion would be doubled. In fact, if we calculate the proportion properly, says Kilvington, we must arrive at the conclusion that a proportion of 9 to 1 is a double proportion of 3 to 1 while a proportion 6 to 1 is not. A proportion of 6 to 1 is less than 9 to 1 and, if we take for granted Averroes' and Aristotle's theorem, the speed of a body twice as heavy, in the same medium, would not be twice as fast: $v2 < 2v1$ and not $v2 = 2v1$.

We can also prove that in the same medium, the speed of a body twice as heavy might be greater than twice the previous one. Suppose that an acting power exceeds resistance in a proportion of 6 to 4; hence, in accordance with Averroes' theorem, when an acting power is doubled we would have a proportion of 12 to 4, while the proper calculus gives a proportion of 9 to 4, which is a proportion double of 6 to 4, while 12 to 4 is not double the proportion of 3 to 2. Since the proportion of 12 to 4 is greater than 9 to 4 the speed of motion would be greater than the double the previous one.

However, it can be noticed, at first glance, that such a calculation of ratios forces changes of interpretation of Aristotle and Averroes, Archimedes, Euclid and Jordanus Nemorius, who state that speed is proportional to the geometrical proportion between a moving power and a resistance of the body moved. While noticing the contradiction, Kilvington confirms that the value of speed depends on the proportion

49 Ricardus Kilvington, *Utrum in omni motu*..., §§ 60–122, pp. 238–262.

of active power to resistance, and only in a case when the proportion of F : R is 2 : 1 would the doubling of F result in double speed.[50]

Conclusion II: A mobile, e.g., a piece of earth, which moves in water will not move twice as fast in air twice as rare. "This conclusion follows from what has been said above" says Kilvington, since if the proportion of the heaviness of a simple body (F) downward motion to the resistance of a medium (R) is greater than 2 : 1, then in a medium doubly rare, e.g., in air the body would move slower than twice. If F : Rw > 2 : 1, and Ra = ½Rw, then F : Ra < 2v.[51] Thomas Bradwardine says: "If the proportion of the power of the mover to that of its mobile is greater than two to one, when the motive power is doubled the motion will never attain twice the speed."[52] If F : Rw < 2 : 1, and Ra = ½Rw, then F : Ra > 2v.[53] These proposition are against Aristotle and Averroes (*Phys.* IV, com. 71, 72) and Archimedes' *De ponderibus* conclusion III. In Part II, however, Kilvington definitely confirms the validity of these conclusions.[54]

Conclusion III: The following theorems of Aristotle and Averroes are false:

a) "If a given power moves a given *mobile* through a given distance in a given time, the same power will move twice the same mobile through half the distance in an equal time, and through the same distance in twice the time." If F1 = F2 and R1 = 2R2, then t1 = t2 (t-time) if s1 = 2s2 (s-distance), or if s1 = s2 then t1 = 2t2.

b) "If a given power move a given mobile through a given distance in a given time, double the power will move that mobile through double

50 For conclusion and discussion see Ricardus Kilvington, *Utrum in omni motu...*, §§ 62, pp. 239–240.

51 See Ricardus Kilvington, *Utrum in omni motu…*, § 63, p. 241.

52 See Thomas Bradwardine, *Tractatus de proportionibus…*, Theorem IV, p. 112, 64–66, English transl., p. 113. The same situation is when the resistance is halved. See *Descriptive and critical analyses* by L. Crosby, [in:] "Thomas of Bradwardine…", p. 39.

53 See Ricardus Kilvington, *Utrum in omni motu…*,§ 63, p. 241. Thomas Bradwardine, *Tractatus de proportionibus*, Theorem V, pp. 112, 68–71, English trans., p. 113: "If the proportion of the power of the mover to that of its mobile is less than two to one, when the resistance of the mobile is halved the motion will never attain twice the speed."

54 See Ricardus Kilvington, *Utrum in omni motu...*, § 124, p. 262.

the distance in an equal time."[55] If F1 = 2F2, R1 = 2R2, and t1 = t2, then s1 = 2s2.

These theorems are false, says Kilvington, since when the proportion of F1 : R1 = 3 : 1, thus, in accordance with the rule presented above, a moving power does not move a body with a speed v2 = 2v1, but with v2 < 2v1, because if F1 : R1 = 3 : 1 then double F1 (read multiplied by 2) results in the proportion F2 : R2 = 6 : 1, if R1 = R2, and 6 : 1, as it was said, is not the proportion double of 3 : 1, but 9 :1 doubles 3 : 1, and 6 : 1 < 9: 1. On the other hand, when F : R = 3 : 2 then: v2 > 2v1, because if F2 = 2F1, F2 : R2 = 6 : 2, when we multiply F1 by 2, this is not proper calculus, since 9 : 4 doubles 3 : 2, and 9 : 4 < 6 : 2, so v2 < 2v1.[56]

This calculation seems to be confusing, but, what Kilvington does here is the adoption of the new calculus of ratios within Aristotle's old theory: a speed is proportional to a F : R ratio. Thus in both cases v2 is not equal to 2v1. In the first case, speed is proportional to a proportion v2 ~ 2F1 : R1 = 6 : 1 and this does not equal 3 : 1 doubled, and a ratio 6 : 1 is less than 9 : 1. That is why he states that speed v2 < 2v1. In the second case when speed – v1 is proportional to 3 : 2, doubling the ratio would result in the speed v2 > 2v1.

In his comment to the above Conclusion III, Kilvington first declares that these calculations of ratios are contrary to the rules of Aristotle, Aristotle, Archimedes, Euclid and Jordanus, which are valid only in one specific case when the ratio of F : R = 2 : 1, because multiplying F by 2 and doubling 2 : 1 gives the same 4 : 1 ratio.[57] In this case the general "modern" rules meet those of Aristotle since: "if the proportion

55 See Ricardus Kilvington, *Utrum in omni motu...*, § 64, p. 242; Thomas Bradwardine, *Tractatus de proportionibus*, p. 96, 205–209, English transl., p. 97.

56 See Ricardus Kilvington, *Utrum in omni motu...*, § 64, p. 242; Aristotle, *Physics*, VII, 250a1–4, 250a25–28; Averroes, *Com, in Phys.* VII, com. 36, 335va, com. 39, 337va; Thomas Bradwardine, *Tractatus de proportionibus*, p. 96, 215–219.

57 See Ricardus Kilvington, *Utrum in omni motu...*, § 126, pp. 262–263. See also Thomas Bradwardine, *Tractatus de proportionibus*, Theorem III, p. 112, 60–62, English trans., p. 113. Bradwardine refers to Anonymous *De proportionitate motuum et magnitudinum* as a source for his theory: "The author of the *De proportionalitate motuum et magnitudinum* (truly much more penetrating than the others) claims that, of equal straight lines moving in equal times: that which traverses the great area and the greater termini."

of the power of the mover to that of its mobile is two to one, the same power will move half the mobile with exactly twice the speed."[58]

And then he explains how the term 'double proportion' (*proportio dupla*) should be understood:

> To the third conclusion "if a given power moves a given mobile through a given distance in a given time, it can move half of this mobile through double the distance in an equal time", I say that it is not valid. But I say that the Philosopher understands by "half the mobile", a part of the mobile which is in a subdouble proportion (*subdupla proportio*) to the ratio of a whole mobile to its power. And in the other rules Aristotle understands by double power a mover which is in a double proportion to the mobile. With this understanding all rules of motion can be verified. (…) And thanks to this *glosa* one can explain Archimedes', Jordanus's and Euclid's theorem which were quoted to support the contrary theory.[59]

Conclusion IV: "Two heavy simple bodies, one of which moves in water one foot deep and the other in subdouble dense air two feet deep, would move downward with the same speed in the same time, since in both cases the proportion of acting power to the mobile is the same, and thus the speed is the same."[60]

Kilvington rejects this conclusion since in his opinion the resistances of air and water are not intensively the same. He distinguishes an internal (intensive) resistance which depends on the structure of the medium, and external (extensive) which depends on the distance traversed.[61] Bradwardine also distinguishes three ways:

> in which things may be said to be of equal resistance, namely, qualitatively, quantitatively and in both senses at once. Qualitatively, there may be equality of resistance in three further ways: intrinsically, extrinsically (…) Intrinsically, those things are said to

58 See Ricardus Kilvington, *Utrum in omni motu*..., § 126, p. 263; Thomas Bradwardine, *Tractatus de proportionibus*, Theorem II, English transl., p. 113.

59 See Ricardus Kilvington, *Utrum in omni motu*..., § 126, pp. 262–263.; Thomas Bradwardine, *Tractatus de proportionibus*..., p. 100, 323–338.

60 See Ricardus Kilvington, *Utrum in omni motu*..., § 67, pp. 243–244.

61 See ibidem.

be equal resistance which are moved with equal ease by virtue of equal density, rarity and other intrinsic conditions. Extrinsically, those things (…) which are equally resistive by virtue of some external assistance. (…) Applying the preceding distinctions to the present difficulty we should say that the given portions of air and earth are of equal quantitative resistance, but not qualitatively-intrinsically equal…"[62]

Conclusion V: "An object (a heavy mixed body) may fall in the same dense medium faster than another (a pure, i.e., elementary heavy body)." To prove this conclusion Kilvington presents two arguments based on the new calculus of ratios and on Aristotle's and Averroes's statement that a medium can be rarefied *in infinitum*. He also proves that a heavy mixed body may not fall in another medium twice as fast as it moves now, and that a heavy mixed body may fall in another medium twice as fast as it moves now. In his reply to conclusion V he confirms these statements.[63] Thomas Bradwardine presents these conclusions in a shorter version as theorem XI: "An object may fall in the same medium both faster, slower, and equally with some other object that is lighter than itself."[64]

Conclusion VI: "A heavy mixed body will move with equal speed in a medium and in a vacuum." By using the new calculus of ratios, Kilvington offers a possible proof of this statement. In Part II of his question, he presents the final conclusion: *omnia talia mixta proportionalis compositionis aeque velociter moverentur in vacuo*, which is the same as Bradwardine's theorem XII: *omnia mixta compositionis consimilis aequali velocitate in vacuo movebuntur*.[65]

Conclusion VII: "If the speed varies in accordance with the ratio of an acting power to resistance, a heavy pure body may move infinitely fast." To support this conclusion Kilvington shows that in a downward

62 Thomas Bradwardine, *Tractatus de proportionibus*..., p. 120, 223–241, English transl., p. 121.

63 For conslusion and discussion see Ricardus Kilvington, *Utrum in omni motu*..., § 68–73, pp. 244–246; § 129, pp. 263–264.

64 For conslusion and discussion see Ricardus Kilvington, *Utrum in omni motu*..., §§ 74–75, pp. 246–247; §§ 130–132, pp. 264–265.

65 See Ricardus Kilvington, *Utrum in omni motu*…, § 130, p. ?264; Thomas Bradwardine, *Tractatus de proportionibus*..., p. 116, 127–128.

motion the speed of motion of a pure heavy body increases infinitely, since at the beginning of the second part of a distance traversed, a body has to overcome half as great a resistance of the medium, because at the beginning F : R = 2 : 1, and in half the distance F : R = F : ½R = 4 : 1 = $(2 : 1)^2$, and thus a body moves twice as fast; at the beginning of the third part F : R = $(2 : 1)^3$, etc., thus before it attains its "natural" place it will move with infinite speed. In Part II of his question, Kilvington affirms that we can say that a body might move with infinite speed when the term 'infinite' is understood syncategorematically, which means that a body would move twice as fast and as three times and four times and *sic in infinitum*, since there is no *maximum quod non-* limit for the speed of motion. Moreover, since there is no proportion between the speed of accelerated motion and a uniform motion, one can say that sincategorematically taken the speed of accelerated motion is infinite in comparison to uniform motion.[66]

To sum up, each of the mentioned thinkers, Kilvington and Bradwardine, claim that every local motion is effectuated with a ratio of a greater inequality between active and passive factors. Each of them maintains that his theory is only a new interpretation of Aristotle and Averroes's statements, and in order to convince their readers they both quote the same fragments, which then they analyze and criticize. They both also use the same way to justify their theories: they criticize the opinion they disagree with from the point of view of their own, and here deemed as correct, solution to the problem. The appropriate way of measuring speed of motion *tamquam penes causam* is to state that it follows the ratio of F to R. Kilvington is aware that the use of Euclid's definition of operations on proportions necessitates a new interpretation of Aristotle's and Averroes' rules of motion. On the one hand, in Euclid's and Archimedes' theory of proportions doubling a ratio means 'compounding' it with an equal ratio, which effectively corresponds to multiplying it by itself, or squaring the fraction we form from the ratio to use our modern terms. On the other hand, Aristotle's and Averroes' statements clearly point to a proportion between an active power and resistance, which is not squared but simply multiplied by two. Having noticed the contradiction of these two views, Kilvington first presents two main arguments against the Aristotelian proposition and finally concludes

66 For conclusion and discussion see Ricardus Kilvington, *Utrum in omni motu…*, §§ 76–80, p. 248; §§ 133–135, pp. 265–266.

that while talking about a power moving one half of a mobile Aristotle means precisely a double ratio between F and R; when talking about a power moving a mobile twice as heavy Aristotle means the resulting resistance when compared with that power gives a "half" of the initial ratio (*medietas proportionis*). The general mathematical rules correspond to those of Aristotle only in one case: if the ratio of the power of the mover to that of its mobile is two to one, the same power will move half the mobile with exactly twice the speed. Kilvington's calculus provides values of the ratio of F to R greater than 1 : 1 for any speed down to zero, since any root of a ratio greater than 1 : 1 is always a ratio greater than 1 : 1. And, with the additional assumption, he accepts, that any excess, however small, of an acting power over resistance is sufficient to initiate motion and to continue it, and here he might have described a very slow motion with a speed greater than 0 and less than 1 ($0 > v < 1$). Hence, he avoids a serious weakness of Aristotle's theory, which cannot explain the mathematical relationship of F and R in very slow motions, when speed is lesser than 1. Kilvington introduced Johannes Campanus of Novara's theory of compounding ratios in the interpretation of Aristotle's laws. The main, and possibly the most important, advantage is the concept of the "halved" proportion (*proportio subdupla*). On the basis of Campanus' statements, to "halve" any ratio is to find its "middle" ratio (*medietas proportionis*), that is a ratio that when multiplied by itself will give the initial ratio. In our, modern terms, to "halve" a ratio means to find its square root. Such an understanding of "halving" ratios guarantees the mathematical consistency of the description of a continuously diminishing motion. Since, when it is first assumed after Aristotle that for a motion to start and continue a force must exceed the resistance, even if the initial ratio of these two factors is successively "halved", in the resulting ratios the force will be always greater than the resistance.[67] What is more, adopting such an interpretation of the relations between changes in the factors of motion and the effects, i.e., changes of speed in local motion allows one to explain why such a motion cannot occur when the motive force equals the resistance, even though their ratio is equal to a 1 : 1 ratio (*proportio equalitatis* in medieval terms), having thus a "positive" value.

67 E. Jung, R. Podkoński, *Richard Kilvington on Proportions*, pp. 91–94. It is obvious that if we take any ratio $F : R > 1 : 1$, then always $\sqrt{(F : R)} > 1 : 1$.

As has been already clearly shown, Richard Kilvington was perfectly aware that the proper understanding of Euclid's definition of a double proportion necessitates a new interpretation of Aristotle's and Averroes' "rules" of motion, which reveals that the speed cannot be described by sole multiplication. On the new interpretation "double" the ratio of 3 : 2 (or the ratio 3 : 2 *duplicata*) was the ratio 9 : 4, not 6 : 2, and the ratio of 3 : 1 *dupla* or *duplicata* was equal to 9 : 1, not 6 : 1. It seems, however, that Kilvington did not realize the significance of his discovery. It was Thomas Bradwardine, who was the greatest beneficiary of Kilvington's work and, who, with great skill used his arguments to formulate the New Rule of Motion.

In the presented question on local motion, Kivington also discusses the problem of accelerated downward motion giving five possible explanations of this phenomena. He mentions the following causes which generate the increasing of velocity in downward motion: 1) diminishing of the resistance of the medium which has to be overcome in a continuous downward motion; 2) approaching the natual place; 3) continuity of motion; 4) constant weight gain (the closer to a natural place, the heavier the body); 5) the pressure of the medium above the moving body. Kilvington claims that the primary cause of acceleration in a downward motion is the dimishing of the resistance of the medium to be overcome, but also the increasing pressure of the medium can play an important role as well as the continuity of motion.[68]

In the dynamic aspect of motion, when the speed is proportional to the F to R ratio, we can only determine its value in an instant or describe uniform motion with equal speed in every instant but not the successive changes of speed in time. In order to characterize the changes of the speed of motion one must analyze the problem of local motion in its kinematic aspect.

1.2. Motion with respect to its Effect – the Distances Traversed and Time

Kilvington's attempt to understand the effects of motion as caused by smaller and greater resistances brings him to a distinction, posed also by

68 See Ricardus Kilvington, *Utrum in omni motu*..., §§ 81–89, pp. 248-251; §§ 136–138, p. 266.

Bradwardine, between its intensity, i.e., the rarity and density of a medium, that are responsible for the faster or slower speed of motion, and its extent, determining the longer or shorter time consumed in motion.[69] Kilvington correctly recognizes that to measure the speed of a uniform motion that lasts some time, it is enough to establish a relation between time and the distance traversed. In his opinion the same distances traversed in equal intervals of time characterize uniform motion. Accelerated motion is described by the same distance traversed in shorter and shorter intervals of time, and decelerated motion is characterized by the same distances traversed in longer and longer intervals.[70] It is also possible to describe difformly difform motion by, for example, unequal distances traversed in unequal interval of times.

Kilvington applies his rules of local motion in order to describe both natural and violent motions when debating its various kinds, such as the uniform and difform motion of a mixed body and the motion of a simple body in a medium and in a void. For as overcoming the resistance with a moving power is the necessary condition for motion and herein as established by Aristotle, this became the indispensable factor medieval natural philosophers were looking for while describing different types of motion. The simplest motion to be described is the violent motion of a mixed body in a medium, when the acting power has to overcome the external resistance of a medium as well as the internal resistance of the element being moved away from its natural place. The local motion of a simple body in a medium is not problematic either, since it can be explained by a natural desire to attain the natural place caused by the "heaviness" (*gravitas*) or the "lightness" (*levitas*) of that body and the external resistance of a medium. Nor does Kilvignton have a problem with the explanation of the natural motion of a mixed body in a vacuum, which is caused by the lightness and the heaviness of the elements constituting this body. Since in a void there is no external resistance, there is only internal resistance left to be overcome. The temporal motion of a mixed body in a void is a result of the natural

69 For Kilvington's discussion of the problem see Ricardus Kilvington, *Utrum in omni motu...*, § 63–89, pp. 241–251; Thomas Bradwardine, *Tractatus de proportionibus...*, pp. 120–122; E. Sylla, *The Oxford Calculators*, p. 293, n. 175.

70 See, E. Jung, *The concept of time*, pp. 196–202. See also J. Murdoch *Infinite Times and Spaces in the Later Middle Ages*, „Miscellanea Mediaevalia" 1998, Bd. 25, pp. 194–204.

inclination of heavy or light elements to direct to their natural places, since in the void the qualities of natural places are the same as in a medium: light elements strive to their natural places upwards while heavy elements downwards. The heaviness or lightness, respectively, play the roles of an acting power and resistance. Although there would be no external resistance in a void, the motion of a mixed body could occur without any contradiction.[71]

The most perplexing explanation concerns the temporal motion of a simple body in a void.[72] In the opinion of Averroes a simple body, e.g., a clod of earth, has an elementary form, prime matter and different quantitative parts, since, being a continuum, it can be divided into parts. Because form cannot resist matter, the resistance cannot come from the qualitative parts. The resistance, however, can come from quantitative parts resisting one another.[73] Kilvington maintains that the temporal motion of a simple body in a void is made possible by internal resistance that results because the peripheral parts of the simple body are more distant from their natural place, i.e., the centre of the Universe, than its central parts. It is obvious, since the line drawn perpendicularly from the centre of a body to the centre of the Universe is always the shortest one, compared to the oblique lines drawn from the centre of the Universe to the other parts of the same body. What is more, the peripheral parts of this body are forced to move parallel to that shortest line, which means their movement is not wholly "natural", yet – at least partially – violent, which makes them resist. Such an internal resistance promotes motion and does not impede it; nevertheless it guarantees temporal mo-

71 See E. Jung[-Palczewska], *Motion in a Vacuum and in a Plenum*, pp. 186–189.

72 Ibidem, pp. 190–192.

73 Richard Kilvington, q. *Utrum corpus simplex possit aeque moveri in plenu et in vacum*, Ms. S. Marco IV, 72 (2810), f. 103ra: "Et ad dictum Commentatoris III et IV *De coelo*, (...) dico quod in corpore simplici non est aliquid (...) preter formam nisi materia et partes quantitative. Et materia non resistit (...) tamen corpus simplex in vacuo resistit sue forme moventi ipsum et etiam eius partes quantitativas, ut post modum patebit etc. (...) Et hic intelligit <Commentator> (Averroes, *Phys.* IV, com., 71, 161va) quod materia prima non resistit forme. Quod tamen totum compositum potest resistere forme non vult dicere et ideo elementum non dividitur in per se movens et in per se motum totalis et distincte a motore sicut animalia, quorum una pars est precise movens et reliqua per se mota. Et hoc loquendo de parte qualitativa".

tion.[74] Consequently, if a void had existed, the natural motion of a simple body would be possible. Moreover the speed of such a motion would be the fastest, compared to the corresponding motion in any, equally great medium, since there is no resistance to be overcome.

To sum up, local motion can occur everywhere if only the active power is greater than the external or internal resistance, or both. The speed of motion is properly described by the continous geometrical proportion of active powers to resistive ones. If one classified motion with regard to its speed, it could be concluded that the fastest motion is the motion of a simple body in a void, while the speeds of other motions would depend on the ratios of F to R, where R corresponds to a total resistance – internal and external.

2. Thomas Bradwardine's Treatise on Local Motion

Thomas Bradwardine, in his *Tractatus de proportibus* following on from William of Ockham, pointed out that a "proportion" (*ratio*) taken in a strict, narrow sense concerns quantities and can be applied only within the sciences that deal with quantities. In a wide, general sense, however, one can determine ratios or proportions between all beings or qualities that can be taken as equal, greater or smaller, or simply similar; there can be fewer or more of them, they are more or less intense, etc. In order to validate the use of the Euclidean-Eudoxean theory of geometrical proportions within the science of local motion, Bradwardine argues further that if determining proportions between forces and resistances would be methodologically incorrect, then establishing proportions between sounds and tones in music should be seen as wrong

74 For the quotes from Kilvington's question see E. Jung[-Palczewska], *Richard Kilvington on local motion*, pp. 192–193, n. 55–58. It seems that Kilvington's solution of the problem was inspired by Robert Grosseteste, who claims that while falling down the parts of a body do not move along the same line, but those which are in the center of a body move along the straight line and those which are not central move along oblique converging lines and resist the former. The different angles of fall in the motion are caused by different "desire" of elements in a body. See R. Grosseteste, *De calore solis*, L. Baur (ed.), [in:] L Baur, "Die Philosophie des Robert Grosseteste, Bishofs von Lincoln", Beitrage zur Geschichte der Philosophie des Mittelalters, Bd 18, H. 4–6, Munster 1917, p. 79; P. Duhem, "Le system du monde", vol. 8, Paris 1918, p. 68.

too. Consequently, harmonics should not be considered a science in the proper meaning.[75]

Bradwardine's treatise is still commonly recognized as the basic source for the whole Oxford Calculators' tradition in the development of mathematical natural philosophy. The treatise was presumably planned from the outset and executed as a systematic handbook, presenting step by step the proper, that is mathematically and logically consistent, interpretation of Aristotelian rules of local motion. Preserved are at least thirty three manuscript copies of this work, not mentioning two early printed editions from the turn of the sixteenth century.[76] Besides the full version of this treatise, there appeared in the later Middle Ages its abbreviated version, entitled simply: *Tractatus brevis de proportionibus abbreviatus ex libro de proportionibus de Thomae Braguardini Anglici*, and the commentary to Bradwardine's *De proportionibus* written by no less famous a person than Bradwardine, the Parisian philosopher Albert of Saxony (*ca.* 1316—1390), who in fact produced a paraphrasis of Bradwardine's treatise.[77]

Thomas Bradwardine was himself a mathematician, philosopher and a famous and influential theologian. His interpretation of Aristotle's "rules" of motion as included in the above-mentioned treatise is still commonly called "Bradwardine's law" in the secondary literature.[78] Obviously, this reinterpreted law always has been recognized as his own invention, since at the very beginning of his work Bradwardine stated firmly that nobody before him had scrutinized fully and adequately the

75 See Thomas Bradwardinus, *Tractatus de proportionibus*, p. 66, 108–110.

76 See H. Lamar Crosby, *Introduction*, p. 9; J.A. Weisheipl, *Repertorium Mertonense*, "Medieval Studies" 31 (1969), p. 180.

77 See Thomas of Bradwardine, *Tractatus de Proportionibus…*, p. 184, note 46; E.D. Sylla, *The Origin and Fate of Thomas Bradwardine's* De proportionibus velocitatum in motibus *in Relation to the History of Mathematics*, [in:] "Mechanics and Natural Philosophy before the Scientific Revolution", W.R. Laird, S. Roux (eds), Dordrecht 2008, 97. On Albert of Saxony biography and works, see Biard, Joél, "Albert of Saxony", *The Stanford Encyclopedia of Philosophy* (Spring 2019 Edition), Edward N. Zalta (ed.), URL = <https://plato.stanford.edu/archives/spr2019/entries/albert-saxony/>.

78 See J.A. Weisheipl, *The Interpretation of Aristotle's Physics and the Science of Motion*, [in:] "The Cambridge History of Later medieval Philosophy. From the Rediscovery of Aristotle to the Disintegration of Scholasticism 1100–1600", N. Kretzmann, A. Kenny, J. Pinborg (eds), Cambridge 1982, pp. 533–535.

important yet very difficult problem of the proportions of speeds in local motion.[79] Nevertheless, as is shown above it was Richard Kilvington who first reinterpreted Aristotelian "laws" of local motion introducing the calculus of ratios (*calculationes*) in such a way that these rules became mathematically and logically consistent; and thus giving the first impetus for the whole Oxford Calculators' method of dealing with natural philosophical issues.

Thomas Bradwardine's *Tractatus de proportionibus* is divided into four chapters, the first of which is devoted to a detailed presentation of the Eudoxean-Euclidean theory of proportions.[80] In the second chapter, Bradwardine presented and criticized four previously formulated theories concerning the relations between factors of motion. The criticism of the first two theories – as is shown above – Bradwardine's repeats after Kilvington.

The third part of Thomas Bradwardine's treatise, where he present his own solution, commences with the boastful *dictum* mentioned above.[81] And next, twelve successive theorems are formulated; the first of which – being rather an axiom – introduces continuous "geometrical proportionality" as the proper description of the relations between forces, resistances and speeds in motion, as follows: "The proportion of the velocities of motions follows the proportion of the force of the mover to that of the moved."[82]

The remaining eleven theorems are drawn as the combination of the above statement together with the mathematical theory of the proportions presented in Chapter I of the same treatise in a "geometrical" manner modelled obviously on the methodology adopted in Euclid's

79 Thomas Bradwardne, *Tractatus de proportionibus*, p. 64: "Omnem motum succesivum alteri in velocitate proportionari contingit; quapropter philosophia naturalis, quae de motu considerat, proportionem motuum et velocitatum in motibus ignorare non debet. Et quia cognitio illius necessaria est et multum difficilis, nec in aliqua parte philosophiae tradita est ad plenum, ideo de proportione velocitatum in motibus facimus istud opus".

80 For the detailed description of this part of Thomas Bradwardine's *Tractatus de proportionibus*..., see. H.L. Crosby, *Introduction*, pp. 18–31.

81 See above, p. 71.

82 As translated by H. Lamar Crosby in *Introduction*, p. 38. Thomas Bradwardine, *Tractatus de proportionibus*... p. 110: "Aequalitas proportionis motorum ad mota est prima et praecisa causa aequalitatis velocitatum in motibus, igitur ad variationem istius causae primo sequitur variatio proportionis in motibus".

Elements. Theorems II and III, proven on the basis of the above statement and Campanus's aforementioned definition of the continuous proportion are aimed at demonstrating the validity of both the first "rules" of motion presented by Aristotle when the ratio of force to resistance initially equals the double ratio (*proportio dupla*, 2 : 1). The next four theorems – i.e., Theorem IV to VII – again based on the above statement and two other conclusions from the first chapter, consequently concern the cases when although the motive power will be doubled or the resistance reduced by its half, the resulting speed will not equal two times the initial speed.[83] With Theorem VIII, Bradwardine proved that motion cannot occur when the motive force equals or is lesser than the resistance.[84] Again, this is proven on the basis of the mathematical conclusions formulated in the first chapter of Bradwardine's treatise and the above-quoted first theorem of the chapter herein presented.[85] The ninth theorem in this chapter aims to show that any excess of the motive force over resistance is enough for motion to commence and last.[86] In turn, the tenth theorem, in a sense conclusive for the former, can be seen as a generalization of the "new rule of motion." It reads that: "For any given motion it is possible to contrive a motion that is twice faster and a motion that is twice slower."[87]

83 Thomas Bradwardine, *Tractatus de proportionibus*..., p. 110: "Quarta conclusio: Si potentiae moventis ad potentiam sui moti sit maior quam dupla proportio, potentia motiva geminata motus eiusdem duplam velocitatem nequaquam attinget. (...) Quinta conclusio: Si potentiae moventis ad potentiam sui moti sit maior quam dupla proportio, eadem potentia movente medietatem eiusdem moti velocitas motus nullatenus fiet dupla. (...) Sexta conclusio: Si potentiae moventis ad potentiam sui moti sit minor quam dupla proportio, dupla potentia movente idem motum, motus ultra duplam velocitatem excrescet. (...) Septima conclusio: Si fuerit potentiae moventis ad potentiam sui moti minor quam dupla proportio eadem potentia movente medietatem eiusdem moti motus ultra duplam velocitatem transibit". See also H.L. Crosby, *Introduction*, p. 39.

84 Thomas Bradwardine, *Tractatus de proportionibus*..., p. 114: "Ex nulla proportione aequalitatis vel minoris inaequalitatis motoris ad motum sequitur ullus motus".

85 See H.L. Crosby, *Introduction*, pp. 39–40.

86 Thomas Bradwardine, *Tractatus de proportionibus*..., p. 114: "Omnis motus ex proportione maioris inequalitatis producitur et ex omni proportione maioris inaequalitatis potest fieri motus"; H.L. Crosby, *Introduction*, p. 40.

87 Ibidem: "Quocumque motu dato, potest motus in duplo velocior et motus in duplo tardior inveniri."

With this theorem Bradwardine proved that for any given motion speed it is possible to find such a specific ratio of its factors that when "doubled" will cause this speed to rise twice, and when "halved," the speed will decrease to a half of the initial speed.[88] This theorem, in fact, expresses the inevitable consequence of adopting the continuous proportion to describe changes in local motions, namely the division of such motions into distinct "species" according to the initial ratio of the motive force to resistance. On the grounds of this theory these distinct "species" are mutually incommensurable, since no speed resulting from any successive "doubling" or "halving" of the given initial ratio of the motive force to resistance will be equal to any speed resulting from the other ratio, for as long as any of these ratios is not the integer multiple of another.[89]

The last two theorems included in the third part of Thomas Bradwardine's *Tractatus de proportionibus*, namely the XIth and the XIIth concern the local motion of bodies that are characterized by some "internal resistance," that is bodies that are "mixed" (*mixta*) from the elementary matter in such a way that they feature contrary qualities – in the case of local motion "heaviness" (or "gravity": *gravitas*) and "lightness" (or "levity": *levitas*) are of significance here.[90] One of the objections raised by Bradwardine himself when discussing these is here of some significance, since it leads him to distinguish between a "qualitative" and "quantitative" description of motion in terms of proportions.

In short, he argued, that if equal speeds are the effects of equal ratios of the motive force to resistance then, if a large quantity of earth mov-

88 See H.L. Crosby, *Introduction*, p. 40.

89 See S. Drake, *Bradwardine's function...*, pp. 54–60. Simply speaking, any speed resulting from the ratio, say 3 : 2 will never be equal to any speed resulting from a 4 : 1 ratio or a 2 : 1 ratio.

90 Thomas Bradwardinus, *Tractatus de propotionibus...*, p. 114–116: "Quantumcumque gravius alio in eodem medio tardius et velocius illo et aequali velocitate potest descendere. (…) Omnia mixta compositionis consimilis aequali velocitate in vacuo movebuntur". Interestingly enough, the last theorem concerns the movement of mixed bodies in a void space, presenting the solution contrary to Aristotle's statements. In the Book IV of his *Physics*, in the chapter presenting arguments against the existence of a void space, Aristotle argued that a motion in such a space would be instantaneous for any body, independently of its weight, since there will be no resistance of a medium (see Aristotle, *Physics*, Bk. IV 215b12–216a26).

ing downwards through air has the same ratio to a large quantity of this medium as a small quantity of earth to a small quantity of air, the speeds of these two quantities of earth should be equal. But this is impossible – argues Bradwardine – since the larger piece of earth, being bigger in quantity, will traverse more than a smaller piece in the same time and medium. Consequently, their speeds must be unequal.[91] In answering this dilemma Bradwardine explained that in a "qualitative" description of motion, analogically to a modern "dynamical" description, the ratio of the active and passive forces is taken into account, while in a "quantitative" description – a "kinematical" one – relations between distances and times are considered. Nevertheless, he gave no hint as to how these two descriptions are related to each other.[92]

The last and fourth part of his *Tractatus de proportionibus* Bradwardine devoted to the description of circular motions and, finally, to the question of the dimensions of the last sphere of the created world, as well as its internal, elementary spheres.[93] The issues discussed by Bradwardine in this chapter were later also deliberated over by William Heytesbury and the Anonymous author of the treatise *De sex inconvenientibus*.[94]

3. William Heytesbury's Contribution to the Oxford Calculators' Science of Local Motion

William Heytesbury's *De motu locali*, a chapter of his *Regulae solvendi sophismata*, is perhaps the best known work originating from the Oxford Calculators' school, at least among historians of science. This is a result of the famous "mean speed theorem" therein formulated: the theorem that determinates the mean speed of a motion that is uniformly accelerated or uniformly decelerated in a given time period. In fact Heytesbury provided there also the second version of this theorem, relating to the

91 See Thomas Bradwardine, *Tractatus de propotionibus...*, pp. 116–120; H.L. Crosby, *Introduction*, p. 44.

92 See ibidem.

93 See Thomas Bradwardine, *Tractatus de propotionibus...*, pp. 124–140; H.L. Crosby, *Introduction*, pp. 45–48.

94 See below, pp. 93, 101–104.

distances traversed in the first and the second half of the duration of such a motion, when one of its limits marks a state of rest. Both these theorems will be presented in detail below, but first let us point to the fact that *Regulae solvendi sophismata* were, most probably, finished in the year 1335, that is only about seven years after Bradwardine's *Tractatus de proportionibus* and around ten years after Richard Kilvington's *Commentary on the "Physics"*.[95] This fact alone clearly demonstrates the level of interest and the impetus that was given to this new method of scientific inquiry, namely the *calculationes*, within the intellectual milieu of Oxford University during the first half of the fourteenth century. Here it is worth mentioning that Swineshead's monumental "Book of calculations" was finished no later than five to fifteen years after Heytesbury's *Regulae*.[96]

In the last, sixth chapter of the *Regulae solvendi sophismata*, commonly quoted as *De tribus predicamentis*, Heytesbury shifted his attention to the problem of the "measurement" of speed in every type of the continuous changes enumerated by Aristotle, i.e., in local motion, with respect to augmentation, and to alteration.[97] The first type of continuous change discussed here is local motion, since, according to Heytesbury's own words: "local motion by nature precedes other kinds [of change] just as the first."[98]

Subsequently he distinguished between uniform and difform motions, defining the former as those in which equal distances are traversed in equal periods of time continuously, being thus characterized with a constant intensity, i.e., speed.[99] With regard to difform motion, he limited himself here to the statement that such a motion can change in infinitely many ways both in respect to distance, as well as with respect to time.[100] In successive paragraphs, however, he introduced a more elaborated distinction of difform motions, namely into uniformly difform ones (*uniformiter difformia*), that is – in our terms – uniformly ac-

95 See chapter I, pp. 20–22.

96 See ibidem, p. 31.

97 See Aristotle, *Physics*, Bk. V, 226a24–226b9; Guilelmus Heytesbury, *Regulae solvendi sophismata: De tribus praedicamenti: De motu locali* (Editions), § 1, p. 269.

98 Ibidem, § 2, p. 270.

99 Ibidem, § 3, p. 270.

100 Ibidem, § 5, p. 270.

celerated/decelerated motions, and those that were difformly difform (*difformiter difformia*).[101]

Obviously, in these accounts we will not find the notions of 'acceleration'/'deceleration.' In describing uniformly difform motion, Heytesbury referred instead to the concept of "latitude" (*latitudo*) that medieval natural philosophers had adopted from the works of Avicenna. Usually, in their discussions, the values of all "latitudes" were expressed in arbitrarily established "degrees" (*gradus*). With regard to a given quality the *latitudo* could be used both to determine its "absolute" intensity from the "no-grade" (*non-gradus*) of this quality – simply, its absence in a considered subject, as well as its "relative" intensity, calculated from the initial to the final degree in the course of the described qualitative change. Consequently, for medieval thinkers, when some quality changed continuously from a degree equal to one to a degree equal to three through, obviously, a degree equal to two, the increases from one to two and from two to three were seen as equal and unequal to each other at the same time. Arithmetically the differences in both cases are the same, but, when understood "physically," these changes are different, since there is more "intensity" introduced into the subject when the quality, say of heat, changes from the degrees two to three than from one to two.[102] However, with regard to the speed in uniformly difform motions, William Heytesbury took into account only the arithmetical differences between the degrees of its "intensity," saying simply that in such motions equal "latitudes" are acquired or lost in equal periods of time.[103]

On the basis of these statements he formulated perhaps the most famous theorem commonly associated with fourteenth century Oxford natural philosophy, namely the "mean speed theorem." In his own formulation it reads as follows:

> Certainly, every latitude [of speed] that is acquired or lost uniformly, whether commencing from a non-degree or from any degree, as long as it is limited to some finite degree, it corresponds exactly

101 Ibidem, §§ 21–23, p. 275.

102 For a more detailed account on the concept of "latitude" and its use in the later Middle Ages see, E.D. Sylla, *Medieval Quantification of Qualities: the 'Merton School'*, "Archives for History of Exact Sciences" 8(1971), pp. 7–39.

103 See ibidem.

> (*equaliter*) to its own middle degree in such a way, that a moving body that acquires or loses uniformly its speed in any given time will traverse the distance perfectly equal to the one it would traverse in the same time when moving with [a speed corresponding to] this middle degree.[104]

Next Heytesbury provided the mathematical proof of this theorem together with a corollary that when a speed will be continuously uniformly acquired from rest, that is from a no-degree (*non gradus*) to employ his nomenclature, for a given period of time its mean speed will correspond to exactly half of the final speed.[105] The more interesting, and in a sense more important corollary formulated here is the one where Heytesbury presents the calculations of the distances traversed in the first half and in the second half of a duration of a motion that is uniformly decelerated to a rest:

> Certainly, the whole motion in a whole [given] time will correspond to its middle degree, namely to the one it will gain in the middle instant of this time, and the second half of this motion will correspond to the middle degree of the second half of this motion that equals one fourth of the degree that is the [initial] limit of this latitude. Therefore, since this half [of motion] will last for half the time, exactly one fourth will be traversed during this second half compared to [the distance] traversed thanks to the whole motion. Consequently, of the whole distance traversed with the whole motion three fourths will be traversed in the first half of this whole

104 William Heytesbury, *De motu locali*, § 26, pp. 276–277. Obviously, Heytesbury took into account the uniformly accelerated/decelerated motion, but since he did not use these notions, we are not introducing them here, in order to avoid anachronistic interpretations that could, eventually lead to some false conclusions regarding the conceptual apparatus of the Oxford Calculators. For example, notwithstanding such consequences Ernest Moody had presented the following translation/paraphrasis of this passage: "The moving body which is accelerated uniformly during some assigned period of time, will traverse a distance exactly equal to what it would traverse in an equal period of time if it were moved uniformly at its mean degree of velocity" (see E. Moody, *Laws of* Motion..., p. 195). It is worth noting that William Heytesbury unambiguously identified the state of rest with the lack or "no-degree" of speed.

105 See Guilelmus Heytesbury, *De motu locali*, § 29, pp. 278–279.

> motion and the last fourth will be traversed in the second half of that [motion].[106]

On the basis of the above reasoning, Heytesbury formulates the general rule that should be recognized, in our opinion, as the second formula of the "mean speed theorem":

> with this kind of uniform increase or decrease of a motion from some degree to no-degree, or from no-degree to some degree, exactly three times more will be traversed with the more intense half of this latitude than with the one less intense (*per … intensiorem quam remissiorem*).[107]

It is worth noting that with the above formula Heytesbury determines the ratio of the distances traversed in the uniformly difform motion relative to its intensity presumably taken "physically," that is in a different sense than he did within the context of the first formula of the "mean speed theorem", where he considered only the "quantitative" relations between the distances traversed in such a motion.

Even though Heytesbury provided a simple and effective algorithm for calculating the ratio of such distances relative to any initial and final degrees of the latitudes of speed, it is symptomatic that he finally stated that: "such a calculation would be more troublesome than useful."[108] This remark, though rather surprising on first glance, assures us in fact that for Heytesbury, as well as for all Oxford natural philosophers contemporary for him, all considerations on the "rules" of local motion were conducted only on the theoretical level, with no reference to everyday or practical applications. On the one hand, such an attitude is of course a consequence of the acceptation of Aristotle's division of sciences, where "physics" is purely a theoretical science.[109] On the other

106 Ibidem, § 36, pp. 281–282.

107 Ibidem.

108 Ibidem, § 38, p. 283.

109 See Aristotle, *Metaphysics*, Bk. VI, 1025b19–22. Since natural science, like other sciences, is in fact about one class of being, i.e. to that sort of substance which has the principle of its movement and rest present in itself, evidently it is neither practical nor productive. See also, E.D. Sylla, *The Oxford Calculators and the Mathematics of Motion 1320 –1350*...., p. 42.

hand, as it seems, it is hard to find or imagine any reason for which fourteenth-century people would desire or need to know the "real" speed of anything expressed in units analogical to those we are used to.[110] This is further confirmed by the fact that all the Oxford Calculators, when pondering mathematically the relations between factors and speeds in local motions used only relative terms, like "faster" (*velocius*), "slower" (*tardius*) or "equally fast" (*eque velociter*).

Finally, with regard to difformly difform motions, that is those that cannot be characterized by uniformly increasing/decreasing speed, William Heytesbury stated authoritatively that with regard to such motions no rule can be formulated.[111] As it seems, this statement was later taken up by Richard Swineshead as a kind of intellectual challenge, since in his treatise "On local motion" included in the "Book of calculations" he formulated some rules that concern a specific kind of difformly difform motions, namely those characterized by a speed increasing/decreasing "faster and faster" or "slower and slower." In our modern terms, he was referring here to motions with an uniformly increasing/decreasing acceleration.[112]

The mentioned treatise "On local motion" by Swineshead – as we have already noted above – should be seen as the final stage in the development of the science of local motion within the Oxford Calculators' school. Richard Swineshead, however, limited himself there only to those considerations "with regard to cause." In both the short trea-

110 Such an interest, but again only on a purely theoretical level, arose among medieval natural philosophers with regard to the speed of light or other such "spiritual" qualities. Actually, the question was whether such qualities are propagated in any medium instantaneously, i.e., with infinite speed, or not. See e.g., John Dumbleton, *Summa logicae et philosophiae naturalis*, ms. Cambridge, Gonville & Caius 499/268, f. 69rb: "Ulterius dubitatur in presenti utrum agentia spiritualia agunt succesive vel subito"; ibidem, 70ra: "Item, potest argui, quod aliquid potest infinite (ms.: infinitum) velociter moveri per tempus".

111 See William Heytesbury, *De motu locali.*, § 40, p. 284. Further on Heytesbury presents four exemplary sophisms with their solutions that are based on the formerly presented conclusions and rules (see ibidem, §§ 42–63).

112 See Ricardus Swineshead, Richard Swineshead, *Liber calculationum*, *Tractatus de motu locali*, [in:] R. Podkoński, "*Suisetica inania*. Ryszarda Swinesheada spekulatywna nauka o ruchu lokalnym", Concl. 52, § 154, p. 335; Concl. 53, § 157, p. 338; Concl. 54, § 159, p. 339; Concl. 55, § 161, p. 339; Concl. 57, § 166, p. 340; Concl. 58, § 168, p. 341. See also, R. Podkoński, *Suisetica inania*, pp. 115–122.

tises on local motion commonly ascribed to this philosopher, he also introduced the description "with regard to effect." Some of the more or less explicit statements in these treatises lead one to the conclusion that they were composed before the *Liber calculationum* but after the "Rules for solving sophisms," with Richard Swineshead's considerations being clearly inspired by the latter.

4. The Theory of Motion in the Anonymous Treatise: *De sex inconvenientibus*

At the beginning of Chapter IV of *De sex inconvenientibus* where the discussion is of methods in determining the speed of local, presented in the question: *Utrum in motu locali sit certa servanda velocitas*, the author, as it is customary in the other questions of this work, presents three opinions. This time, however, he first presents the difficulties (*inconvenientia*) that are to speak against the third position, with which he agrees. Because the author's arguments will be easier to understand when we have first presented the initial two opinions and their criticism, and only later the third opinion, we will start with the first one

As the anonymous author writes, the first view is "already criticized by many, and more precisely by two most famous, i.e., Thomas Bradwardine and Adam of Pipewelle.[113]" This position recognizes that the speed of motion should be determined by an excess of acting powers over passive ones. In this case, the speed is proportional to the ratio between the acting power F and the resistance R to be overcome. This is the second opinion mentioned and criticized by Kilvington and Bradwardine, although – as the anonymous author claims – that is the first opinion discussed by Bradwardine in his treatise on motion. Kilvington and later Bradwardine give many numerical examples proving the falsity of this theory. However, their criticism is conducted from the point of view of the right solution, i.e., the correct understanding of continuous proportion. But the anonymous author does not use Kilvington's and Bradwardine's examples, and all the arguments (difficulties) against this theory are of his authorship. He argues that if we were to adopt this position, the six following difficulties are justified:

113 See Anonimus, *De sex inconvenentibus*, q.: *Utrum in motu locali sit certa servanda velocitas*, (Editions), § 2, p. 299.

The first difficulty assumes that despite the fact that a body constantly accelerates its motion, because its force constantly more and more exceeds the resistance, the speed of motion of this body would result from the ratio of the equality between force and resistance. In justification, the author states that the moving force is equal to resistance and continues to increase with increasing resistance, therefore the force is still greater than that which the body had at the previous moment, so the motion occurs, although the resistance has the same value as the force.[114] In response, he notes that the proportion does not refer to the ratio of the value of force at a given moment to the value of force at the previous moment, but to the ratio of total force to total resistance.[115]

The second difficulty is surprising because it is entirely contrary to the Aristotelian concept of natural motion which is always an accelerated one. First of all, it is stated here that no heavy body moving naturally towards its natural place increases the speed of its motion; secondly, speeding up the motion is the result of the so-called smaller inequality, i.e., when the force is less than the resistance to be overcome. To justify this difficulty, the anonymous author constructs a sophisticated case that will later finds its use in Richard Swineshead's treatise.[116] Let's presume that a heavy body moves towards its natural place, which is the center of the Earth, and around the center of the world there is a medium that uniformly resists the moving body, and that, at the beginning of the movement this body is above the center and moves in time in such a way that in the first half of time it touches the center, and in the second half of time it moves until its center coincides with the center of the world. Then the speed of this body would not increase, because in the second half of the time the ratio of force to resistance would constantly decrease, so the speed would decrease. This is because half the body below the center of the world would resist the half that is above, so the total resistance will increase constantly.[117] In response, the author confirms the conclusions, i.e., he recognizes that the accepted case is not absurd.[118]

The third difficulty is this: despite the fact that one body will act upon the other one infinitely fast, and yet it can act on the other even

114 Anonimus, *Utrum in motu locali…*, § 36, p. 314; § 42, p. 315.

115 Ibidem, § 192, p. 387.

116 See, R. Podkoński, *Suisetica inania…*, pp. 163–173.

117 See Anonimus, *Utrum in motu locali…*, § 37, p. 314; §§ 43–45, pp. 315–317.

118 See ibidem, § 193, pp. 387–388.

faster. To prove this difficulty, the author assumes that two warm bodies, while heating, act unequally fast, since one is hotter than the other. Consequently, the warmer body, while heating, acts at a higher speed because the ratio of its strength of action to the resistance of the heated body is greater than the ratio of the less warm body to its resistance, and yet the less warm body while heating acts infinitely fast, because it first heats the proximal part then the remoted part and the first half of the proximal part, and the first half of the half of the proximal part, and so on in infinity, and the heated ever smaller proportional parts are less resistive, so the less warm body acts at infinite speed.[119] In reply, the author points out that even if the body would act infinitely fast, all interacting bodies would act in the same way.[120]

The fourth difficulty is constructed as a sophism: despite the fact that something begins to act infinitely fast, it will continue to act faster and faster than it begins to act. The case outlined here is similar to the previous one and is based on the assumption that a body heating another body that does not resist, acts infinitely fast, as it is in the case of heating an unevenly warm body by a fire applied to its end which is as hot as fire; at this end the body does not resist, so the movement is infinitely fast.[121] In reply, the author says that the conclusion that the heated body does not resist is false.[122]

The fifth difficulty assumes that although two points will move in a straight line, a point moving faster will not traverse a greater distance in the same time. The justification of this difficulty is argued for with an interesting example that Kilvington had previously presented in his question on continuity, namely the so-called shadow cone. We take into consideration two light sources and two constantly diminishing obstacles that cast the same shadows, and we assume that one glowing body will intensify and shine brighter and the other will not change, and we take two points in two different shadow cones that move in such a way that they are always embraced by the cones. These points will reach their ends at the same moment of time and will move until the shadow cones are destroyed, and yet the point that moves in the shadow cone

119 See ibidem, § 38, p. 314; § 46, pp. 317–318.

120 See ibidem, § 194, p. 388.

121 See ibidem, § 39, p. 314; §§ 47–48, pp. 318–319.

122 See ibidem, § 195, pp. 388–389.

cast by the intensifying light will move faster because the stronger light will destroy the shadow cone faster.[123]

The sixth difficulty has also the form of a sophism: two bodies moving with rectilinear motion, equidistant from their endpoints, will reach them equally fast, and one of them all the time will move faster than the other, and yet the latter will never move slower than the first one. It is assumed here that in some moment of time during which two bodies move, they are not equidistant from their endpoints, and thus the body more distant from its end will traverse a greater distance in the same period of time, so it will move faster. This inference is correct at any moment of time, so it is correct at all moments of time, thus it is correct in all time - says the anonymous author of the treatise.[124] He responds to both the fifth and sixth difficulties, providing arguments showing that the conclusions are false and the entailments are incorrect, so the difficulties are poorly constructed.[125]

The second theory states that speed is determined by the ratio of the excesses of forces to resistances. It is difficult to understand and clearly explain what the author meant by that, because neither Kilvington nor Bradwardine present such a position. It seems that this theory assumes that one should always compare two movements or two stages of one movement and determine the ratios of force to resistance, i.e., the excess, to which is proportional the speed of motion at one moment or one motion, relative to the excess, which is the result of the ratio of force to resistance at another moment of motion or another motion. According to the anonymous author, this opinion is also criticized by many, including Thomas Bradwardine and Adam of Pipewelle.[126] We do not know any of Adam's text, perhaps he advanced such a solution which he then criticized.[127] For sure neither Kilvington nor Bradwardine mention this theory. Against this position the author also puts forward six difficulties, which are constructed in ways that seem to confirm that this opinion should be interpreted as above.

The first difficulty assumes that even if two same bodies with the same value of active power were to move in a medium with the same re-

123 See ibidem, § 40, p. 315; § 49, pp. 320–321.

124 See ibidem, § 41, p. 315; § 50, pp. 321–322.

125 See ibidem, § 196, pp. 389–390.

126 Ibidem, § 2, p. 299.

127 On Adam de Pipewelle see Chapter I, p. 24.

sistance, and their speed is proportional to the same difference between F and R, with the other conditions unchanging, then one of them would move twice as fast as the other. The justification of this argument is as follows: if we assume that the ratio of forces of the same, homogeneous bodies to the resistance of the medium is equal to 2 : 1 and the medium, in which one of these bodies moves, goes up with the same speed as this body moves, so this body, supported by the medium, descends twice as fast as the other.[128] In response to this argument, the author states that it is false because it was assumed that "other conditions are unchanging", and in the example an additional assumption is made about the upward motion of the medium. If, however, we accept such an assumption, then the argument is correct.[129]

The second difficulty assumes that, despite the fact that the body overcoming the resistance of a medium will increase its power in the middle of its movement, it will still move down in this medium slower than before and that this would not be caused by the increasing density of the medium. Its justification is as follows: we keep the previous case with the difference that we assume that one body increases its power of movement, and we assume that the medium in which this body moves, goes upward and this ascent is opposite to the body's downward motion, so it slows down the body's motion.[130] In response, the author notes that the upward motion of the medium, which interferes with the downward motion of the body, is balanced by increasing the power of this body, so the ratio of force to resistance remains the same throughout the movement. If, however, the medium in which the body moves still rarefies then the movement would slow down.[131]

The third difficulty assumes that the speed of the body resulting from the ratio of the moving force to the resistance of the medium will increase, although during its motion the body decreases its power of movement. To justify it, a similar case with a medium going up is assumed, but this time the medium moves slower in the second part of time. Therefore, in this case the losing of strength of the moving power would not cause the diminishing of the speed of motion.[132] The answer

128 See Anonim, *Utrum in motu locali…*, § 17, p. 308; §§ 23–25, pp. 309–310.

129 See ibidem, § 186, pp. 383.

130 See ibidem, § 18, p. 308; §§ 26–27, p. 310–311.

131 See ibidem, § 187, p. 384.

132 See ibidem, § 19, pp. 308–309; § 28, pp. 311–312;

to this argument is very short: "as in the previous case, this case is also badly constructed."[133]

The fourth difficulty is as follows: a heavy body, such as a clod of earth, moves downwards in a natural motion, yet it does not itself desire such a movement. The justification for this difficulty is sophisticated and it plays with terms. Firstly, since the anonymous author claims that, when we say that the body by nature desires to move downwards, and this requires some speed, yet it strives with all its power, so it will immediately be in its natural place, that is, it will move at an infinite and not determined speed. Secondly, because there is no reason for such a body to move at a certain speed rather than at any other, it can be assumed that it moves faster, slower, uniformly or non-uniformly.[134] In reply, the author states that no degree of speed could be determined for a natural downward motion, thus any speed is proper.[135]

The fifth difficulty is this: let us assume that some force cannot overcome some resistance and a resistance twice as great. This seems to be obvious, however, the anonymous author gives an example of fire affecting water and air and then, *secundum imaginationem*, reduces the original properties of air, replacing cold with heat in the same proportion of cold to moisture in which at first heat was proportional to moisture.[136] Finally, in the answer, he says that in this case the proportions are not the same, so the conclusions are not well justified.[137]

The sixth difficulty: some force overcomes some resistance, which continues to increase until it doubles, and yet this force can cause an increasing speed of action or at least the same speed as in the beginning. The example justifying this conclusion assumes that the hottest fire, i.e., with the greatest power to heat, heats the air in a ratio of 2 : 1 and that the air is cooled in a ratio 1 : 2 less than the original ratio of heat to cold in the air, then the ratio of heat to cold at the end of this heating process will be greater than 2 : 1, but the resistance will continue to increase.[138] The answer is short: neither the example given, nor the conclusion are

133 See ibidem, § 188, pp. 384–385.
134 See ibidem, § 20, p. 309; 29–31, pp. 312–313.
135 See ibidem, § 189, p. 385.
136 See ibidem, § 21, p. 309; §§ 32–33, p. 313.
137 Ibidem, § 190, pp. 385–386.
138 See ibidem, § 22, p. 309; § 34, p. 314.

true, which is confirmed by examples similar to those cited in the previous, fifth argument.[139]

As was already mentioned, the anonymous author of the treatise *De sex inconvenientibus* began his question by presenting six difficulties against the position that he himself accepts, according to which the speed is determined by the ratio of ratios of forces to resistances.[140] He is convinced, however, that the solution to the problems pondered in three other articles will help us better understand this theory, so we will now present the content of these three articles.

4.1. The Causes of Accelerated Motion

The first article discusses the causes of the increasing speed of a heavy body in its downward motion ("Does increasing the speed of a heavy body have a specific reason?"). Thus, the author is interested, in modern terms, in how to measure the speed of the uniformly accelerate motion of free fall. As usual, at the beginning of the article we find six difficulties, which this time are actually different ways of explaining the cause of the increase of the speed in downward motion. A great part of these arguments are to be found in Kilvington's question discussed above.[141]

As this time the author does not answer all the difficulties presented, we shall first discuss them and then we will present his final opinion.

The first difficulty results from the assumption that the decreasing of the resistance of the medium is the cause of the acceleration. The author constructs an interesting, provocative argument to substantiate this inference: if the speed were increased in such a way, Socrates could jump over the sphere of the Moon. The argument presented here is quite clever: if Socrates's power to perform the jump does not weaken, then it can be assumed that he jumps from Earth towards the Moon, first traversing a small distance and at the end of this distance he has the same power as at the beginning of the motion, and thus Socrates will be able to jump higher; moreover, from this second point he will have

139 See ibidem, § 191, pp. 386–387.

140 See ibidem § 127, pp. 346–347.

141 See Ricardus Kilvington, *Utrum in omni motu…*, §§ 81–89, pp. 248–251.

to cover a smaller distance to the Moon, which is less resistive, so the speed of his motion will increase, and so on by every subsequent jump. Therefore, Socrates reaches a speed sufficient to jump over the Moon sphere.[142]

The second opinion recognizes that increasing the speed of downward motion is caused by continuity of motion. If this were the case, then such an inconvenient conclusion could be drawn: the speed of motion increases, and yet the ratio of force to resistance decreases. To justify this, the anonymous author gives the following examples: 1) A heavy body in its downward motion from the sphere of fire must overcome the increasing resistance of continuously denser media such as fire, air and water, and then the longer the movement lasts, the more resistance it has to overcome, and thus the speed decreases; 2) Earth constantly warmed by the Sun should move faster and faster so then finally all buildings would fall on to the Earth; 3) Since the motion of the sky and of all stars is continuous, they should move with a constantly increasing and not a constant speed; 4) If a heavy body slows its motion, and continuation is a sufficient cause of acceleration, then the body slows down and accelerates its motion at the same time.[143]

The third opinion recognizes that the acceleration in motion is caused by approaching to the natural place; the fourth opinion states that the enlarging medium located above the moving body while pushing it down accelerates its motion; the fifth opinion claims that the acquisition of an additional accidental resistance is the cause of acceleration; and the sixth finds the cause of acceleration in motion in the increasing desire to be placed within the natural place of a heavy body.[144]

The anonymous author of *De sex* gives many examples to support these opinions and finally he states that: there is no single cause of increasing the speed in free fall, the most important one is the decrease of resistance of the medium that has to be overcome to move down-

142 See Anonim, *Utrum in motu locali…*, § 54, p. 323; §§ 60–62, pp. 323–325; §§ 97–100, pp. 335–337.

143 See ibidem, § 55, p. 323; §§ 63–69, pp. 325–327.

144 See ibidem, §§ 54–59, p. 323; §§ 70–93, pp. 327–334.

wards; but also the five other causes listed above are partial causes that play an important role in different situations.[145]

4.2. The Motion of a Sphere

The second article is also presented in the form of a question: "Is the speed of motion of any sphere determined by some point or space"? The author initially cites six opinions.

The first position, which the author recognizes as the commonly accepted one, states that the motion of an orb depends on its lowest point. He rejects this, arguing that if that were the case, the orb of fixed stars would move as fast as the Earth and faster. Thus, the speed of a celestial body's motions is determined by its lowest point, i.e., the center of the orbs, that is the Earth, which – according to Ptolemy – is considered to be a point. Then, the Earth would move at the same speed as the orbs, and this movement should be perceptible. But the center point – the Earth – is motionless, so consequently the celestial orb would not move, which is also denied by observation.[146]

The second opinion refers to Gerard of Brussels's – as Rommevaux-Tani writes – "Treatise on the proportions of movements and size" (*Tractatus de proportionibus motuum et magnitudinum*) quoted by Thomas Bradwardine in his *Tractatus de proportionibus.* This position assumes that the speed of motion of the celestial orb is determined by the midpoint between the lowest and highest point. Gerard from Brussels says: "Any part as large as one wishes of a radius describing a circle, [which part of it is] not terminated at the center, is moved equally as its middle point. Hence the radius [is also moved equally] as its middle point. From this it is clear that the radii and the motion have the same ratio."[147]

The anonymous author of the treaty offers three difficulties, which indicate that if this opinion were adopted, the orb of fixed stars would move as rapidly as its midpoint, and thus as fast as its middle sphere or

145 See ibidem, §§ 95–96, pp. 334–335; Ricardus Kilvington, *Utrum in omni motu…*, §§ 136–138, p. 266.

146 See ibidem, § 102, p. 337; §§ 108–110, pp. 337–339; See also S. Rommevaux-Tani, *The study of local motion…*, (forthcoming).

147 For English translation see M. Calgett, "Archimedes in the Middle Ages", vol 5, Part I, p. 111.

the body of the sphere, which is either the Sun or the body below the sphere of the Sun. Thus, the orb of Saturn or Mars would move more rapidly than the orb of fixed stars. This goes against what all astronomers say.[148]

The third opinion states that since the speed of motion of the sphere cannot be determined by the lowest or middle point, then it is determined by the highest point. In the opinion of the anonymous author, this is the position which Thomas Bradwardine accepts in his treatise *On proportions*, Chapter IV. Against this position the anonymous author raises three doubts indicating that if the speed of motion of the sphere depended on the speed of the point on its circumference, then such a speed would be uniform. And then, when we consider all the points of such a sphere, it turns out that the motion is not uniformly difform, because the father the point is from the center, the faster is its motion.[149]

The fourth quoted position recognizes that the speed of the sphere is not determined by some point, but by the space described during the motion. The author argues against this position with three main objections, similar to those presented above, and based on the statement that the sphere would move with infinitely variable motion, because the sphere can be "divided" into smaller spheres less distant from the center, and the sphere closest to the center would move slower, therefore, the motions of the total sphere, which consists of internal spheres, would have a uniformly difform speed, one which is contrary to observation. The author states: "Infinitely many other [difficulties] can be demonstrated, but I keep going because I consider this position to be entirely false."[150]

The fifth difficulty indicates the problems that arise when we assume that the speed of motion is determined by the space that has been covered in the motion. The main objection raised against this position points out that in this case the comparison of the speed of motion of a point plotting a section in motion with the speed of a section plotting

148 See Anonimus, *Utrum in motu locali…*, §§ 111–114, pp. 339–340. See also S. Rommevaux-Tani, *The study of local motion…*, (forthcoming).

149 See Anonimus, *Utrum in motu locali…*, §§ 115–118, pp. 341–344. See also S. Rommevaux-Tani, *The study of local motion…*, (forthcoming).

150 See Anonimus, *Utrum in motu locali…*, §§ 119–123, pp. 344–345. See also S. Rommevaux-Tani, *The study of local motion…*, (forthcoming).

a square in motion, lies in there being a section and a surface, which i.e., a square and a section cannot be compared.[151] This argument is also found in the Bradwardine treatise.[152]

The sixth opinion assumes that: "the speed of motion of a sphere moving the fastest around its center is described by the distance traversed by the fastest moving point or by distances traversed by the fastest moving points in the same period of time, as Master Thomas Bradwardine claims."[153] The author further says: "I consider this position necessary, true and the right view, which should be maintained, and since it is consistent with the third view, I reject the other opinions."[154]

Despite this declaration, however, the author, in accordance with the adopted outline of the treatise, offers four objections to this position. The arguments are fairly elaborate, so as discussing them would take up a lot of space we shall briefly present the one that refers to Euclid's theorem. The author asks the question as to whether the radii of two spheres, one of which is twice as large as the other, are in a double ratio. According to Euclid, the ratio between two spheres is the triplicate ratio to their diameters. Therefore, the anonymous author concludes that the ratio between the radii is smaller than the double ratio. Undoubtedly, this shows that the author is aware of the usefulness of mathematics in the final resolution of problems within the field of astronomy, but his efficiency in using this tool leaves much to be desired,[155] which is easy to see in the following quote:

> Therefore, to answer the main question of this article I say that the speed of the sphere moving around its center is described by its fastest moving point, so that the entire sphere moves as fast as this point, and not faster, and such motion is described by the motion of that point. Similarly, in relation to two spheres rotating uniformly at the same or equal time, I say that what will be the

151 See Anonimus, *Utrum in motu locali…*, §§ 124–126, p. 346.

152 See Thomas Bradwardine, *Tractatus de proportionibus …*, p. 128.

153 See Anonimus, *Utrum in motu locali…*, § 127, pp. 346–347; §§ 128–132, pp. 347–351.

154 Ibidem, § 134, p. 353.

155 See S. Rommevaux-Tani, *The study of local motion…*, (forthcoming).

ratio of the largest circuits, this will be the ratio of the speeds of these spheres.[156]

4.3. The Mean Speed Theorem

The third question asks: "Whether the speed of each uniformly difform local motion, beginning at no-degree is equal to its middle degree?" It discusses the above presented Heytesbury's Mean Speed Theorem. In modern terms the question would be: "Is the speed of accelerated motion starting at zero speed equal to the speed of uniform motion achieved at the midpoint of the duration of the motion?" In this article, its author frequently uses terminology, characteristic for the Calculators, such as *latitudo velocitatis*, *gradus velocitatis* or *non-gradus*. Therefore, the main problem is whether the velocity starting at zero in uniformly variable motion corresponds to the velocity value at the midpoint, i.e., that reached in the middle moment of the motion.

This time the author, like Heytesbury before, considers the problem due to the effects of such motion, i.e., the distance traversed and the time consumed, and he is not interested in the causes of motion such as force and resistance, about which he said much earlier. Thus, by presenting a basic issue in yet another form, we are interested in the answer to the question: "Will the same distance be traversed at the same time as the distance that would have been traversed if the body were moving at a constant speed equal to that which one has in the middle point of the duration of its motion?"

The author explains what he means by "middle point" by saying:

> And then, if someone asks about what I call the middle point, which is equal to the whole motion, I say that if any latitude of motion that lasts for some time begins with a non-degree and ends to some extent, then in the middle of this time a certain degree of motion is obtained, which is equal to the whole latitude, and this is called the middle degree of motion, which is equidistant from the ends of time, i.e., from the first, zero – a beginning of motion and the last moment of motion. Therefore, with regard to time it

156 Anonimus, *Utrum in motu locali…*, § 134, p. 353.

> is said that in equal time the motion of such a latitude will reach a degree two times greater and a degree two times smaller.[157]

The "middle speed" equidistant from both ends of the latitude of the speed is their arithmetic mean.

The anonymous author, in accordance with the adopted method of presenting material, first presents six difficulties. In concluding this part, the author states:

> There are many other arguments that I omit for the sake of brevity. I only touch on some problems, giving others material for a broader analysis and defense of their position. Due to the above-mentioned and other similar arguments, according to some, when it comes to the latitude of local motion ending on a zero degree of speed, the entire latitude is equal to the most intense degree of speed and not to its middle degree, and the ratio of motions is described by the ratio of the most intense degrees of these motions.[158]

In order to justify his own opinion, that: "in any uniformly difform local motion beginning at no degree, the latitude of the whole latitude of motion is equal to its middle degree and only to it", the anonymous author gives six arguments.

The first argument is based on – as the anonymous author claims – Averroes' commentary to Aristotle's *Nicomachean Ethics*, Book II, which states that in all continua there is something that is the greatest and something that is the smallest, and therefore there must be something that is equal.[159] The latitude of the speed of uniformly difform motion is a continuous quantity, therefore divisible, and its middle point divides this latitude into two halves, one of which is bigger, because the speed in this half is more intense, i.e., it has a higher value, and the other is smaller, i.e., the speed has a lower value. From this, he infers that in this whole latitude of speed there is a degree that is equal to its whole, be-

157 Ibidem, §. 176, p. 377.

158 Ibidem, § 163, p. 365.

159 See Aristotle, *Nicomachean Ethics,* II, 6 (1106a); Awerroes, *Com. in Eth. Nic.* II, com. 10, f. 24vb.

cause no part of the whole latitude of the speed of motion can be either greater or less than that produced by the whole latitude.[160]

The second argument outlines the following situation: Socrates and Plato move with uniformly difform motion; Socrates with an accelerated motion from zero speed to a speed of a certain value, Plato vice versa, with a delayed motion starting from the value of the speed at which Socrates ends the motion to no-degree. The points at which these motions end are not equal, so these motions can be compared only with regard to the midpoint, because the speed they both have in the middle point of the time of their movement is equal. As the author states: "they do not seem to correspond to or be equal to other [degrees of motion]".[161]

As Rommevaux-Tani rightly points out:

> It should be noted that in this argument, and it will be the same in those which follow, the author considers only two positions as possible: either the latitude of a uniformly difform motion is equal to its middle degree, or it is equal to its extreme degree. So, to prove that it is equal to its middle degree, he takes it to be enough to show that it is not equal to its extreme degree. Here, the author has in mind the model of the motion of a sphere of which he spoke in preceding article. Two hypotheses among those he examined made the motion of a sphere depend on the motion of one point: Gerard of Brussels' opinion, according to which the motion of a sphere is measured by the motion of the midpoint of its radius and that of Bradwardine for whom the motion of the sphere is measured by the motion of the extreme point of its radius. The author transposes these two hypotheses to uniformly difform local motion which intensifies from no degree, the motion which he considers here. But, while for the uniformly difform local motion beginning at no degree, the author of the *Tractatus de sex inconvenientibus* accepts the middle degree theorem, we have seen that, as far as the motion of a sphere is concerned, he follows Bradwardine's opinion rather than that of Gerard of Brussels.[162]

160 See Anonimus, *Utrum in motu locali…*, § 164, pp. 365–366. See also S. Rommevaux-Tani, *The study of local motion…*, (forthcoming).

161 See Anonimus, *Utrum in motu locali…*, § 165, pp. 366–367.

162 See S. Rommevaux-Tani, *The study of local motion…*, (forthcoming).

In the opinion of Rommevaux-Tani, this seems contradictory, so the anonymous author makes some comments which clarify his opinion declaring that the motion of a sphere and the uniformly difform local motion are not the same, so their latitudes need not to correspond to the same point. In local motion, whose speed is constantly increasing, also the points of the moving body will have a constantly higher speed, while in the motion of the sphere all points located on its radius the farther from the center will have an increasing speed.[163] These two motion are not comparable, so they do not have to be affected by the same rules.

The third argument presents a situation that links the latitude of motion, i.e., its speed and the distance traversed. The author considers such a possible case, that Socrates moves uniformly difformly from no degree to degree 8 of the speed, and Plato moves uniformly at a constant speed of 4. To prove the mean speed theorem, the author has to show that both of them will traverse equal distance. He proves that in the first half of the time Socrates will cover half of the distance that Plato will cover, and at the same time in the first half of time Socrates will cover one third of the distance he will cover in the second half of time, so both of them will traverse the same distance equal to 4, because Socrates will in the first half of time 1, and in the second 3, and Plato in the first 2 and in the second 2. This argument shows once again that speed is considered with regard to the midpoint of time.[164]

The fourth argument is based on the *reductio ad absurdum* and indicates the difficulty that results from the opinion that the speed of uniformly difform motion is described by the highest degree of speed. The example here is outlined as follows: Socrates moves from zero degree to the C value of speed and at the end of the first half of time he has the speed B, and in the first half of the second half he has the speed D. As shown above, in this case Socrates in the first half of time traverses 1/4 of the whole distance, and in the second half 3/4, and when the ratio of D to B = 3 : 2, so if we assume that the final degree of the speed determines the entire speed, then the speed in D is 3/2 of the speed in B, so Socrates acquires in D 3/2 of the speeds B, i.e., 1/4 + 1/8, and in the second half of the second half he reaches a speed of 3/4 - 1/4 - 1/8 = 1/4 + 1/8, i.e.,

163 Ibidem.

164 See Anonimus *Utrum in motu locali…*, §§ 166–167, pp. 367–370.

he moves with the same speed, and thus with uniform motion, something that is contrary to what was assumed.[165]

The fifth argument, which presents extremely chaotic considerations – as Rommevaux-Tani claims – should be understood as follows. The author compares here two forces that overcome the same resistance of 4; one force uniformly increases in the first half of the time from 4 to 6, and the other from 6 to 12. If the speed of motion is determined by the fastest motion, then the force at the end of the second half of the time acts twice as fast as at the end of the first, because 12 is twice greater than 6. The second force, however, also uniformly increases in the first half of time from 4 to 6, and in the second from 6 to 9. The author notes that the ratio of 6 : 4, which equals 3 : 2, is the same as the ratio of 9 : 6 = 3 : 2, so the ratio 9 : 4 is a doubled ratio of 6 : 4. And this is in line with the understanding of proportions that we talked about at the beginning of this chapter, because (9 : 4) = (9 : 6)(6 : 4) = (3 : 2)(3 : 2), but the author does not bother to explain the calculus. He concludes, however, that two unequal forces will cause the same speed of motion.[166]

The sixth argument deals with the same situation as the one in the fourth argument and concludes that Socrates will still be moving at the same speed, because the ratio of the next to the previous speed will still be the same.[167]

We already know that the anonymous author of the treaty adopts a third position, which recognizes that speed in local motion should be determined by the ratio of ratios of force to resistance.[168] Although he accepts it, he, again, presents six difficulties at the beginning of this question, which he, however, does not solve. Let's look briefly at the way of arguing against this position. The difficulties are as follows:

First: two bodies, composed of heavy and light elements, in which the ratio of lightness – which is the primary quality of light elements, such as fire and air – to their heaviness – the primary quality of heavy elements, such as water and earth – is the same, and yet if these bodies moved in a medium that would resist them equally, then one would

165 See ibidem, §§ 168–169, pp. 370–371. See also S. Rommevaux-Tani, *The study of local motion…* (forthcoming).

166 Anonimus, *Utrum in motu locali…*, § 170, pp. 371–373. See also S. Rommevaux-Tani, *The study of local motion…* (forthcoming).

167 Anonimus, *Utrum in motu locali…*, § 171, p. 373.

168 See ibidem, § 179, p. 379.

move and the other would not move in it. To demonstrate this difficulty, the anonymous author constructs the following case: the heavier part of the heterogeneous mixed body A is placed below the center of the world, and the homogeneous body B, which has as many heavy elements as it does light, is placed above the center the world. In this case, body A would move, because its light elements would tend to their proper place, i.e., upwards, and heavy ones, that are below the center of the world, would strive to come into contact with the center of the world, which would also cause an upward movement. However, in the homogeneous body B, the ratio of heaviness to lightness is still the same, and it is this ratio of power to resistance that causes the speed in motion, and this ratio is 1, because the strength here is equal to resistance, so B does not move. In response, the author states at the beginning that this conclusion is not ridiculous, because in motion not only is the ratio of heavy and light elements that make up the body important, but also their location, because the location decides whether the elements will be supported in motion or disturbed. However, since in this case it was assumed that there are as many heavy as light elements, the ratio of the force of those which tend to their natural place to the resistance of those which tend to their natural place is the ratio of equality, i.e., $F : R = 1$, so in this case neither body A nor B would move.[169]

Second: two heavy bodies with the same amount of earth and water, move in a medium which resists equally to them both, thus the ratio of their moving power to the resistance is the same, but while taking into account only the resistance of the medium we can conclude that one of them will move faster than the other. To justify this conclusion, the author uses the same example as presented above, but he additionally assumes that these bodies move towards the surface of the water, and that the equal parts of these bodies are under the surface, but not the same parts. Namely, while part of the water in one body is above the surface of the water, and part of the earth in this body is below this surface, the parts of the earth and the water of the other body are located in the opposite direction. Thus, in the downward motion, the first body moves faster, because the water contained in it tends to the surface of the water, and in the case of the second body the water contained in it slows its downward motion, since it tends upwards to touch the surface

169 See ibidem, § 3, p. 299; § 9, pp. 301–302; §§ 180–181, pp. 379–380.

of the water. In reply, the author states that this objection can be refuted in the same way as the first.[170]

Third: no matter what the ratio of the weight of the body, such as a clod of earth, is to the resistance of the medium in which it moves, it will move infinitely slowly. The rationale for this conclusion is as follows: let the heaviness, the acting power, of a clod of earth tending to its natural place, i.e., the center of the Earth, have a value of 3, and let the resistance of the medium be 2, and let presume that during the downward motion the internal resistance increases. Thus, the total resistance, the sum of internal and external resistance, will also increase reducing the speed of the body's motion. The author's reply suggests that this argument is a polemic with someone else, perhaps his student, when he discussed these issues in class. The anonymous author states that the assumptions made are in contradiction.[171]

Four: two of the same capable moving powers, equal to their resistance, operating within an hour, will be equally intense at the end, although one of them intensifies faster than the other. To justify this difficulty, the anonymous author assumes that the speeds of motion of two identical bodies are the results of the same ratios of their force to resistance, and that one force steadily increases to a value equal to twice the strength of the initial forces. The example outlined above shows that in the middle of the process, the speed of the body, whose strength increases, determined by the ratio of force to constant resistance, increases by half in relation to the speed that was at the beginning, and is twice as low as the final speed; while the speed obtained as a result of the constant ratio of force to resistance is still the same and equal to the speed of a body with a constant ratio of force to resistance. In reply, the author states that the conclusion being drawn from this example cannot be confirmed, because it is impossible that the intensification or weakening of a force occurs at a uniform speed, because such a process can occur only as a result of a change in a uniformly difform motion. It is so, because if we assume that a force is intensifying, starting from 2 to 8, then between these values it has a value of 4 and 6, and the speed of motion is determined by the ratio of force to resistance, so if we assume, for example, that the resistance is constant and equal to 1, the subsequent ratios are like 2 : 1, 4 : 1, 6 : 1 and 8 : 1 and the latter 8 : 1 is

170 See ibidem, § 4, p. 299; §§ 10–11, pp. 302–303; § 182, p. 380.

171 See ibidem, § 5, p. 300; § 12, pp. 304–305; § 183, pp. 380–381.

greater than 6 : 1 in a ratio of 4 : 3; 6 : 1 to 4 : 1 = 3 : 2, 4 : 1 to 2 : 1 = 2 : 1, thus the speed determined by these ratios is variable, although not uniformly variable.[172]

Five: two bodies with the same power for motion move overcoming the same resistance, and yet, if they need to overcome an extra resistance, one of them will be able to overcome it and move faster, while the other will not. To justify this, the anonymous author uses such an example: two bodies, which are clods of earth capable for moving with a force equal to 6, move, and when we add respectively: to the first body composed of earth and fire, in which the ratio of the power of the earth capable of moving is 3 and the resistance which is caused by fire is also 3; and to the second body we add a body with a resistance of 2. Then the first enlarged body would move at a speed proportional to the ratio 6 + 3 = 9 to 3, i.e., 9 : 3 = 3 : 1; while the other enlarged body would move at a speed proportional to the ratio of force 6 to resistance 2, i.e., 6 : 2 = 3 : 1. That is, both of these bodies would move at the same speed, although one moves thanks to a force of 9 and the other moves thanks to a force of 6, and consequently – as the author concludes – the forces that produce the same effect, i.e., the same speed of motion, are equal, so 9 is equal to 6. This seems ridiculous, but in response the author states that the conclusion is possible and true in a given example, if we consider the ratios of forces to resistances. However, the ratio between the forces causing motion is not the same and these forces do not exceed the same resistance with the same ratio, because a force equal to 9 has an excess over the resistance 3 equal to 6 (9 - 3 = 6), and a force equal to 6 has an excess over the resistance 4 equal to 2. The author believes, however, that if we consistently recognize that speed is determined by the ratio of force to resistance, then only this ratio is important, not the value of the excess of an active power over resistance.[173]

Six: two bodies move in the same resistive medium at a speed proportional to the same ratio of active power to resistance, and yet, when the resistance increases twice, one body will be able to move and the other will not; and if we reduce the resistance of the medium by two, then one body will move faster than the other. To justify this difficulty, we assume that the air in which the two bodies move puts up a resistance of 2 and one body moves at a speed proportional to the ratio of

172 See ibidem, § 6, p. 300; § 13, pp. 305–306; § 184, pp. 381–382.

173 See ibidem, § 7, p. 300; § 14, pp. 306–307; § 185, pp. 382–383.

force to internal resistance as 8 to 2, and the other body is a clod of earth with a force of motion of 4. In this case, both bodies move at the same speed determined by the ratio 8 to 2 + 2 = 4, and 8/4 = 2/1, and 4 to 2, gives also the ratio 2/1. And if we double the resistance of the medium, then the first body would move at a speed proportional to the ratio 8/(4 +2) = 8/6 = 4/3, and the second would not move, because the force would be equal to a resistance of 4. If the resistance of the medium decreases twice, then one body would move at a speed proportional to the ratio 8 to 2 + 1 = 8/3, and the other proportional to the ratio 4/1, and thus it would move faster than the first. In reply, the author confirms the conclusion of the fifth difficulty, because the argumentation is the same.[174]

5. John Dumbleton on Local Motion

In the very beginning of Part III of his *Summa*, devoted to the problem of motion in general, John Dumbleton, like his older colleagues Kilvington, Heytesbury and the anonymous author of the treatise *De sex icnonvenientibus*, declares that motion should be considered with respect to four predicaments; in the case of three of these, namely with regard to quality, quantity and place, it is considered properly (*proprie*); the motion of substance, however, is considered improperly (*improprie*), since a substance moves with regard to these three, above-mentioned predicaments, that are the accidents of the substance, thus the substance moves because it is a proper subject of these three types of changes.[175] According to the actual title, Dumbleton conceived his treatise as a *Summa* of the logic and natural philosophy of his times, so he elaborately discussed all these kinds of motions. Since the present book focuses on the Oxford Calculators' theory of local motion, we limit ourselves here to the contents of chapters 5 to 12 of Part III of Dumbleton's *Summa*, from which we provide a working edition in Part II of this volume (Editions).

In Chapter 5, Dumbleton takes up a model of discussion employed by his predecessors such as Richard Kilvington and Thomas Bradwardine,

174 See ibidem, § 8, p. 300; § 15, pp. 307–308.

175 See Johannes Dumbleton, *Summa logicae et philosophiae* naturalis, Part III: *De motu locali* (Editions), § 2, p. 393.

and commences the debate with arguments against two theories concerning the relations of forces, resistances and speeds. The first opinion was traditionally ascribed to Avempace, according to which the speed of motion is proportional to the arithmetical difference between an acting power and resistance (v ~ F – R). Hence, with regard to this theory, a motion can also occur in a void when there is no resistance at all. Obviously, that is contrary to Aristotle, who claimed that motion in a void must have been infinitely fast.[176] To refute this opinion Dumbleton gives the six following arguments:

1). The first proof is based on *ad absurdum* reasoning. Since every motion can be either infinitely fast or infinitely slow let us presume that an acting power F has now a value 4 and constantly diminishes up to 2 and that resistance R has now a value 2 and also diminishes to 2 in such a way that the excess of F over R is always the same, equal to 2. Thus the motion is uniform, since its speed is a result of the excess equal in any moment of its lasting; and, therefore, the motion would not be infinite.[177]

2). The second argument is based on the assumption that there is no limit for the speed in local motion, so it can increase infinitely. The current theory violates this statement because in relation to it a finite force would not cause an infinite speed of motion, since an acting force can only exceed resistance to the extent that it has value.[178]

3). Here Dumbleton repeats the argument, one already formulated by Kilvington and Bradwardine, that a body would move in a medium and in a void with equal speeds. This would be so because if a body moves in a void with no resistance thanks to the power F = 2, it moves with a speed = 2 (v~ F – R, 2 – 0 = 2), and it would move with the same speed in a medium with the resistance 2 and a moving power of 4 (4 – 2 = 2).[179]

4). This argument states that an acting force equal to 6 would move the resistive body equal to 2 faster in the medium than an acting force equal to 2 would move the resistive body in the void, although in this case it does not have to overcome any resistance. This is because in the first case the speed proportional to the difference between F and R is equal to 4 (F – R = 6 – 2 = 4), and in the second the speed is equal to

176 See ibidem, § 9, p. 395.
177 See ibidem, § 12, p. 396.
178 See ibidem, § 13.
179 See ibidem, § 14, pp. 396–397.

2 (F – R = 2 – 0 = 2). It seems that Dumbleton wants to point out here the very fact that it should be always easier to act if there is no resistance to be overcome than to act when there is a resistance.[180]

5). Here Dumbleton states that for Avempace a motion in a void is most natural motion, but if this were so the value of the speed of such a motion should be determined for any heavy body. We can assume, however, any value of such a speed since it is only proportional to the acting power.[181]

6). This theory concludes that the speed of motion in a void can be infinite, since a power equal to 2 moves slower than that of a value of 4 and this slower than that of a value of 6 and so on in infinitum. On the other hand the motion in a void is most natural, thus motion with an infinite speed is most natural, something which contradicts both Aristotle and Averroes.[182]

According to the second theory invoked by Dumbleton a speed of local motion or alteration is proportional to the ratio of the moving power to resistance, and it varies proportionally to the variation of the intensity of the power while the resistance remained constant. Thus, if the power is doubled, the speed is doubled, if the power increases by 3 : 2 also the speed increases by 3 : 2. The speeds of motion are equal when the ratios of power to resistance are equal. The discussion on this opinion spreads over 18 paragraphs (17–35) of Chapter 6. Sylla notices that this is "the traditional Aristotelian view",[183] while Dumbleton, however, does not refer to Aristotle here at all.

The foundation for the Aristotelian view, Sylla says, is the following:

> Let 3 act on 2 and another 3 on another 2, each according to the ratio 3 : 2. Each of these motions induces a certain latitude of heat or local motion. If, therefore, one makes a single agent from these two and applies it to one of the previous bodies acted on, since one agent does not impede the other, indeed it more likely assists, it follows that the two agents together will produce twice what

180 See ibidem, § 15, p. 397.

181 See ibidem, § 16; § 17, p. 397.

182 See ibidem, § 18, p. 398.

183 See E. Sylla, *The Oxford Calculators and Mathematical Physics*..., p. 147.

one produced before, and consequently cause a motion twice as fast.[184]

Dumbleton begins his discussion against this position with some, at least unconventional, arguments which have nothing to do with the calculus of ratios. He points out that on the basis of this very opinion, i.e., that the speed of motion varies in accordance to the increasing or decreasing of the value of the moving power only, one can draw the conclusion that a motion would not slow down to the rest when a part does not belong to the whole. The example is as follows: Socrates's power to move is assigned as 4 and the resistance of a medium as 2, thus while decreasing Socrates's power down to the intensity of the resistance, i.e., down to 2, during this motion the speed would diminish to its half. He proves, sophistically, that a power equal to 5 would move as fast as a power 4, since they both would move with the same speed twice faster than before.

In reply Dumbleton states that only a voluntarily acting agent can diminish the speed of its motion infinitely, like a man who can walk or move something infinitely slow. Such a situation can occur only because a man has the volition to do something with some speed. If we take into account the agents which do not act voluntarily, however, the speed of motion they cause must be proportional to the ratio of the force to resistance.[185]

Dumbleton argues against such a conclusion and proves that either Socrates, who acts voluntarily and a fire, which acts by natural neccesity, would produce the same speed of motion during their action, if we presume that it is possible to compare the ratio of forces to resistances despite the fact the active powers act differently.[186]

Next the argument points out that in the circular motion different points move with different speeds, so part of a circle does not move with the same speed as a whole circle.

Dumbleton tacitly assumes that if it is not possible to preserve a relation between the parts of an active power and the speeds they produce, whether they act independently or together, then it is the ratios of power to resistance that preserve their identification with certain speeds produced,

184 See ibidem, p. 148; Johannes Dumbleton, *De motu locali*, § 20, p. 398.

185 See Johannes Dumbleton, *De motu locali*, §§ 21–22, pp. 398–399.

186 See ibidem, § 23, p. 399.

whether the ratios are combined or not. Thus it is the ratios that produce, or cause, the speeds and not simply the powers, assuming that the resistances are constant.[187]

Dumbleton argues that:

> if the Aristotelian position asserts that 8 produces in 1 twice the speed that 4 produces in 1 then this must be related to the fact that the ratio 8 : 1 contains the ratio 4 : 1 one and half times [8 : 1 equals 4 : 1 to the three-halves power]. If this is the case, then a power A should move a resistance 4 twice as fast as 3 moves 2 if the ratio A : 4 equals the ratio 3 : 2 to the three-halves power. But in such a case A will be less than 9 (because the ratio 9 : 4 is equal to 3 : 2 squared, which is more than 3 : 2 to the three-halves power), whereas according to the standard Aristotelian position, it should take a power 12 to move 4 twice as fast as 3 moves 2 (or 6 moves 4).[188]

Consecutive arguments are aimed at showing that if we accept this opinion it follows that Socrates would not be able to cause any difficulty in acting although such a difficulty has its middle value between the minimum he cannot do and the maximum he can do. This is proved by the calculus of ratios.[189]

The next argument reveals the weakness of this position, since Dumbleton clearly notes that when the variations of speed depend only on variations of the acting power then the speed would increase infinitely faster than the ratio F : R, because if we presume that F1 = 4 thus the doubled power F2 = 2 x F1 = 8, so the speed proportional to F2 : R = 8 : R > (2 x 4): R, but a proper double ratio of F2 : F1 = 8 : 4 = 2 : 1. Thus in the constantly accelerated motion the ratios would be doubled. i.e., constantly twice as the previous one, but the speed would be multiplied only by 2 in the respective periods: 4/1, 8/1, 16/1 and so on. The consequent is false, since:

187 See ibidem, § 24, pp. 399–400.

188 See ibidem, § 25, p. 400, English translation E. Sylla, *The Oxford Calculators and Mathematical Physics...*, pp. 148–149.

189 See ibidem, § 26, pp. 400–401.

> If some latitude of motion is produced by three ratios of 2 : 1, if some fixed part of the motion corresponds to the part of the latitude of proportion from the ratio of 2 : 1 down to the ratio of equality, it follows that just as great a part of the same latitude of motion will correspond to just as much of the other latitude of proportion.[190]

As Sylla interprets it: "this means that if three ratios of 2 : 1 or, in other words, if $(2 : 1)^3 = (8 : 1)$ produces a velocity of 3n, then the latitude of ratio from 1 : 1 to 2 : 1 produces the velocity n, the latitude from 2 : 1 to 4 : 1 produces the velocity n, and the latitude from 4 : 1 to 8 : 1 also produces n."[191]

With two consecutive arguments it is pointed out that if latitudes of speeds were not "parallel" to the latitudes of the ratios of F to R, thus there would be a part of the latitude of speed unpaired with any part of the latitude of ratio.[192]

A B Q

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C D

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In the subsequent argument about exploiting the calculus of ratios Dumbleton says:

> It follows from this position that 9 acts just as quickly in 4 as 12 in 4, for 9 acts in 4 according to two ratios of 3 : 2 or to a double ratio of 3 : 2. Therefore 9 acts twice as much in 4 as 6 acts in 4 because 6 acts [in 4] only according to one ratio of 3 : 2. This follows from the foundation of this position because one ratio compounded or joined (*coniuncta*) with another does not hinder the other as one agent hinders another joined to it.[193]

190 See ibidem, § 27, p. 401; English translation E. Sylla, *The Oxford Calculators and Mathematical Physics*..., p. 149.

191 E. Sylla, *The Oxford Calculators and Mathematical Physics*..., p. 149.

192 See Johanes Dumbleton, *De motu locali*, §§ 28–30, pp. 401–402.

193 See ibidem, § 31, pp. 402–403; E. Sylla, *The Oxford Calculators and Mathematical Physics*..., pp. 149–150.

And then Dumbleton proposes three other arguments "treating the mathematical compounding of ratios as if it involved a physical compounding of actions," as Sylla notices.[194] By the end of Chapter 6, Dumbleton argues against this position in the following way:

> If from a ratio 2 : 1 there arises a motion A, which cannot arise from a smaller ratio, it follows that a motion double A cannot be produced except by two ratios 2 : 1, that is by a ratio compounded of two double ratios and a subdouble motion A must be produced by *medietas* of a double ratio, because the effect is not doubled unless the cause is doubled.[195]

From this, concludes Dumbleton it: "follows that as much as a motion can infinitely diminish, its speed can infinitely slow down." He concludes that the infinite latitudo of motion corresponds to a double ratio of F to R.[196]

In Chapter 7 and 8 Dumbleton discusses the third, previously mentioned opinion, which – as he declares – is the theory of Aristotle and Averroes. He says:

> The third opinion expressed by Aristotle and Averroes, claims that a speed of motion results from a geometrical proportion and it intensifies or diminishes because the next ratio of force to resistance is greater or smaller than the previous one, in such a way that the previous speed exceeds the next one with regard to the proportion of the first to the second ratio.[197]

From this opinion it is stated that:

- The speeds of different types of motion with regard to augmentation, alteration and local motion are equal if they are caused by the same ratio of F to R.

194 See Johannes Dumbleton, *De motu locali*, §§ 32–34, pp. 403–404; E. Sylla, *The Oxford Calculators and Mathematical Physics...*, p. 150.

195 See Johannes Dumbleton, *De motu locali*, § 36, pp. 404–405, English transl., E. Sylla, *The Oxford Calculators and Mathematical Physics...*, p. 150.

196 See Johannes Dumbleton, *De motu locali*, § 37, p. 405

197 See ibidem, § 11, p. 396.

- The speed which is caused by a greater ratio is faster than that caused by a smaller ratio.
- Despite the fact that F is intensifying or diminishing, if the ratio of F to R remains the same, the speed, caused by that ratio, is the same.
- The latitude of speed and the latitude of ratios correlate in the same way, i.e., they are getting bigger (*adquiruntur*) or smaller (*deperduntur*) concurrently, like the distance traversed in one day: the more space traversed, the faster the speed. Hence the proportion of equality corresponds to no-speed (a state of rest) while greater proportions correspond to greater speeds.[198]

Consequently, a ratio that is in a double or triple proportion with respect to a given, initial ratio corresponds to the doubled or tripled speed, respectively. In the next argument Dumbleton explains this correspondence with numbers. This function is illustrated also with two parallel lines (in ms. Magdalen 32), the one representing the latitude of ratios and the other the latitudes of speed.

A16	B8	C4	D2	E1	latitudes of the speed of motion corresponding to the ratio of F : R assignated below
H16	F8	G4	I2	K1	latitudes of the speed of motion corresponding to the speed of F : R assignated below

Where the line ABCDE illustrated the changes of speed and the line HFGIK the changes of rations. Thus Dumbleton explains – as Sylla claims – Bradwardine's function using the calculus of ratios, saying for example:

> Also the proportion of 16 to 1 contains the same latitude of proportion up to the proportion of equality, which is the limit of the proportion of major inequality. And when 16 diminishes to 8 it loses the double proportion, i.e., the proportion between 16 and 8, and since a double proportion is a quarter part of a whole

198 Ibidem, §§ 39–42, p. 406.

> proportion of 16 to 1, as line AB is a quarter of the line AE, the speed loses the latitude of the AB line.[199]

The following calculus presented in this argument operates on geometrical representation of doubled proportions, represented by ratios and speeds of motion. Dumbleton notices that:

> When one adds a double proportion to the proportion represented by the AE line, the speed of motion increases by one quarter, because the proportion is greater by one quarter. And if a half of a double proportion is added, thus the speed of motion AC increases by one quarter of a double, and so *in infinitum*. This is obvious in numbers, because 20 is greater than 16 by 4 (a quarter) like 32 adds to the speed of motion only by one quarter, i.e., that which is represented by the line between A and B, because 20 contains 16 and it quarter part, like the ratio 32 : 1 contains a ratio 16 : 1 and its one-fourth part, i.e., a ratio of 2 : 1. And because the number to number is 6 : 4 (*sesquiquarta*), thus the ratio to ratio is [a proportion] 6 : 4, since the ratio 32 : 1 is 6 : 4 of 16 : 1.[200]

What is most notable here is Dumbleton's proclamation: "These calculus are obvious from Campanus of Euclid's [*Elements*], Book V and the same is stated by Aristotle and Averroes in the *Physics* Book IV and VII."[201]

The next example: the line OTQM represents the latitudes of ratios and the line IXG the latitudes of the speed of motion, and the section QM of the whole latitude between 9 and 4 represents the speed G of motion:

16	12	9	4	
O	T	Q	M	latitudes of ratios
I	X	G		
				latitudes of speed

199 See ibidem, § 45, pp. 407–408; See also E. Sylla, pp. 402–406.

200 See Johannes Dumbleton, *De motu locali*, § 45, pp. 407–408.

201 Ibidem: "Ista patent per Campanum V Euclidis. Istum intellectum habet Aristoteles IV et VII *Physicorum* cum Conmentatare super eundem textum."

Dumbleton says:

When the speed of motion results from the ratio 9 : 4, thus, speed G is as far distant from rest as the ratio 9 : 4 is distant from a proportion of equality (1 : 1). So when 9 is augmented up to 16, it acquires, with respect to 4, two equal proportions, *id est*, twice a ratio of 4 : 3, because 12 : 9 = 4 : 3 and 16 : 12 = 4 : 3. Thus the whole ratio of 16 : 9 is composed of two ratios of 4 : 3 [(16 : 12)(12 : 9) = (4 : 3)(4 : 3)], like the line IG, which is composed of the two halves of this line: lines IX and XG. And because the ratio 12 : 9 is a middle ratio between 16 : 12 and 12 : 9, thus the ratio of 12 : 9 contains a half of the whole ratio of 16 : 9. Consequently an acting power of the value 12 acquires and has a half of the whole latitude of speed of motion and the proportion between G and I and between 9 and 16. But it is wrong to claim that the whole latitude of the ratio between 12 and 9 is like the latitude between 9 and 4, i.e., like the ratio between 16 and 4 is like the ratio between 12 and 4, because they have to be taken uniformly with regard the point in which an acting power increases or the resistance decreases. Like when Socrates uniformly traverses the distance AB in one hour, in the second half of his motion he traverses the parts CD and DB, equal to one quarter of the whole distance. But DA is not in the same ratio to CA as BA is to DA, but in both, in the third and last quarter of the hour, he traverses the same distance. The same refers to the uniformly acquired latitude of ratios.[202]

A E C D B

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The following consequences can be drawn from the above: 1) if the resistance is not uniform the speed of motion would diminish in the same way despite the unequal acting of the powers that cause the motion; 2) increasement of the ratio of F : R is always dependent on the initial value of the acting power with which the motion commenced; 3) the latitude of the speed of motion to be added either to a more intense or remiss degree equally relates to the distance traversed; 4) the halves of each latitude ending at rest have the same intensity, not with regard

202 See ibidem, § 46, p. 408. See also E. Sylla "The Oxfrod Calculators and the Mathematics of Motion 1320–1350…", pp. 404–405.

to the duration of time, but with regard to the latitude, this equally concerns the latitudes of speed and the latitude of the ratio of F : R; 5) the speed of motion, and the ratio and all such qualities, either in reality or in the imagination consist of actual qualitative parts. Since all qualities consist of divisible parts and the parts of the same quality can be added, thus more intense speed results from a greater ratio of F : R. It is faster, simply because the new ratio was added to the previous one. The motion begins with the proportion of equality when F is equal to R, so the speed of motion starts from rest up to infinity, since Dumbleton is of the opinion that the line, representing the latitudes has no end.[203]

As Sylla rightly alleges:

> In Dumbleton's terms, the representation of the latitude of proportion is the only natural one, because he regards the compounding of proportions as addition and thus regards the latitude of proportion as "linear" scale of proportion, where equal differences of proportion or equal proportions are represented by the equal lengths of the line or latitude. This is all in accordance with the Calculator's understanding of proportions <as additive qualities> discussed in the first part of this chapter.[204]

This way of pairing latitudes of proportion and speed of motion has few advantages over a description by geometrical proportions of the ratios of Fs to Rs. First, it clearly illustrates the possible variety of speeds within the range of zero to one speed, because a motion begins with the proportion of equality which corresponds to no-speed. Thus, Dumbleton develops one of the most important conclusion that can be drawn from Richard Kilvington's theory, namely that any excess of the acting force over the resistance is not only sufficient for motion to occur, but also to start the motion, and that in order to describe the motion properly one must use the continuous proportion. This theory, later mathematically elaborated by Thomas Bradwardine and accepted by John Dumbleton in his geometrical representation of proportionalities, as is seen above, "involves only paring the variables and not equating them

203 See Johannes Dumbleton, *De motu locali*, §§ 47–55, pp. 408–411.

204 See E. Sylla, "The Oxfrod Calculators and the Mathematics of Motion 1320–1350…", p. 406.

as in a proportionality, so the question of dimensionality can be sidestepped if desired" as is pointed out by Sylla.[205]

Secondly, Dumbleton is the first to perfectly explain how to understand a double, a triple, a quadruple proportion and the corresponding variations of speed. He says that to double proportion means that we take the same ratio twice, i.e., we "add" these two equal ratios one to another, the triple proportion means that we add the same ratio three times, and so on, therefore it perfectly shows the changes of the speed being caused by these ratios. Dumbleton emphasizes the additivity of the constituents of local motion.

Thirdly, Dumbleton considers not only the cases when ratio to ratio is proportional with regard to the integer (2 : 1, 3 : 1), but also with regard to fractions like 3 : 2, 3 : 4, and it correctly illustrates the constant changes of ratios in the causes of motion and the effect of their mutual action – the speed of motion.

The limitation to the pairing of latitudes of ratios and speeds is that each latitude must represent the whole range of possible variations of the given variable, and "it was considered necessary – as Sylla notices – that latitudes be matched in their entirety and not part of one latitude to the whole of another."[206] The latitude of ratios of greater inequality has only one limit, i.e., its lower limit – the ratio of equality; it does not have the upper limit, since there is no greatest ratio. It also refers to the speed of motion, which does not have the upper limit, but has the lower limit equal to no-speed (no-degree of its intensity, in actual medieval terms), since there is no minimal speed just above rest, because any excess of F to R is sufficient for motion.

From all this is follows, as Dumbleton argues in §§ 60–69, Chapter 8 that the latitude of minor inequality does not represent any motion, since in such motion the speed would be valued by less than zero. He argues that it is right to say that the proportion of minor inequality becomes greater and greater, which means that, for example, a proportion of 1 : 2 is greater than 1 : 4 and this is greater than 1: 8, but this does not mean the ratios of the factors of motion, i.e., of active power and resistance become greater, and in a proper way: "a latitude when it is the latitude of motion must be described by the intensification of the quality and enlarging the quantity." Thus, although, the latitude of quantity,

205 See ibidem, p. 407.

206 See ibidem, p. 408.

represented by the line describing the proportion between, e.g., (1 : 2) … (1 : 16), becomes longer, the latitude of quality – the speed of motion, does not increase. Therefore, the speed of motion is not caused by the proportion of minor inequality.[207]

In Chapter 9 of Part III of his *Summa* Dumbleton goes back to the argument presented in the beginning of the discussion about the "measurement" of local motion.[208] He argues against the Aristotelian rule, as presented in his Book VII of the *Physics*, which states that a doubling of an acting power would cause the double speed of motion, i.e., multiplied by 2. Dumbleton is of the opinion that "in every action the whole action is the action of the whole agent and all of its parts."[209] This means that the result of action of the parts does not have to be the same as the result of the operation of the whole composed of these parts. Dumbleton agrees with Aristotle hereby, while claiming that the motion is the result of action of the whole acting power on the whole resistance.[210]

5.1. The Mean Speed Theorem

In Chapters 10–12 of Dumbleton's *Summa* herein described he also presents his proofs of the so-called Mean Speed Theroem, offered first in William Heytesbury's works.[211] Recently, the theorem is called the mean degree theorem.[212] Dumbleton states as a supposition the additivity of degrees of motion that he had argued for in Chapter 7. If there are two equal or unequal degrees of uniform motion R and D, then together they will lead to exactly as great a distance being traversed in an hour as a degree which adds above D as much as R

207 See Johannes Dumbleton, *De motu locali*, §§ 60–69, pp. 413–416.

208 See ibidem, § 70, p. 416.

209 See ibidem, § 74, p. 417.

210 See ibidem, § 75.

211 See above, pp. 89–91.

212 See E. Sylla, *The Oxford Calculators' Middle Degree Theorem in context*, "Early Science and Medicine" 15 (2010), p. 353; S. Rommevaux-Tani, *The study of local motion…* (forthcoming).

was distant from a no-degree.[213] By this means, any difform motion starting with the speed above zero can be divided into two motions, one uniform motion at the degree from which it began to increase its speed, and the other increasing its speed from zero in the same way that the original increased its speed over the starting velocity. The two conclusions of Chapter 7 are the key to Dumbleton's proof of the middle degree theorem, which involves a division of a uniformly difform motion into parts, so that in the more intense half of the motion, there is a uniform motion at the middle degree, plus an accelerated motion just like that in the first half. Then any choice of an equivalent uniform degree to the whole motion must be consistent with the choice of uniform degrees equivalent to the two identical accelerated parts of the motion in the first and second halves of the motion. Any choice but the middle degree leads to inconsistencies.[214]

6. Richard Swineshead's Speculative Science of Local Motion

The description of the style and character of reasonings included in the "Book of calculations" presented in the first chapter of this monograph, that is generally it contains only complex logicomathemathical analyses with no relation to natural, observable phenomena fits especially well the chapter "On local motion" (*De motu locali*), which given the context interests us most here. This section of *Liber calculationum* is generally a sequence of 58 consecutive conclusions or "rules" (*regulae*) concerning mainly the "measurement" of the changes of speeds relative to varying motive forces and resistances. The initial 27 conclusions are derived from the precedent, already proven ones *more geometrico* on the basis of the first and only laconic assumption (*suppositio*) that: "motion is measured in terms of geometrical proportion."[215] The remaining conclusions (28 to 58 in order) are, for the most part,

213 See Johannes Dumbleton, *De motu locali*, §§ 78–83, pp. 418–422; E. Sylla, *The Oxford Calcultors' Middle*, p. 356.

214 See M. Clagett, "Science of Mechanics," pp. 305–325; see also Idem, *The Place of John Dumbleton in the Merton School*, "Isis" 50 n.4 (1959), p. 452–453.

215 Richard Swineshead, *Liber calculationum: Tractatus de motu locali*, § 1, [in:] R. Podkoński, *Suisetica inania*, (referred further as *Tractatus de motu locali*), p. 271: "Suppo-

accompanied with more or less extensive reasonings. The manner the treatise was constructed imitates clearly the method of Euclid's "Elements", and had been employed earlier also by Thomas Bradwardine in the third chapter of his *Tractatus de proportionibus*.[216] This very fact suggests that the treatise *De motu locali* was intended from the outset to be an exemplary complete realization of the speculative science of local motion observing the postulates formulated by Aristotle in this respect.[217] While remaining within the boundaries of Aristotelian natural philosophy, Richard Swineshead, it seems, considered all the possible imaginary configurations of changes in the factors of motion that could be drawn and proven *more geometrico* from the first assumption explicitly and Aristotelian "equations" concerning local motion implicitly. It is worth here noting that in none of his texts concerning local motion did Richard Swineshead bother to explain the reasons as to why he accepted the "geometrical proportion" as the adequate one in his account of the "rules" of motion, neither did he provide the details of the method of *calculationes* in this regard. Only in the short treatise (*opusculum*) "On motion" did he briefly present the general summary of the calculus of ratios with respect to the specific of motive powers and resistances:

> it follows that universally for the [speed of] motion to be doubled it is required that the proportion is doubled (…) But since not always when the [motive] power is doubled is the speed doubled in result, but in a certain case, that is in a case when the motive power is precisely in a double proportion to the resistive [power], and the same must be understood with regard to dividing the resistancy in half.[218]

nendo motum attendi penes proportionem geometricam quedam hic de motu locali regule exarantur."

216 See Thomas Bradwardine, *Tractatus de proportinibus*..., pp. 110–116.

217 See Aristotle, *Posterior analytics*, 71b8–72b4, I.2, G.R.G. Mure (transl.), [in:] "Basic works of Aristotle", pp. 111–113. The same manner was adopted by Benedict de Spinoza (1632–1677) in his *Ethics* and Christian Wolff (1679–1754) in his *Ontology*. See also, G. Olson, *Measuring the Immeasurable: Farting, Geometry, and Theology in the Summoner's Tale*, "The Chaucer Review" Vol. 43, no. 4, pp. 414–417.

218 Richar Swineshead, *Opusculum de motu*, §§ 12–13, p. 139.

Next he explained that this is so because: "not always when the [motive] power is doubled or a resistance is halved is the proportion doubled in result, but the [speed of] motion is doubled if and only if the proportion is doubled", with the final, additional remark, that; "these statements are obvious on the basis of many well known conclusions on proportions."[219] This seems to be an unambiguous reference to Thomas Bradwardine's *Tractatus de proportionibus* and suggests that the "new rule of motion" was already commonly accepted in Oxford at the time of the composition of this *opusculum*. This remark aside there follow additional explanations with regard to "doubling" the speed only in effect of the increasement of the motive power, relative to the initial ratio of this proportion to resistance. In such cases:

> it is necessary that [a motive power] increases proportionally above its [initial] degree as much as it is now greater compared to a given resistance. If it is twice greater (*in duplo maior*), then it is necessary that it is doubled (*dupletur*); if it is three times [greater] (*in triplo*), then it is necessary that it is triplicated (*tripletur*), and so on *in infinitum*.[220]

The algorithm introduced here guarantees that the resulting ratio of motive power to resistance will be "doubled," (i.e., squared in our terminology), relative to the initial ratio. Rather more difficult, but an equally effective algorithm is further provided with respect to resistive power:

> if the resistance is now half [the motive power] (*subdupla*), it is necessary that it will be halved (*subdupletur*) [i.e., a half of the initial resistance remains finally]; if the resistance is three times less [compared to the motive power] (*subtripla*), it is necessary it is divided into three (*subtripletur*) [i.e., the third part of the initial resistance remains]; and so on *in infinitum*.[221]

Surprisingly enough, the second short treatise "On local motion" ascribed to Richard Swineshead contains no explanation as to the method

219 Ibidem, § 14, p. 139.
220 Ibidem, § 15, p. 139.
221 Ibidem, § 16, p. 140.

of inquiry employed, even though the "geometrical proportion" is obviously implicitly assumed in all the conclusions presented there.[222]

The treatise *De motu locali* commences with the "rules" concerning the action of a single power relative to a single resistance, next the description of the results of the action of two varying powers relative to the same resistance or the action of one power relative to two different resistances changing in an uniform manner are introduced:

> (Conclusion 1) Wherever a [motive] power increases with respect to constant resistance, it acquires such a ratio with respect to this constant resistance, by which this [power] will be greater.[223]
>
> (Concl. 2) If a [motive] power diminishes with respect to some constant resistance, it loses such a ratio with respect to this resistance, by which this power will be lesser.[224]
>
> (Concl. 3) Wherever a resistance increases or diminishes with respect to some constant [motive] power, this power loses or acquires such a ratio with this resistance, by which ratio this resistance will be greater or smaller.[225]

222 It must be stressed here that both these *opuscula* should be seen as Richard Swineshead's consecutive approaches to the problem of the "proper" description of the "rules" of local motion, mutually independent in such a sense that they cannot be taken as parts of a greater whole. This can be confirmed by a fact that in both these short treatises we encounter the very same rules, formulated a bit differently, and proved in a bit different manner. See R. Podkoński, *Richard Swineshead's...*, pp. 53–57, 77–78; J.E. Murdoch, E.D. Sylla, *Science of Motion*, p. 206.

223 Richard Swineshead, *Tractatus de motu locali*, § 2, p. 271: "Ubicumque aliqua potentia crescit respectu resistentie non variate, tantam proportionem acquiret respectu illius resistentie non variate per quantam ipsa fiet maior."

224 Ibidem, § 4, p. 272: "si aliqua potentia decrescit respectu alicuius resistentie non variate, tantam proportionem deperdet respectu illius resistentie, per quantam ista potentia fiet minor."

225 Ibidem, § 5, p. 272: "Ubicumque aliqua resistentia crescit vel decrescit respectu alicuius potentie non variate, tantam proportionem deperdet illa potentia vel acquiret cum illa resistentia, per quantam proportionem illa resistentia fiet maior vel minor."

> (Concl. 4) Wherever a [motive] power increases or decreases with respect to two resistances, equal or unequal, it acqures or loses its motion equally fast with respect to any of them.[226]
>
> (Concl. 5) Wherever a resistance increases or decreases with respect to two constant [motive] powers, equal or unequal, both these powers lose or acquire the same ratio, and equally fast will lose or acquire their motions with this resistance.[227]

Three initial conclusions, in a sense, can be seen as a conscious presentation of how the *calculationes* should be employed with respect to changes of motive power and resistance in local motion, of course, inasmuch as one is already acquainted with the calculus of ratios in general. Richard Swineshead simply stated there that the changes of the speed of motion are relative to the "addition" or "substraction" of ratios. This assumption he employed already in the proof of Conclusion 4:

> This follows evidently from the first and second conclusion taken together with the initial assumption. It [i.e., a motive power] acquires or loses the same ratio (*equalem proportionem*) with respect to each [of these resistances] which with respect to the very self, and the [speed of] motion universally follows the ratio therefore, etc.[228]

Further on more and more complicated configurations are considered, and the conclusions are alternated in the same manner as shown above: growing-diminishing, motive power(s)-resistance(s), and so on. Finally in the treatise "On local motion" we find the conclusions deter-

226 Ibidem, § 7, p. 272: "Ubicumque aliqua potentia crescit vel decrescit respectu duarum resistentiarum, sive equalium sive inequalium, eque velociter intendet vel remittet motum respectu utriusque."

227 Ibidem, § 9, p. 273: "Ubicumque aliqua resistentia crescit vel decrescit respectu duarum potentiarum equalium vel inequalium non variatarum, equalem proportionem ille potentie deperdent vel acquirent, et eque velociter remittent vel intendent motus suos cum illa resistentia."

228 Ibidem, § 8, p. 272: "Hec per primam et secundam conclusiones addita suppositione prima concluditur evidentissime. Equalem enim proportionem acquiret vel deperdet respectu utriusque, quantam respectu sui ipsius, et motus universaliter sequitur proportionem, ideo etc."

mining the (approximate) mean speed of motions that change "faster and faster" or "slower and slower", that is of the specific subgenus of difformly difform local motions.[229]

It must be stressed here that with all the 58 conclusions formulated in this chapter Richard Swineshead did not exhaust the full range of the imaginable cases of changes of factors of local motion with respect to speed and the ensuing mathematical relations between these variables. In the next treatise of the "Book of calculations," i.e., in *De medio non resistente* quite a substantial number of even more complicated cases is further considered.[230] Contrary to the suggestion included in the traditionally accepted title of this chapter, Swineshead here did not ponder on the motions in a void, which were impossible in the Aristotelian worldview.[231] The conclusions discussed concern either the cases when the resistance of the medium increases from a no-degree (*a non gradu*) of intensity, or the motive power increases from a no-degree of its intensity, or both factors increase from a no-degree simultaneously.[232] Thus, the non-resisting medium appears here always, and only, as a theoretically assumed initial, instantaneous stage of a given, considered case. Most probably Swineshead introduced here such assumptions in order to make the subsequent calculations somewhat simplier. It seems obvious, and had been explicitly stated before by William Heytesbury, that it is much easier to consider the cases where only one terminus of a change is taken into account, the other being a no-degree, since the

229 See footnotes 254–259 below.

230 I follow the order of treatises according to the printed versions of Richard Swineshead's *Liber calculationum* here. In most preserved manuscript copies of this work treatise *De medio non resistente* is transcribed after the treatise *De loco elementi*. See R. Podkoński, *Richard Swineshead's* Liber calculationum *in Italy. Some Remarks…*, pp. 312–313, 337–338; Idem, *Richard Swineshead's* Liber calculationum *in Italy. The Codex Bibl. Naz. San Marco, lat. VI.226 and its Significance*, "Recherches de Théologie et Philosophie médiévales" LXXXIV 2 (2017), pp. 407–421.

231 See Aristotle, *Physics*, 215b12–216a11, IV.8.

232 The contents of this treatise are much better described in the "table of contents" included in the Vatican, BAV, vat. lat. 3095 copy of the "Book of calculations", where the treatise is divided into two chapters entitled: *Conclusiones de motu locali ubi in medio non resistente est generatio latitudinis resistentie partibilis quo ad subiectum*, and: *Conclusiones de motu locali quando medium est uniformiter difforme ad non gradum terminatum a cuius extremo remissiori incipit crescere potentia a non gradu*, respectively. See R. Podkoński, *Richard Swineshead's* Liber calculationum *in Italy. Some Remarks…*, p. 316.

whole latitude of this change effectively equals the degree of intensity in this, first or last, terminus.[233] These circumstances notwithstanding, Richard Swineshead in the treatise *De medio non resistente* more or less purposedly slightly crossed the boundaries of Aristotelian natural philosophy, while introducing the imaginary case of a local motion that commences when both motive power and resistance increase simultaneously from a no-degree of their intensities.[234] It has been already stated that one of the basic conditions for motion to begin and occur as commonly accepted in Aristotelian medieval natural philosophy was the dominance of the motive power over the resistance, but in this very case these seem to equal each other in absolute terms. Similarly, within the course of one of the reasonings included in the same section of the "Book of calculations" Richard Swineshead stated arbitrarily that something could begin to move infinitely slowly when the ratio of factors of its motion would be a *proportio equalitatis*, that is the values, or degrees of the intensities of motive power and resistance would be as 1 : 1.[235] We must not forget, however, that such instances were assumed there only *secundum imaginationem*, as purely hypothetical, initial stages of motions taken as, and in Swineshead's own words, external termini of the motion considered.[236]

Thus, in addition to 58 "rules" concerning the different configurations of changes in the factors of motion contained in *De motu locali*, Richard Swineshead formulated 26 consecutive conclusions in the trea-

233 See William Heytesbury, *De motu locali*, §§ 33–38, pp. 280–283.

234 See Richard Swineshead, *Liber calculationum: De medio non resistente*, Venetiis 1520, f. 52ra: "Si in medio uniformiter difformi terminato ad non gradum incipiat potentia crescere a non gradu uniformiter continue crescens, et ad aliquem punctum intrinsecum inciperet aliqua potentia crescere a non gradu uniformiter sicut alia, ipsa continue intendet motum suum."

235 Ibidem, f. 50ra: "Et per consequens, cum resistentia extremi intensioris sit equalis sue potentie, sequitur quod infinite tarde incipit moveri, quia a proportione equalitatis seu infinite modica proportione maioris inequalitatis."

236 Ibidem, f. 54rb: "Ideo pro solutione huius argumenti est notandum quod quando aliquid movebitur ab aliquo instanti continue remittendo motum suum vel intendendo, nulla est maxima velocitas que immediate post tale instans movebitur, sed aliquis est gradus a quo exclusive incipiet motus intendi vel remitti (…). Consimiliter deducitur, quod si motus alicuius incipiat intendi vel remitti, qui continue tardius et tardius intendetur vel remittetur velocius et velocius, nullus est gradus intensissimus quo immediate post hoc intendetur vel remittetur."

tise *De medio non resistente*. These, however, were not derived nor proven *more geometrico*, with most probably this being the main reason why they were compiled in a distinct chapter. Still, with respect to the manner of the presentation of these conclusions, we recognize Richard Swineshead's typical "pattern" of formulating consecutive discussed cases. The very first "rule" in *De medio non resistente* is as follows:

> If there would be a non-resisting medium in which something mobile moves locally, and from one end of this medium a latitude of resistance would begin to increase partially uniformly difformly, in such a manner that the whole latitude extends uniformly difformly in that part of the medium where it is extending, and its every degree moves [i.e., increases] uniformly, while the less intense end remains unchanged; the movement of this mobile beginning from this end of this medium would remain continually uniform, when all the external impediments or assisting factors are omitted.[237]

In fact, the above case, or conclusion, could at the same time be seen as the initial assumption of the whole chapter of the "Book of calculations" presently described. The next discussed case that, in Swineshead own words: "is derived from the above one" (*ex isto deducitur*), differs only in that the mobile begins to move from the most intense end of the same medium. Interestingly enough, he proved further that in consequence this mobile will move faster and faster.[238] And in the consecutive case it is assumed that the resistance of the medium diminishes in such a manner that it is continually distributed uniformly difformly

237 Ibidem, f. 48vb: "Si sit medium non resistens <in> quo aliquod mobile movet localiter a cuius uno extremo incipiat esse partibilis acquisitio latitudinis uniformiter difformis resistentie, extremo remissiori quiescente, manente continue tota illa latitudine uniformiter extensa per partem, per quam extendetur, omnique gradu ipsius movente uniformiter; motus illius mobilis incipientis ab extremo eiusdem medii manebit continue uniformis, deductis aliis impedimentis et iuvamentis extrinsecis."

238 Ibidem, f. 49va: "Ex isto deducitur, quod ubicumque in tale vacuum seu medium non resistens inducitur latitudo uniformiter difformis, extremo remissiori quiescente, cuius omnis gradus continue intendat motum suum, ita tamen quod latitudo maneat uniformiter difformis, mobile incipiens motum in illo extremo movebitur continue velocius et velocius."

from the no-degree of its intensity, and – similarly to the precedent case – there is a mobile that begins to move from the most intense end of this medium. It is proven that in this case the mobile will move more slowly.[239]

Following his typical pattern of formulating the consecutive conclusions Swineshead next discussed the cases of two motions in the same medium, either resulting from different motive powers, or beginning in differently resistant points of the same medium. What is common for these cases is that the assumed motive power(s) is always constant.[240] Only the twelfth conclusion of this part of the "Book of calculations" concerns a case where not only does the resistance increase in the described way, but also the intensity of the motive power increases concurrently from the no-degree. The case is obviously similar to the one assumed in the first, above-presented, "rule" of *De medio non resistente*; the only difference being that here the motive power is supposed to change. The resulting motion will be accelerated, according Richard Swineshead's reasoning herein presented, yet the rate of the acceleration will be continuously lesser and lesser.[241]

239 Ibidem: "Et si quilibet gradus remittat motum suum, illud movens remittet motum suum."

240 See e.g., ibidem, f. 50ra: "Si ad unum punctum vel gradum in huiusmodi medio progrediente latitudine intendatur motus aliqua velocitate ad gradum minorem resistentie pro eodem instanti tardius intendetur motus quam ad illum gradum magis resistentem, sive illa latitudo resistentie terminetur ad non gradum sive ad certum gradum in extremo suo remissiori" (...); f. 50rb: "Ubi intendit una potentia motum suum ad aliquem gradum resistentie, mota latitudine modo dicto, omnis potentia maior que ibi intenderet motum suum tardius intenderet quam potentia data. Omnis que minor velocius intenderet ad illum gradum quam maior. Et infinite tarde intenderet aliqua potentia motum suum cum illo gradu."

241 Swineshead himself did not use the notion of acceleration, neither the 'rate of acceleration', but for the sake of clarity we introduce these notions here, as equivalents of the actual formula. Richard Swineshead, *Liber calculationum: De medio non resistente*, f. 51ra: "Si ab extremo medii non resistentis generaretur latitudo resistentie quiescente extremo remissiori, cuius quilibet gradus uniformiter a non gradu intendat motum suum manente illa latitudine continue uniformiter extensa; et in eodem extremo incipiat potentia motiva crescere a non gradu, que etiam uniformiter crescat, illa tardius et tardius intendit motum suum deductis aliis impedimentis et iuvamentis extrinsecis."

In terms of the assumed initial conditions, this is perhaps one of the most complicated cases discussed in this part of the "Book of calculations" as well as in every treatise of this work dedicated to the description of local motion.[242] All the subsequent "rules" in the treatise *De medio non resistente* concern more "static" situations in the sense that all describe the motion in the uniformly difformly resistant medium relative to the distance from the non-resisting point of this medium.[243] The factor that undergoes changes there is (or are) motive power(s) only.[244] It must be pointed out here also that, in the terms of the complexity of the *calculationes* involved in the proofs of the conclusions, the treatise *De*

242 The other one being the above-mentioned case of a long, heavy rod traversing through the centre of the Earth. The *calculationes* provided there, both in numbers and in general terms, are perhaps the most advanced ones in all the Oxford Calculators's scientific writings (see R. Podkoński, *Suisetica inania...*, pp. 165–170). We will not present here the contents of the treatise *De loco elementi* from the "Book of calculations" deliberately, even though – in a broad sense – it concerns the local motion too. Firstly, the contents of this section of *Liber calculationum* are quite well described in the secondary literature. Secondly, despite these truly advanced *calculationes* developed within this text, they were conducted *gratia artis* in fact, since the final conclusion – accepted quite arbitrarily – is opposite to the statement that follows from these calculations. See Richardus Swineshead, *Liber calculationum: De loco elementi*, [in:] M.A. Hoskin, A.G. Molland, "Swineshead on Falling Bodies", pp. 168–182.

243 According to order of treatises and chapters of the "Book of calculations" included in the above-mentioned manuscript copy Vatican, BAV, Vat.lat. 3095 all these conclusions belong simply to the separate chapter of this work, see footnote 201 above.

244 For example, we find there the following conclusions. *Liber calculationum: De medio non resistente*, f. 51rb: "Si sit medium uniformiter difforme terminatum ad non gradum a cuius extremo remissiori incipit potentia moveri crescens uniformiter a non gradu et movens continue secundum proportionem potentie ad resistentiam sibi immediatam, ista potentia continue movebitur uniformiter"; f. 53rb: "Si ad aliquem punctum medii uniformiter difformis terminati ad non gradum incipiat potentia moveri, que continue per uniformem intensionem sue potentie uniformiter continue movebitur; quecumque potentia minor posset ad aliquem punctum incipere moveri, que tamen eque veloci intensione sue potentie intenderetur, sicut erit intensio prime potentie; uniformiter continue movebitur." All these remaining fourteen conclusions of the treatise "On non-resisting medium" in the above-mentioned "table of contents" of the "Book of calculations" included in the ms. Vatican, BAV, Vat.lat. 3095 codex are considered as the separate chapter.

medio non resistente is much more difficult to follow for the modern reader when compared to the treatise *De motu locali*. In this respect Richard Swineshead's chapter "On non-resisting medium" from *Liber calculationum* is similar to the part of John Dumbleton's *Summa…* on local motion, with the important difference that Swineshead's arguments are mathematically and logically consistent and acceptable, even if – as Murdoch and Sylla have remarked – the conclusions included in this treatise are perfect examples of his mathematical ingenousness, not in that he did complex mathematics, but in that he knew how to avoid complex mathematics.[245] We do not mean, of course, that the chapter is easily understandable for a person not well acquainted with the method of inquiry adopted by the Oxford Calculators. However, within the chapter one rarely encounters analyses involving the calculus of ratios, and if there are any such analyses, they are based on general terms. Only once within the whole do we find numerical values of the factors of motion, arbitrarily accepted in fact.[246] For the most part the chapter "On non-resisting medium" includes such lengthy reasonings. What is more, among the conclusions herein discussed we are provided with a (almost) strictly numerically determined final value of the relation between the speeds of assumed motions:

> If one [motive] power increases uniformly from no-degree twice as fast as the other, and [the motion of] the lesser of these will result from the double proportion in such a uniformly difformly [resistant] medium that the limit of which is no-degree, the greater [power] would move more than four thirds faster than the lesser [power] and not one-and-a-half times faster.[247]

245 Johannes Dumbleton, *De motu loclai*, *passim*. J.E. Murdoch, E.D. Sylla, *Swineshead*, p. 204.

246 See Richard Swineshead, *De motu locali*, § 53, p. 285: "Sicut, si medium esset uniformiter difforme a 4 usque ad 8 et sit A ut 12, et habeat B equalem proportionem ad extremum remissius medietatis intensioris, scilicet triplam. Cum ergo illud extremum sit ut 6, patet quod B erit ut 18. Sed proportio 18 ad 8 est maior quam proportio 12 ad 6, quia proportio 12 ad 6 est precise dupla, et proportio 18 ad 8 est dupla sexquiquarta."

247 Richard Swineshead, *Liber calculationum*, f. 52ra: "Si una potentia crescat a non gradu uniformiter in duplo velocius alia in huiusmodi medio uniformiter difformi terminato ad non gradum, quarum minor movebitur a proportione du-

And in the reasoning accompanying the above conclusion it is concluded that the calculated proportion between the speeds of these motions is the "half of the double proportion" (*medietas duple proportionis*) that equals, in our modern terms, the ratio $\sqrt{2} : 1$.[248] What is more, Swineshead pointed out accurately that the value of this proportion lies between one-and-a-half and four thirds (*inter sesquialteram et sesquitertiam*), but at the same time he remarked that calculating the exact value of this proportion would require much more labour than is worthwhile.[249]

The most advanced *calculationes* in the "Book of calculations" we encounter is perhaps its best known treatise: *De loco elementi*.[250] Within this chapter the already mentioned imaginary case of a long heavy rod traversing through the centre of the Earth is discussed. The problem itself could be formulated only within the assumptions of Aristotelian natural philosophy. Simply speaking, if we accept that all heavy bodies by nature move towards the centre of the Earth, then what would happen if such a long body fell through this centre – that is, when some length of it were to traverse beyond this point while the greater part is still moving towards this centre? Obviously, as it seems, these parts beyond would serve as a resistance relative to the remaining ones. Given the resistance would continuously increase, there arises the main question as to when the rod would stop, i.e., how long a period of time would have to pass before its middle point were to coincide with the centre of the Earth?[251] On the basis of these assumptions one could conclude that, paradoxically, the middle point of the rod would never

pla; maior movebitur plus quam in sesquitertio velocius minori sed non in sesquialtero velocius."

248 See A.G. Molland, *Continuity and Measure*, "Miscellanea Mediaevalia" 16/1(1983), p. 138; S. Drake, *Bradwardine's function*..., p. 60.

249 See Richard Swineshead, *Liber calculationum*, f. 52ra: "movebitur velocius quam b aliqua proportione inter sesquialteram et sesquitertiam; que tamen sit ista maius requireret studium quam induceret de profectu". With respect to the numerical values of these ratios, it is obviously true that $4 : 3 < \sqrt{2} : 1 < 3 : 2$.

250 M.A. Hoskin, A.G. Molland, *Swineshead on Falling Bodies: An Example of Fourteenth-Century Physics*, „British Journal for the History of Science", 3(1966), pp. 150–182 (all the following quotations are taken from the preliminary edition of the treatise *De loco elementi* included there); J.E. Murdoch, E.D. Sylla, *Swineshead*, pp. 198–199.

251 The case appears first in Walter Burley's commentary on Aristotle's *Physics*, cf. M.A. Hoskin, A.G. Molland, *Swineshead on Falling Bodies*..., pp. 151–152.

really reach the centre of the Earth, since it would move towards this centre more and more slowly *in infinitum*.[252] Richard Swineshead ingeniously reduces the problem to an analysis of the continuously and mutually dependent changes of lengths that are related strictly to the continuously varying values of motive power and resistance. Finally he arrived at the conclusion that the resultant changes of speed would occur proportionally faster compared to the succesive distances traversed by this rod, and thus it would effectively move infinitely slowly some time before its middle point were to coincide with the centre of the Earth.[253]

252 See Richard Swineshead, *De loco elementi*, [in:] M.A. Hoskin, A.G. Molland, *Swineshead on Falling Bodies...*, pp. 168–169: "Item sequitur quod, licet esset vacuum undique citra centrum vel medium non resistens, et una pars foret ex una parte centri et alia pars minor foret ex alia parte centri, ita quod illud moveretur continue secundum proportionem partis citra centrum ad partem ultra centrum, nunquam posset devenire ad centrum ita quod eius medium foret medium mundi."

253 See ibidem pp. 176–177: "Dividatur ergo distantia inter *c* centrum ipsius terre et D centrum mundi in partes proportionales, et sint minores partes versus D, et arguitur sic. Pertransita prima parte proportionali istius distantie a *c* puncto, deperdetur medietas distantie inter C D puncta; et distantia inter C D continue erit equalis medietati excessus partis citra centrum ad partem ultra. Ergo medietas excessus partis citra D ad partem ultra erit deperdita, et quantum aufertur a parte citra D centrum addetur parti residue precise, ut notum est. Ergo, per primam regulam, proportio partis citra D centrum ad partem ultra minorabitur ultra subduplum, et per consequens motus eius erit tunc remissior quam subduplus ad proportionem prehabitam seu motum prehabitum. Et sic etiam pertransita secunda parte proportionali minorabitur excessus ad subduplum, quia in omni parte proportionali istius distantie minorabitur distantia inter C D puncta ad subduplum, que distantia continue erit subdupla ad excessum partis citra D ad partem ultra. Et sic in principio cuiuslibet partis proportionalis posterioris erit motus remissior quam subduplus ad motum habitum ad principium partis proportionalis istius distantie immediate precedentis. Ergo si super quamcumque partem proportionalem moveretur gradu quo ad principium istius movebatur, in nullo tempore finito deveniret C punctus ad D punctum. Sed nunc super omnem partem proportionalem erit motus tardior quam esset tunc, quia super omnem partem proportionalem remitteretur motus plus quam ad subduplum. Ergo nunc in nullo tempore finito deveniet C punctus ad D punctum. Consequentia ista nota est. Et consequentia proxima etiam patet. Quia, si super omnem partem istius distantie moveretur C punctus equaliter sicut movebitur ad principium istius partis, ex quo secunda pars proportionalis est in duplo minor prima, et motus in secunda esset minor quam subduplus ad motum in prima, maius tempus requireretur ad pertransitionem secunde

The reasonings that led to the above conclusion have already been well discussed and presented with the help of modern mathematical notation and functions by Murdoch and Sylla in their summary of the "Book of calculations" as well as by Hoskin and Molland in their introduction to their scholarly edition of *De loco elementi*.[254] In our opinion one of the most impressive examples of Swineshead's proficiency in the *calculationes* are the conclusions preliminary for the reasonings he formulated and had proved beforehand. For example, in order to confirm the following reasoning:

> If there is a ratio of greater inequality between two [quantities] and the fourth part of the excess of the greater over the lesser is subtracted and added to the lesser, then finally the ratio between these will be less than a half of the initial ratio between them. It will be so, because in effect of the equal increase of the lesser and

partis proportionalis quam ad pertransitionem prime. Et, per idem argumentum, maius tempus requireretur ad pertransitionem tertie partis proportionalis quam secunde, et sic in infinitum. Ergo tunc in nullo tempore finito transiret C totam illam distantiam. Ergo, ut plus, a multo fortiori nunc non sufficit totum pertransire. Quod fuit probandum. Vel sic arguitur brevius. Ille excessus istius partis citra D ad partem ultra D tardius et tardius proportionaliter continue diminuetur. Ergo in nullo tempore finito deperdetur iste excessus ad non gradum. Consequentia tenet, quia si ad non gradum deperderetur aliquando esset aliquantus, et aliquando subduplus, et aliquando subquadruplus, et sic in infinitum, quid non esset nisi infinite proportionabiliter diminueretur, ut constat. Patet ergo consequentia, et antecedens arguitur sic. Proportionabiliter sicut motus erit tardior, ita tardius deperdetur excessus. Sed motus velocius proportionabiliter remittetur quam excessus. Ergo excessus tardius et tardius proportionabiliter remittetur. (…) per ultimam regulam, velocius proportionabiliter minorabitur proportio illarum partium, quarum scilicet una est citra D centrum et alia ultra, quam iste excessus. Et eque proportionabiliter minorabitur motus cum ista proportione, quia motus iste correspondet illi proportioni et continue correspondebit. Ergo velocius proportionabiliter continue minorabitur motus, quam excessus unius partis supra aliam. (…) Sequitur ergo intentum quod tardius et tardius proportionabiliter minorabitur ille excessus, quia si sic nunquam deveniet ad non gradum. Sed, si *c* punctus deveniet ad D, excessus iste ad non gradum diminueretur, quia tunc pars citra centrum parti ultra illud centrum erit equalis. Ergo nunquam deveniret *c* ad *d*, quid fuit principaliter intentum."

254 See J.E. Murdoch, E.D. Sylla, *Swineshead*, pp. 198–199; M.A. Hoskin, A.G. Molland, *Swineshead on Falling Bodies*..., pp. 153–154.

> decrease of the greater the excess of the greater over the lesser will equal the half [of the initial value].[255]

> he formulated and proved the "rule":

> If there are four continuously arithmetically proportional terms, the greatest ratio, namely the one between the smallest two out of these four [terms], exceeds the second (i.e., middle) ratio by more, than this [middle] ratio exceeds the third ratio, which is the smallest among these three ratios that can be established between these three terms.[256]

First, Richard Swineshead provided the confirmation of the above "rule" on the basis of numerical values and showed that with respect to the arithmetical series: 7, 6, 5, 4, the ratio 5 : 4 is greater than the ratio 6 : 5 and this is greater than the ratio 7 : 6, respectively. What is more, he observed further on, the proportion (5 : 4) :: (6 : 5) is greater than the proportion (6 : 5) :: (7 : 6) consequently.[257] And next, he proved

255 Richard Swineshead, *De loco elementi*, p. 169: "Si inter aliqua sit proportio maioris inequalitatis et quarta pars excessus maioris supra minus auferatur a maiori et addatur minori, tunc inter illa in fine erit proportio minor quam subdupla ad proportionem existentem inter ista duo in principio. Tunc enim excessus maioris supra minus minorabitur ad subduplum per equale crementum minoris et decrementum maioris."

256 Ibidem: "Si sint quatuor termini continue proportionales arithmetice, proportio maxima, que scilicet est inter terminos duos minores eorum quatuor, per plus excedit secundam proportionem quam ista secunda excedat tertiam, que est minima illarum trium proportionum que sunt inter illos quatuor terminos."

257 Ibidem: "Ut, si isti essent quatuor termini continue proportionales arithmetice sicut 7, 6, 5, 4, tunc inter 5 et 4 est maior proportio quam inter 6 et 5, et inter 6 et 5 est maior proportio quam inter 7 et 6. Ideo probandum est quod proportio 5 ad 4 per maiorem proportionem excedit proportionem 6 ad 5, quam ista proportio 6 ad 5 excedit proportionem 7 ad 6. (…) Sit enim E terminus se habens ad 5 ut 5 se habet ad 4, scilicet in proportione sexquiquarta, et sequitur quod E continet 6 et quartam unitatis, per unam propositionem probatam ubi primo tangitur de motu locali: Si ad duos terminos inequales sint proportiones equales, sicut unus est alio maior ita excessus termini ad terminum illum maiorem comparati. Ponatur ergo quod F se habeat ad 6 sicut 6 ad 5, videlicet in proportione sexquiquinta, et sequitur per regulam allegatam quod F continet 7 et 5 partem unitatis. Ex his sic arguitur: proportio E ad 5 excedit proportionem

the latter conclusion again, on general terms, as follows (for the sake of clarity we reconstructed Swineshead's proof employing the modern mathematical notation):

(1) Let there be four terms A, B, C, D that are to each other as follows: $A < B < C < D$, and:
$(D - C) = (C - B) = (B - A) = G$; that is they form the arithmetical series.

(2) then let us introduce the term E such that $E : B = B : A$;

(3) from this it follows that $E > C$, since: $C : B < B : A$;

(4) consequently: $E - B > B - A$.

(5) Let us introduce the term F such that $F : C = C : B$;

(6) from this it follows that $F - C > C - B$.

(7) On the basis of the fact that $B : A > C : B$.

(8) It follows that $(E - B) : g > (F - C) : G$.

(9) Consequently: $E - B > F - C$.

(10) Since: $C - B = G$, and: $D - C = G$, then: $E - C > F - D$.

(11) Since $C < D$, then $E : C > F : D$. This is because, if we accepted that: $E - C = F - D$, then it would follow that: $E : C > F : D$, then since $E - C > F - D$, *a fortiori* thus: $E : C > F : D$.

(12) On the basis of that $E : B > C : B$, and $E : B = (E : C) :: (C : B)$; and that $F : C > D : C$, and: $F : C = (F : D) :: (D : C)$,

(13) it follows that: $(E : B - C : B) > (F : C : C - D : C)$, and it is assumed that: $B : A = E : B$, and $C : B = F : C$.

(14) Finally we arrive at the general conclusion that: $(B : A - C : B) > (C : B - D : C)$.[258]

6 ad 5 per proportionem E ad 6, eo quod proportio E ad 5 componitur ex proportione E ad 6 et 6 ad 5, et proportio 5 ad 4 est equalis proportioni E ad 5 per positum. Ergo proportio 5 ad 4 excedit proportionem 6 ad 5 per proportionem E ad 6. Et similiter proportio F ad 6 excedit proportionem 7 ad 6 per proportionem F ad 7. Sed proportio E ad 6 est maior quam F ad 7, quia E excedit 6 plus quam F excedat 7, ut patet, et E est terminus minor quam F. Ergo proportio E ad 5 excedit proportionem 6 ad 5 per proportionem maiorem quam proportio F ad 6 excedat proportionem 7 ad 6. Cum ergo proportio 5 ad 4 sit equalis proportioni E ad 5 per casum, et proportio 6 ad 5 est equalis proportioni F ad 6, patet quod proportio 5 ad 4 per maiorem proportionem excedit proportionem 6 ad 5 quam proportio 6 ad 5 excedat proportionem 7 ad 6. Et in isto exemplo patet veritas istius notabilis."

258 See ibidem, pp. 169–170: "Eadem ergo regula generaliter arguitur sic. Sint A B C D quatuor huiusmodi termini continue arithmetice proportionales, ita quod

Richard Swineshead's proficiency in the method of the *calculationes* is further confirmed here by the fact that in the above-presented reasoning there are few, let us say, "intermediate steps" that are not formulated explicitly. It seems that for Swineshead these were so obvious that there was no need to include them.[259] We encounter no fewer than three

A terminus sit minus et D maius vel maximus. Et arguitur quod proportio B ad A per proportionem maiorem excedit proportionem C ad B quam proportio C ad B excedat proportionem D ad C. Detur enim E terminus qui se habeat ad B sicut B se habet ad A, et sit excessus omnium G inter istos quatuor terminos. Et notum est quod E est maius C, eo quod proportio C ad B est minor proportione B ad A. Sicut ergo B est maius A ita excessus E super B est maior excessu B supra A, per regulam preallegatam. Sit ergo F se habens ad C sicut C se habet ad B. Et sequitur quod proportionaliter sicut C est maius B ita excessus F supra C est maior excessu C super B. Cum ergo maior sit proportio B ad A quam C ad B, sequitur quod excessus E supra B se habet in maiori proportione ad G excessum quam excessus F supra C se habet ad G excessum. Et, si sic, per plus excedit E B quam F C. Consequentia satis patet, quia omne quid se habet ad aliquid in maiori proportione, idem est maius illo quid se habet ad idem in minori proportione. Cum ergo inter C B sit G excessus et inter C D similiter, sequitur quod E plus excedit C quam F excedat D. Et C est minus D. Ergo proportio E ad C est maior proportione F ad D. Consequentia tenet, quia si esset equalis excessus precise E super C, et F supra D, proportio E ad C esset maior proportione F ad D, per hoc quod ex equali excessu inter minora resultat proportio maior quam inter maiora. Ergo a fortiori, quando maior est excessus E super C quam F super D, erit inter E C maior proportio quam inter F D. Cum ergo proportio E ad B excedat proportionem C ad B per proportionem E ad C, quia tota proportio E ad B componitur ex proportione E ad C et ex proportione C ad B, et similiter proportio F ad C excedit proportionem D ad C per proportionem F ad D, ergo proportio E ad B per maiorem proportionem excedit proportionem C ad B quam proportio F ad C excedat proportionem D ad C. Et equalis est proportio B ad A sicut E ad B, per positum, et equalis etiam est proportio C ad B sicut F ad C per positum. Sequitur ergo quod proportio B ad A excedit proportionem C ad B per maiorem proportionem quam proportio C ad B excedat D ad C. Consequentia tenet per hoc: si sint duo equalia quidcumque tertium ab illis duobus equaliter excedetur. Et sicut arguitur de istis quatuor terminis A B C D ita de omnibus aliis est arguendum."

259 These intermediate steps, in our reconstruction, can be as follows. From step (2) to step (3) there can be assumed such a line of thought: $E : B = B : A$, and: $C : B < B : A$, therefore: $C : B < E : B$. Consequently: $E > C$. From (3) to (4): if $E > C$. then it is true that: $E - B > C - B$. Since it is assumed that: $C - B = B - A$, therefore: $E - B > B - A$. From (5) to (6): it is accepted that: $F : C = C : B$, and: $D : C < C : B$, thus: $D : C < F : C$. Consequently: $F > D$. If it is so then it is true that:

subsequent, equally advanced lines of thought in the same chapter.[260] What is most surprising here is the fact that all these *calculationes* were developed *gratia artis*, since eventually Richard Swineshead states the necessity for the middle point of the rod to coincide with the centre of the Earth.[261] And with respect to the "rules" so laboriously proven he remarked explicitly that although they were employed in order to argue for the opposite, false solution, they can be useful in many other cases.[262] No wonder, then, that Italian humanists recognized the "Book of calculations" in general as *sophisticas quisquilias et suisetica inania*.[263] At

$F - C > D - C$, and it is assumed that: $D - C = C - B$, therefore: $F - C > C - B$. The most difficult to reconstruct is the line of thought from step (7) to (8). We did it analogically to the actual reasoning employed by Richard Swineshead in the step (11) actually: if it is accepted that: $B : A > C : B$, and that: $E : B = B : A$, and also that: $F : C = C : B$, then consequently: $E : B > F : C$. From the first assumption: $C > B$. Even if: $B = C$, then: $E > F$; thus *a fortiori* if it is assumed that: $B < C$, then still: $E > F$; and consequently: $E - B > F - C$.

260 See Richardus Swineshead, *De loco elementi*, pp. 170–175.

261 See ibidem, p. 178: "Ideo forte ponitur alia positio, videlicet quod pars ex una parte centri non appetit resistere nec resistit parti ex alia parte centri, neque toti ne ipsum descendat. Sed quia est pars totius, ideo appetit ut medium totius sit medium mundi'; ibidem, pp. 180–181: "Pro isto dici potest quod nulla pars resistit alteri in descensu. Et dicitur concedendo quod idem est appetitus partis cum alteri parti coniungitur sicut a toto separaretur; diversimodo tamen se habet in effectu. Et conceditur quod per eundem appetitum moveretur ad centrum, ipsa separata a toto, per quam quando est pars totius recedet a centro, ut apparet. Unde huiusmodi corpora non habent aliquem complexum appetitum: ideo dicetur quod non appetunt movere, nec appetunt quescere, nec appetunt esse in centro, nec non esse in centro; immo appetunt locum infimum in comparatione ad totum universum. Quid ipsum maxime habet quando medium eius est medium mundi, sicut dicimus quod aliquid est propinquius alteri quando secundum se totum tangit illud quam si solum secundum punctum ipsum tangeret, et etiam quando tangit cum puncto medio dicimus idem esset propinquius quam quando solum tangit secundum extremum. Ideo medium eius esse medium mundi, et ipsum esse in loco, et quiescere in loco, et moveri ad locum, sunt talia que effective consequuntur eius appetitus in complexum quid est ad centrum. Unde, sicut quelibet pars partis est pars totius, ita appetitus cuiuslibet partis erit pars totius appetitus; et, quia est pars totius appetitus ideo iuvat totum ut totum situetur in loco suo naturali, et non sibi resistit."

262 See ibidem, p. 177: "Quamvis iste regule precedentes quodammodo ad probationem huius conclusionis hic ponantur, possunt tamen ad multa alia deservire."

263 See J.E. Murdoch, E.D. Sylla, "Swineshead", p. 209.

least with respect to the chapter *De loco elementi* it is obvious that for Swineshead constructing and solving the complicated cases was a self-justified activity and needed no application to any practical, external purposes. In this respect Richard Swineshead also conformed to the Aristotelian model of theoretical science.[264]

The final remark of the treatise "On non-resisting medium" that calculating the exact value of a proportion would require much more labour than it is worthwhile reminds us of the *dictum* we find in William Heytesbury's *Regulae solvendi sophismata* where, in almost an identical wording, he discarded as totally purposeless the calculation of the intermediary distances traversed with uniformly accelerated/decelerated motions.[265] The dependence of the conclusions Richard Swineshead formulated when developing his 'science of local motion' on Heytesbury's statements, the latter included in the section *De motu locali* of his *Regulae*…, is clearly visible in the above-mentioned short treatises on motion (*opuscula de motu*) ascribed to Swineshead. A detailed examination of the contents of these *opuscula*, allows us additionally to understand some of the specific features of the conclusions Swineshead included in the sections of the "Book of calculations" dedicated to local motion in general. But before we take a closer look at these short treatises and compare them to the treatise "On local motion," it is worth mentioning the particularly interesting conclusions we find in the other part of *Liber calculationum*, namely in the treatise *De maximo et minimo*.

Interestingly enough, Richard Swineshead in taking up the issue of determining maxima and minima within qualitative changes, a problem widely discussed by the first Oxford Calculators, reduced it in his considerations only to questions on the limits of motive powers or resistances in local motions. In this regard, he simply accepted and repeated the solutions already established in his predecessors' works, that with respect to a constant active power it is possible only to specify its maximum intensity that cannot move something in a given resistant medium, while with respect to resistance it is possible to specify its minimal intensity that prevents the given active power from producing a motion within it.[266] The original issue we encounter within the chap-

264 See Aristotle, *Metaphysics*, 982a30–32, A(I).2, pp. 691–692.

265 See footnote 218 above. Guilelmus Heytesbury, *De motu locali*, § 38, p. 283.

266 See Richardus Swineshead, *Liber calculationum*, f. 33vb: "Quando igitur hec divisio est assignanda respectu potentie active uniformiter, dabitur per affirma-

ter *De maximo et minimo* is the notion of the 'weakening power' (*potentia debilitabilis*) considered in the context of local motion.[267] Swineshead provided here few typical cases of differently "configured" resistant media and determined the relation between motions effectuated in these media by constant power (i.e., not weakening one, *potentia indebilitabilis*) and by a weakening power that at the beginning of change equals the former in terms of intensity (or degree).[268]

tionem de maximo et negationem de minimo (…). Eo ipso enim quod potentia activa potest in aliquod agere, a fortiori in quodlibet minus potest. Ideo non est dare minimum quod potest facere potentia activa nec maximum in quod non potest, quia quod in b non potest agere, nec in maius b potest a fortiori. Ergo, hoc non est maximum in quod non potest. Et similiter non eo ipso quod potest in aliquod, potest in omne maius, nec eo ipso quod potest in quodcumque minus illo certo dato, potest in illud, quia potentia equalis sue resistentie non potest agere in suam resistentiam, sed in om nem minorem. Ideo illa est minima in quam non potest agere. (…) Quando tamen sit divisio respectu potentie passive, illa est danda per affirmationem de minimo et negationem de maximo, non enim potest esse affirmatio maximi."

267 It is worth noting here that the notion '*potentia debilitabilis*' appeared already in William Heytesbury's *Regulae solvendi sophismata* in the section *De maximo et minimo* as well. Yet in Heytesbury's account the action of 'weakening powers' was considered only in the context of lifting or carrying weights. See Guilelmus Heytesbury, *De maximo et minimo*, [in:] J. Longeway, "William Heytesbury on Maxima and Minima…".

268 See e.g., Richardus Swineshead, *Liber calculationum*, f. 33vb: "Sive fiat divisio respectu potentie debilitabilis sive indebilitabilis, existente medio uniformi universaliter est pars negativa sustinenda. Sit enim casus ille quod a sit aliqua potentia indebilitabilis, et sit b equalis sibi potentia debilitabilis, et sit c resistentia uniformis et sibi equalis; tunc a non potest dividere c medium uniforme. Sed omne medium uniforme minus resistens eo potest dividere, quia ad omne tale medium se habet in proportione maioris inequalitatis; ergo potest aliquam partem eius dividere, et sic totum cum sit potentia indebilitabilis. Similiter b poterit illud idem medium pertransire, quia ex quo se habet ad illud in proportione maioris inequalitatis, potest eius aliquam partem pertransire. Et quantumcumque debilitetur, potest minus debilitari; ergo totum sufficit pertransire. Et nullam partem c sufficit a aut b pertransire; igitur c est minimum in quod non sufficit pertransire"; ibidem, f. 34va: "Si fiat divisio respectu medii difformis quod potentia indebilitabilis sub gradu extremali existens sufficeret pertransire, vel in quo illa potentia cessaret a motu propter defectum proportionis (…) ergo dato tali medio est dare minimam potentiam indebilitabilem, et potentia debilitabilis illi potentie equalis est maxima potentia que illud medium non potest pertransire"; ibidem, f. 34va–b: "Si fiat divisio respectu medii difformis, quod poten-

We can only speculate, but quite plausibly it seems, that the notion of the 'weakening power' was introduced by Richard Swineshead into his considerations on local motion as conforming in a broad sense the natural, observable phenomenon that most local motions that occur in the sublunar realm become slower relative to the distance traversed.[269] This remark found in the context of the motions caused by non-weakening and weakening powers seems to provide the explanation that such a deceleration can be either an effect of the increasing resistance of a medium or of the weakening of the motive power only, since – as we read: "a distance (*quantitas*) does not resist."[270] This explanation assures us, in fact,

tia indebilitabilis sub gradu extremali existens sufficeret pertransire, vel in quo illa potentia cessaret a motu propter defectum proportionis (…) est sustinenda pars affirmativa, et respectu debilitabilis pars negativa. Verbi gratia, si fiat talis divisio huiusmodi medium potest aliqua potentia pertransire et non qualibet, vel ergo est dare minimam que potest, vel maximam que non potest, et hoc loquendo de medio in quo potentia equalis maxime eius resistentie cessaret a motu propter defectum proportionis. Dicendum est quod est dare minimam potentiam indebilitabilem qua illud medium sufficit pertransire et illa potentia est sub gradu terminante illud medium, ubi est maxima eius resistentia. Illa enim potentia potest totum medium pertransire et non minor, eo quod omnis minor haberet proportionem equalitatis ad aliquem punctum citra extremum ut notum est. Ergo, dato tali medio est dare minimam potentiam indebilitabilem, et potentia debilitabilis illi potentie equalis est maxima potentia que illud medium non potest pertransire. Quia, ex quo illa potentia debilitatur, cessaret a motu antequam ad extremum deveniret, et omnis maior illud totum sufficit pertransire, quia omnis maior ad omnem punctum habet proportionem maioris inequalitatis. Et quantumcumque modicum debilitetur, minus posset ipsum debilitari; ergo sequitur quod illa potentia est maxima potentia debilitabilis, que totum non sufficit pertransire."

269 The acceleration of the free fall motion was noticed already by medieval natural philosophers, but not properly described yet. For Kilvington' theory see above, p. 79.

270 Richard Swineshead, *Liber calculationum*, f. 33vb: "Item falsitas vel verificatio subcontrariorum ad huiusmodi divisionem requisita, que facta est respectu pertransitionis spatii, scilicet sic dividenda illa potentia potest aliquod medium pertransire et aliquod non. Potest haberi duobus modis secundum intellectum, aut ratione virtutis, aut ratione quantitatis. Primo modo sic: est aliquid medium tantum resistens quod hec potentia illud non poterit pertransire. Aut intelligitur secundo modo, quod est aliquid tante quantitatis quod ratione sue magnitudinis potentia illa non potest pertransiri. Sed illud est falsum. Ex quo quantitas sibi non resistit, non est aliqua magnitudo quin maior quecumque potentia

that when developing his logico-mathematical 'science of local motion' Richard Swineshead strove to remain within the boundaries of Aristotelian natural philosophy, providing the additional conditions that are coherent with its basic assumptions. In fact, in Aristotle's works we find no remark on the above-mentioned phenomenon, and his "equations" and "rules" of local motion lead to the conclusion that motion caused by a constant motive power acting on a constant resistance should last *in infinitum* with a constant speed. Consequently, any change of speed must be correlated with a change in the intensity of these factors.

The same reasons, in our opinion, help to explain one of the salient features of Richard Swineshead's account on local motion, that is the fact he generally provided the descriptions "with respect to cause" (*tamquam penes causam*), introducing the "kinematical" rules (*tamquam penes effectum*, i.e., those correlating the distance traversed in a given motion to its duration) only where he felt it absolutely necessary.[271] Interestingly enough, even though both short treatises (*opuscula*) on motion ascribed to him are from the outset divided into sections *penes causam* and *penes effectum*, his understanding of the latter description is different than the one adopted by his predecessors, i.e., by Richard Kilvington and William Heytesbury.[272] In both texts Swineshead took specifically the "degree of intensity" of a given motion as "the effect" in the description *penes effectum*, not the distance traversed in a given time. By the "degree of intensity" he meant, of course, the speed of a given motion:

> To whichever degree in local motion there corresponds the certain length of a line that in such and such a [period of] time can be drawn [with the motion characterized by a given degree], all other [circumstances] being equal. Similarly in the whole, just as to the certain degree of a ratio of an agent to the passive [power] there

quantumcumque modica pertransiri potest, eo quod cum velocitate data potest quantumcumque modica potentia movere, ut notum est."

271 See Richard Swineshead, *Tractatus de motu locali*, § 45, pp. 281–282; § 59, p. 287; §§ 63–66, pp. 288–290; § 83, p. 302; § 88, p. 304. Most of these fragments concern the corrollary of William Heytesbury's Mean speed theorem that establishes the ratio of distances traversed in the first and the second half of the duration of the uniformly accelerated/decelerated motion, the limit of which is rest.

272 See Richardus Swineshead, *Opusculum de motu*, § 7, § 61, [in:] R. Podkoński, *Richard Swineshead's science of motion*, pp. 137, 151; Richardus Swineshead, *Opusculum de motu locali*, § 54, [in:] R. Podkoński, *Richard Swineshead's…*, p. 174.

> corresponds the certain degree of motion, so to the certain degree of motion there corresponds the certain distance that would be covered with any of these degrees in such and such a [period of] time.[273]

In the above passage Richard Swineshead definitely established the mutual correspondence between *penes causam* and *penes effectum* descriptions of local motions. If we accept that a certain ratio of the motive power to resistance produces a certain degree of intensity of motion, and with such an intense motion a certain distance can be traversed in a given time, then both these descriptions are perfectly adequate. Keeping in mind, however, that a distance should not be taken as a factor in the description of motion, we can easily explain why Richard Swineshead in his ultimate account on local motion, that is in the treatise *De motu locali* in his "Book of calculations", formulated only *penes causam* "rules." Such a description is simply the proper one from the point of view of Aristotelian philosophy in general, because *scientia sensu stricto* concerns causes, as we read in the very first sentence of the above-mentioned *opusculum*.[274]

Close scrutiny of the contents of both the short treatises on motion ascribed to Richard Swineshead allows us also to find the possible sources and his motivations for pursuing the "calculatory" science of local motion to such an extent.[275] Perhaps the most striking feature of these texts is the

273 Richardus Swineshead, *Opusculum de motu*, § 65, p. 152.

274 See Richardus Swineshead, ibidem, § 1, p. 137. See Aristotle, *Metaphysics*, A(I).2, 982a28–b3, W.D. Ross (transl.) [in:] "The Basic Works of Aristotle", pp. 691–692: "But the science which investigates causes is also *instructive*, in a higher degree, for the people who instruct us are those who tell the causes of each thing. And understanding and knowledge pursued for their own sake are found most in the knowledge of that which is most knowable (for he who chooses to know for the sake of knowing will choose most readily that which is most truly knowledge, and such is the knowledge of that which is most knowable); and the first principles and the causes are most knowable; for by reason of these, and from these, all other things come to be known."

275 In what follows we present only the issues and solutions included in these *opuscula* we consider important in the context of the present monograph. The detailed analysis of the contents of these short treatises on motion is presented in: R.Podkoński, *Richard Swineshead's science of motion*, pp. 45–98. The dependence of these *opuscula* on William Heytesbury's *De motu locali* is described in R. Podkoński, *The* "Opuscula" *de motu ascribed to Richard Swineshead. The testimony of the*

fact that in the initial passages of both discussed are the same six conclusions aimed at determining the changes of velocities relative to uniform changes of motive power or resistance, or both agents concurrently. What is more, four out of these conclusions we find later as the consecutive "rules" in the treatise "On local motion" in the "Book of calculations."[276] What is most important here is the remark, present in both *opuscula* and introductory to these rules, that: "every change of motive or resistive power is either uniform or difform, but with regard to difform [changes] there can be no rule [formulated]."[277]

This remark reminds us clearly of the "condition" formulated earlier by William Heytesbury, that: "With respect to difform increase or remission, be it either from a certain degree to no-degree or the other way round, or from a certain degree to any other, there can be no rule [formulated]."[278]

But while in Heytesbury the above-quoted restriction referred to local motions described *penes effectum* only, that is he had in mind the difformly changing speed of a local motion, Swineshead interpreted it in his own way, namely as a statement concerning the changes in the intensities of the factors of local motion. Consequently, he began his considerations with motions caused by a uniformly changing intensity of factors, that is the motions that with respect to changes in their speed are difformly difform, since their speed is either increasing or diminishing "faster and faster," or

ongoing development of the Oxford Calculators' science of motion, [in:] "Quantifying Aristotle…", (forthcoming).

276 See Richardus Swineshead, *Opusculum de motu*, § 20, p. 140; § 22, § 24, p. 141; § 26, § 28, § 30, p. 142; Idem, *Opusculum de motu locali*, §§ 4–7, pp. 162–163; §§ 11–12, p. 164; Idem, *Tractatus de motu locali*, §§ 27–30, p. 277.

277 See Richardus Swineshead, *Opusculum de motu locali*, §§ 2–3, p. 162: "Et quia omnis variatio potentie motive seu resistive vel est uniformis vel difformis, et de difformi nulla potest esse regula nisi quatenus refertur ad illud quod est uniforme, quia infinitis modis contingit diversitas difformitatis. Ideo primo est advertendum quod sequitur de intentione motus vel remissione de uniformi augmentatione potentie motive respectu eiusdem resistentie et econtra"; Idem, *Opusculum de motu*, §§ 18–19, p. 140: "Et quia solum uniformiter et difformiter contingit fieri mutationem, et de difformi non <potest> poni aliquid certum, <quia> ex infinitis modis contingit fieri difformitas et de infinitis non est scientia. Consequens est dicere quid sit asserendum de uniformi intentione potentie et remissione, et de uniformi intentione resistentie et remissione."

278 William Heytesbury, *De motu locali*, § 40, p. 284.

"slower and slower" – to use Swineshead's own descriptions.[279] Interestingly enough, it seems that Richard Swineshead was perfectly aware of the fact that Heytesbury's "restriction" should be properly taken in the context of the changes of speed, for in the *Opusculum de motu locali* it is repeated in the section dedicated to the *penes effectum* description of local motion:

> since among difform motions some are uniformly difform, some difformly difform, and with respect to these difformly difform there can be no rule as to which uniform degree they correspond.[280]

The term " as to which uniform degree they correspond" relates obviously to the famous formula of the "mean speed theorem" that, as presented above, establishes in fact a relation between some uniformly difform (i.e., accelerated or decelerated) motion and a uniform motion, that is the motion characterized by the constant degree of intensity, in the terms used by the Oxford Calculators.[281]

We have remarked here already that the final conclusions of the treatise "On local motion" included in Richard Swineshead's "Book of calculations" are aimed at determining the mean speed of motions that change "faster and faster" or "slower and slower":

> (Conclusion 52) Every motion that diminishes faster and faster with respect to traversing a distance (*quantum ad pertransitionem spatii*) corresponds to the degree that is more intense than [its] mean [degree].[282]

> (Concl. 53) Every motion that diminishes slower and slower corresponds to the degree that is less intense than [its] mean [degree].[283]

279 See the texts referred to in the footnote 246 above.

280 See Richard Swineshead, *Opusculum de motu locali*, § 63, p. 176.

281 William Heytesbury, *De motu locali*, § 26, pp. 276–277.

282 Richardus Swineshead, *Tractatus de motu locali*, § 154, p. 335: "[O]mnis motus velocius et velocius deperditus quantum ad pertransitionem spatii gradui intensiori medio correspondet."

283 Ibidem, § 157, p. 338: "[O]mnis motus tardius et tardius deperditus, gradui remissiori medio correspondet."

(Concl. 54) Wherever a motion is increased faster and faster, it corresponds to the degree that is less intense than [its] mean [degree].[284]

(Concl. 55) Wherever a motion is increased slower and slower, [it] corresponds to the degree that is more intense than [its] mean [degree].[285]

(Concl. 57) Every motion that is acquired faster and faster with respect to traversing a distance corresponds to the degree that is more intense than the degree it will attain in the middle instant [of its duration].[286]

(Concl. 58) Every motion that diminishes faster and faster with respect to traversing a distance corresponds to the degree that is more intense than is the degree it will attain in the middle instant.[287]

Obviously, then, these "rules" refer to the specific kind of difformly difform motions with respect to the changes in their speeds. What is

284 Ibidem, § 159, p. 339: "Ubicumque velocius et velocius motus intenditur, correspondet idem gradui remissiori medio."

285 Ibidem, § 161, p. 339: "Ubicumque motus tardius et tardius intenditur, correspondebit gradui intensiori medio."

286 Ibidem, § 166, p. 340: "Omnis motus velocius et velocius acquisitus, gradui intensiori quam sit gradus quem habebit in instanti medio quoad pertransitionem spatii correspondet."

287 Ibidem, § 168, p. 341: "Omnis motus tardius et tardius deperditus, gradui intensiori quam sit gradus habitus in instanti medio correspondet." Taking into account the "pattern" of formulating the succesive conclusions adopted by Richard Swineshead both here and elsewhere within the treatise "On local motion" it seems obvious that there should follow another two "rules", namely the ones concerning the motion that is acquired slower and slower and the motion that diminishes slower and slower, respectively. With these lacking conclusions there would be the elegant, in a sense round, number of 60 "rules" formulated within this treatise. In fact, the most complete manuscript copies of this treatise, as well as all its printed editions, end abruptly after the conclusion 57 in the middle of the first sentence of the reasoning accompanying it. The above-quoted conclusion 58 is, in fact, reconstructed on a basis of a self-quotation we find in another part of Richard Swineshead's *Liber calculationum*, namely in the treatise *De inductione gradus summi* (see § 167, p. 341.) See also R. Podkoński, *Suisetica inania*…, p. 136.

more, it does not appear merely a coincidence that the very same motions, i.e., those that change "faster and faster" or "slower and slower" were described in rules common for both short treatises on motion ascribed to Richard Swineshead and his treatise "On local motion."[288] It seems verisimilar that the "restriction" formulated by William Heytesbury, whereby there can be no rule with respect to difformly difform motion, was recognized by Swineshead as a kind of intellectual challenge. And the above-quoted, final rules of the treatise *De motu locali* seem to contradict, broadly, this "restriction". Of course these rules do not allow one to calculate the exact value of the "mean degrees" of such motions. This, however, was not a problem for fourteenth-century natural philosophers, as even Richard Swineshead, following on from William Heytesbury – as noted above –, explicitly recognized the calculating of the exact values of intensities (speeds) as useless.[289] What is more, with respect to this "special" kind of difformly difform motions, as described by the above-quoted rules, Richard Swineshead formulated also the general rule that:

> (Concl. 56) If some latitude [i.e., speed] of motion is acquired in the same way as it is diminishing, it corresponds to the same degree [in both cases],[290]

specifying in due course that by "the same way" of acquisition and diminishing it should be understood in a "symmetrical" sense, i.e., only with respect to cases when the speed was first acquired "faster and faster", and subsequently diminished "slower and slower"; or when the speed was first acquired "slower and slower" and next it diminished "faster and faster".[291]

288 See footnote 248 above.

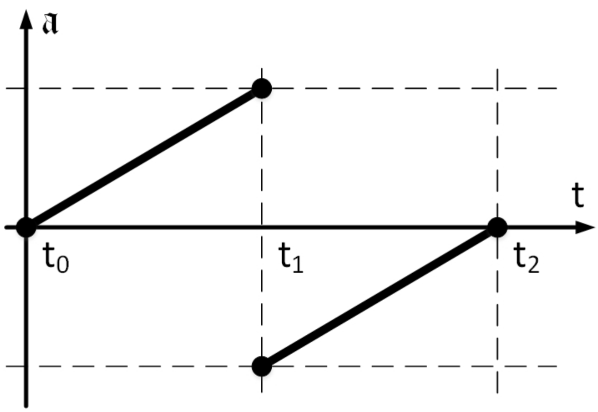
289 See footnotes 220, 236 above.

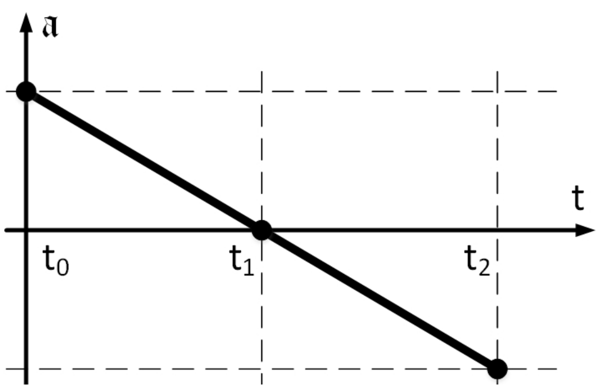
290 Ricardus Swineshead, *Tractatus de motu locali*, § 163, p. 339: "Si aliqua latitudo motus consimiliter acquiritur sicut deperditur, eidem gradui correspondet."

291 Ibidem, § 165, p. 340: "Pro quo intelligendum est, quod si motus sit aliqualiter acquisitus, et per aliquod tempus remittatur ita, quod versus principium secundi temporis sit consimilis deperditio, sicut versus finem primi temporis erat acquisitio, et si sit versus finem secundi temporis consimilis deperditio sicut versus principium primi temporis acquisitio, illi motus eidem gradui correspondebunt, ut acquisitum in uno tempore et deperditum in alio. Sicut etiam in proposito: si in uno tempore aliquis motus velocius et velocius acquiratur, et in alio tempore consimiliter deperdatur, in eodem alio tempore tardius et tardius remittetur.

In our terms these are the cases when the acceleration of a given motion changes uniformly. For the modern reader the cases described by the above-quoted general rule are perhaps best explained, anachronistically, with diagrams of functions representing the changes of acceleration (a) relative to time (t). Thus when the speed first (in the period t0—t1) is acquired "faster and faster" and next (t1—t2) it diminishes "slower and slower" its acceleration changes in the following way:

And, respectively, when the speed is first acquired "slower and slower" and next it diminishes "faster and faster", its acceleration changes as pictured below:





Sed si sic intelligatur 'consimiliter deperdet sicut acquiret', et ita quod versus principium deperditionis sit ita consimilis deperditio, sicut est acquisitio versus sui principium, tunc non est conclusio vera."

The diagrams present our own interpretation of Richard Swineshead's conclusions, yet the most plausible ones as the above-quoted general "rule", i.e., Conclusion 56 of his treatise "On local motion", applies adequately to such cases.[292] Taking into account the limitations of the method Richard Swineshead employed within his speculative science of local motion, together with – the Aristotelian in fact – assumptions concerning the conditions and factors of local motion he presumably accepted here, the above "rule" should be recognized as one of his most ingenious achievements.[293]

The genius of Richard Swineshead, his extraordinary proficiency in the method of "calculations" and the awareness of the limitations of the speculative science of local motion developed by his predecessors is further confirmed by other reasonings and conclusions he included in the treatise *De motu locali*. We have in mind here the section where he discussed the possibility of "adjusting" or "manipulating" the changes of intensity of the motive power in such a way that a resultant motion in a given uniformly difform resistant medium would be uniformly difform (*scil.* uniformly decelerated), while if this power were to remain constant the speed of its motion could not change uniformly in this medium.[294] This reservation, and consequently a problem raised here by Richard Swineshead, is the direct effect of interpreting the Aristotelian "rules" of motion in the terms of the calculus of ratios in general, and the "continuous proportion" in particular. On the basis of the Calculators' "new rule of motion", the changes of speed associated with a series of ratios derived from a particular integral ratio of motive power to resistance through the repeated squaring or square-root extraction of this ratio are incommensurable with the speeds associated

292 Strictly speaking, the equivalent diagrams representing the changes of speed relative to time should be constructed from the sections of parabolic lines.

293 We should not forget also that the symbolic mathematical "language" we are used to, that often helps to notice or explain the relations between the terms considered in a given case, had not been yet introduced in the times of the Oxford Calculators.

294 Interestingly enough, in the already mentioned here "table of contents" included in the codex Vatican, BAV, Vat.lat. 3095 this section is indicated as the distinct chapter: *Numquid si una potentia uniformiter remittat motum suum ad non gradum in medio difformi aliqua potentia maior vel minor per sui variationem poterit remittere motum suum continue uniformiter totum illud medium pertranseundo.* See R. Podkoński, *Richard Swineshead's* Liber calculationum *in Italy*..., p. 316.

in the way decribed with any other integral ratio of motive to resistive power, prime to the first ratio.[295] Consequently, for a given uniformly difformly resistant medium there is only one degree of intensity of motive power that causes a uniformly changing motion in this medium. This phenomenon was first discovered by Thomas Bradwardine, who introduced different "species" (*genera*) of motions to this context, but Richard Swineshead presented it in the treatise *De motu locali* straightforwardly:

> (Conclusion 38) If there is some constant power (*potentia non variata*) that decreases its motion uniformly up to a no-degree or up to [a certain] degree in a given difformly [resistant] medium, there can be neither greater nor lesser constant power that could decrease its motion uniformly when traversing the very same medium.[296]

It is worth noting that he had already formulated the same "reservation" in each of his short treatises on motion:

> (*Opusculum de motu*): If, when traversing some resistance, some power uniformly increases or decreases its motion, [there can be] neither greater nor lesser power that would uniformly increase or decrease its motion when traversing the very same resistance.[297]

> (*Opusculum de motu locali*): There is no difformly resistant [medium] in which, when a certain mover decreased uniformly its motion [in it], some other [mover], namely unequal to the first one, could

295 See S. Drake, *Bradwardine's function, mediate denomination and multiple continua*, "Physis" 12 (1970), p. 55.

296 Richardus Swineshead, *Tractatus de motu locali*, § 80, p. 301: "Si aliqua potentia non variata in medio difformi remittat motum uniformiter ad non gradum vel ad gradum, nulla potentia maior nec minor non variata potest uniformiter remittere motum suum idem medium transeundo."

297 Richardus Swineshead, *Opusculum de motu*, § 78, p. 155: "Ex istis patet, quod si aliquam resistentiam pertranseundo aliqua potentia uniformiter intendet vel remittet motum suum, nulla potentia maior vel minor illam eadem resistentiam pertranseundo uniformiter intendet vel remittet motum suum."

> decrease its motion uniformly acting with full power when the resistance of this medium remains unchanged.[298]

In each instance the above-quoted conclusions are proved with reference to the "mean speed theorem."[299] But only within the treatise "On local motion" in his *Liber calculationum* did Richard Swineshead ponder the possibility of "adjusting" the motive force greater or lesser than the exemplary one in order to effectuate uniformly difform motion. First, he determined what kind of motions would be caused in the given uniformly difformly resistant medium by the motive powers that are either more or less intensive than the one that will move in the same medium with a speed constantly diminishing to the rest. These motions, he concluded, would be remitted infinitely fast or infinitely slow, respectively, close to the most intense limit of the medium.[300] And next Richard Swineshead introduced and discussed four possible methods and conditions for "adjusting" the motive powers that are different from the "model" one:

(1) a continuous intensification of a motive power that is initially greater than the "model" one;[301]

298 Richardus Swineshead, *Opusculum de motu locali*, § 39, p. 170: "Item, quod in nulla resistentia difformi in qua remitteret aliquis motor motum suum uniformiter, remitteret aliquis alius, scilicet inequalis secundum ultimum suum movendo uniformiter motum suum nulla facta variatione in ipsa resistentia." Ibidem § 48, p. 172: "Sequitur igitur propositum principale, videlicet quod in nulla resistentia difformi in qua remitteret aliquis motor uniformiter motum suum, remittit aliquis alius inequalis illi cum paribus uniformiter motum suum."

299 See Richardus Swineshead, *Tractatus de motu locali*, §§ 82–96, pp. 302–309; Idem, *Opusculum de motu*, § 79, pp. 155–156; Idem, *Opusculum de motu locali*, §§ 43–51, pp. 171–173.

300 See Idem, *Tractatus de motu locali*, Conclusio 39, § 98, p. 310: "Si aliqua potentia in medio difformi remittat motum suum uniformiter ad non gradum, omnis maior infinite velociter remittet motum ad extremum intensius deveniendo; Conclusio 40, § 100, p. 311: "Si aliqua potentia in medio difformi uniformiter remittet motum suum ad non gradum, omnis potentia minor in illo medio movendo infinite tarde remittet motum suum."

301 Ibidem, Conclusio 41, § 103, p. 312: "Ubi potentia in medio difformi remittit motum suum uniformiter ad non gradum, potentia maior per continuam intensionem potentie uniformiter poterit continue remittere motum suum illud totum medium transeundo."

(2) a continuous abatement of such a motive power that would cause its motion to diminish to the rest in the end of the given medium;[302]

(3) a continuous intensification of a motive power that is lesser that would cause its motion to diminish to a certain speed in the end of the given medium;[303]

(4) a continuous intensification of such a power that would cause its motion to diminish to the rest in the end of the same medium.[304]

While formulating the last possibility he remarked briefly that it is impossible to cause the uniformly difform motion diminishing to the rest in the end of the same medium by a continuous abatement of a power that is initially lesser that the "model" one, yet in due course he discussed the analogical case, namely:[305]

(5) a continuous abatement of a motive power that is lesser than the "model" one that would cause its motion to diminish to the rest before reaching the end of a given medium.[306]

302 Ibidem, Conclusio 42, § 104, p. 312: "Ubi potentia per medium difforme uniformiter remittit motum suum ad non gradum, potentia maior per continuam remissionem illius potentie uniformiter poterit totum medium transeundo remittere motum suum. Sed hoc non potest esse, nisi remittendo ad non gradum."

303 Ibidem, Conclusio 43, § 105, p. 312: "Ubi potentia aliqua remittit motum uniformiter ad non gradum in medio difformi, potentia minor per continuam intensionem potentie potest remittere motum suum uniformiter totum medium transeundo, et hoc remittendo motum ad gradum."

304 Ibidem, Conclusio 44, § 106, p. 312: "Ubi una potentia etc., potentia minor per continuam eius intensionem ad non gradum uniformiter poterit remittere motum suum totum medium transeundo, sed per remissionem potentie nequaquam."

305 Ibidem.

306 Ibidem, Conclusio 48, § 136, pp. 327–328: "In medio difformi, ubi una potentia uniformiter remittit motum suum ad non gradum non variata, alia potentia minor per continuam remissionem sue potentie uniformiter remittet motum suum ad aliquem punctum medii intrinsecum deveniendo."

We will not present the extensive and sophisticated lines of reasonings provided to determine the details that theoretically would let each of the above conditions be fulfilled. In fact it would be – in Swineshead's own words – more troublesome than worthwhile. It is enough to state here that in each of the above-mentioned cases Richard Swineshead determined in detail in what manner, when and/or at which point of the medium the assumed motive power should be modified in order to cause the uniformly decelerated motion.[307] What is most important here is the fact that these solutions, again, seem to be inspired by the "restriction" formulated by previous Oxford Calculators that Swineshead, in a sense, decided to break.

The exposition, included in the present chapter, of the main problems discussed and solutions offered by Richard Swineshead with respect to the "science of local motion" shows clearly the range and the complexity of the reasonings he provided. In fact, it seems almost impossible to imagine any example of the configurations of the factors of local motion or relations between them that was not considered by the Calculator. It suffices to note here that in Oxford University following the treatise *De motu locali* there were no further accounts on this topic.

307 Ibidem, §§ 109–112, pp. 313–316; § 122, pp. 321–322; § 127, pp. 323–325; §§ 133–135, pp. 326–327; § 138, pp. 328–329.

Chapter IV

Towards Modern Mechanics?

In the first chapter of this book we introduced the *dramatis personae* of our study, the Oxford Calculators commonly recognized as those the most influential i.e., Richard Kilvington, Thomas Bradwardine, William Heytesbury, John Dumbleton and Richard Swineshead. We also have presented here the important anonymous treatise *De sex inconvenientibus*, and we have mentioned the name of Roger Swineshead, who surely was associated with the Oxford Calculators School. The main idea of this chapter was to show, through the short descriptions of the Calculators' works, the scope of their main philosophical interests. Accurate information about the availability of their works, i.e., critical editions, old prints and manuscripts, was intended to show which of the Calculators' works has been the most often examined, since their works were edited, and which has simply been forgotten in the general history of medieval science, for they still remain in manuscript form.

In Chapter II we presented the scientific background and sources of inspiration of the theories of motion as proposed by the Oxford Calculators. Most of their works were composed in order to meet the requirements of the curriculum of the University of Oxford, that, in the fourteenth century, obliged bachelors and masters at the Arts Faculty to comment on Aristotle's *Physics* and *De generatione* as well as to teach logic. That is why Chapter II begins with Aristotle's theories and Averroes's commentaries. The latter, introduced within his comments some new material presented in the context of discussing the ideas of his Arabic predecessors and those of his contemporaries: Avempace's, and Al-Kindi's solutions, to mention the most important ones. In fact, Averroes's interpretation of Aristotle's texts on natural philosophy, gave the impulse to formulate new theories on motion. Latin philosophers in the fourteenth century interpreted Aristotle through Averroes's expositions being absolutely sure that his commentary mirrors and stays in accord with the theory of the Stagirite. The far-reaching moment in the history of "mathematical physics", as developed by the Oxford Calculators, was also the broad use of mathematics, which from the very beginnings of Oxford University was recognized as a demonstrative science and the proper tool of analyses within the philosophy of nature. It was the first chancellor of Oxford University, Robert Grosseteste, who was to introduce mathematics into his philosophical considerations. This attitude was adopted and enthusiastically propagated by Roger Bacon, John Peckham and Robert Kilwardby, among other English philosophers. The teaching of logic and mathematical disciplines such as geometry, arithmetic, optics, music, static and astronomy was far more developed in Oxford than in other medieval universities. This legacy was most obviously also inherited by the Oxford Calculators. In Chapter II of the book we summarized the most significant theories of thirteenth- and early fourteenth-century English thinkers. The most influential, however, was – in our opinion – the original, innovative philosophy of William of Ockham. Ockham was only a bit older than the first Oxford Calculators, and his ideas – as we are convinced – gave them the first impulse to reinterpret Aristotelian theories in natural philosophy.

In Chapter III we present detailed analyses of the theories of local motion offered by the above-mentioned Calculators. The analyses we have included there, indicate also clearly the continuous development of the theory of local motion: revealing the relationships of a varied kind (inspirations, borrowings, controversies, etc.) between the specific opinions of these thinkers.

The purpose of the present, fourth chapter is to answer the question as to whether the achievements of the Oxford Calculators really gave the impulse for the development of the seventeenth century mechanics, or rather if they only provided a new interpretation of Aristotelian philosophy of nature.

The history of studies on the Oxford Calculators commenced at the beginning of the 20th century with Pierre Duhem's monumental works: *Études sur Léonard de Vinci*, t. 1–3 (1906–1913), and: *Le système du monde; histoire des doctrines cosmologiques de Platon à Copernic; L'astronomie latine au Moyen Age*, t.1–10 (1906–1959). Until then, the predominant view was that the period preceding the seventeenth-century Scientific Revolution had not influenced it at all, and consequently should be seen only as the pre-scientific era of false superstitions and ignorance with respect to a worldview. Ignoring this attitude, Pierre Duhem found traces of medieval science in the scientific theories of the 17th century, and in a consequence claimed enthusiastically that modern science was a product of the Middle Ages. In his view the accomplishments of fourteenth-century French philosophers and theologians were instrumental in the development of the theories of Galileo Galilei and René Descartes. Duhem was also convinced that modern science originated in 1277, at the University of Paris, when Bishop Stephen Tempier condemned 219 "errors" in philosophy and theology, thus liberating medieval science from Aristotelian constraints. Edward Grant upheld this last thesis in many of his works.

Pierre Duhem's serious analyses were founded on a large number of medieval sources. In his opinion the most important philosophers were John Buridan and Nicole Oresme, who had introduced mathematics into physics and abandoned Aristotelian natural philosophy. There is no doubt that Nicole Oresme was the most original and inventive "scientist" of his times, who knew how to make the best use of the new calculus of ratios invented by the Oxford Calculators. John Buridan, on the other hand, being a nominalist, was against "mathematical physics" and eliminated mathematics from natural philosophy. The most famous of his achievements, the "impetus theory" had been invented, in fact, already by Al-Kindi and functioned as common scientific knowledge in the Middle Ages.

Nevertheless, Pierre Duhem's careful and thorough studies have brought to light the forgotten innovative theories of later medieval philosophers and mathematicians. Since then, serious research into medieval science has begun. The reaction to Duhem's views was favorable and some historians believed that he had indeed succeeded in discovering the 14th century precursors of Galileo. Lynn Thorndike in his *History of Magic and Experimental Science*, vol. 1–8 (1923–1958) presented the opinion on the evolutionary character of the development of science from the twelfth to the eighteenth century. Also Alistair Crombie in his books: *Augustine to Galileo: the History of Science A.D. 400–1652* (1952) and *Robert Grosseteste and the Origins of Experimental Science 1100–1700* (1953), *Medieval and Early Modern Science*, vol. 1–2 (1959), favored the theory of the evolutionary character of the development of natural science, and pointed out many significant experiments conducted in physics, medicine, pharmacology and biology to prove that science had been continuously developing from the times of St. Augustine to the seventeenth century. Other historians, however, were less enthusiastic about a thesis on the evolutionary character in the development in sciences. It seems that the author of the most important criticism was Annelise Maier, whose extensive studies on medieval philosophy and theology, and research into an enormous number of manuscripts, lead her to the conclusion that: "the conception of nature to be found in the 14th century could be seen as a preliminary stage to, and a preparation for, "classical physics", French and English scholars paved the way for later science by creating assumptions, which were used as the points of departure" (see *Ausgehendes Mittelalters* (1964), *Studien zur Naturphilosophie der Spatscholastik* (1951), *Zwischen Philosophie und Mechanik* (1952). Also an extensive work heralded by Marshall Clagett (*The Science of Mechanics in the Middle Ages* 1959 and *Archimedes in the Middle Ages*, vols 5, 1964–1980) drew the attention to the innovative character of medieval science for which Galileo found application in his theory. Clagett sympathized with a continuity thesis much more than Maier and he believed that Maier's revisions of Duhem's thesis had been correct in the main conclusion that medieval natural philosophers had set the stage for 17th century physical concepts.

In addition to the above-mentioned studies, many works have been published discussing the views of individual Calculators. H. Lamar Crosby has presented the critical edition with an introduction of Thomas Bradwardine's *Tractatus de proportionibus*. Curtis Wilson, John Longeway, and Fabienne Pironet have published books and papers mostly on William Heyetesbury. Sabine Rommevaux-Tani and Joanna Papiernik have published papers based on their critical editions of the anonymous treatise *De sex inconvenientibus*. The Ph. D. dissertation of James Weisheipl was based on the transcription of the whole text of John Dumbleton's *Summa logicae et philosophiae naturalis*. Robert Podkoński has prepared critical edition of five different treatises by Richard Swineshead. Edith Sylla, who has dealt with the Oxford Calculators since her doctoral studies in the 1970s, has transcribed various parts of Dumbleton's and Swineshead's works, and she has used these transcriptions in many of her papers. Many other historians of science, such as John Murdoch, Ernest Moody, and George Molland, to mention the most renowned, have worked on either their own transcriptions of different manuscript copies of Oxford Calculators' works or on old prints from the 16th century.

Given that contemporary researchers still formulate their opinions about the late medieval philosophy of nature on the basis of fragmentary and abbreviated presentations of the Oxford Calculators' works, their incomplete knowledge frequently leads to mutually incoherent or even contradictory statements. Therefore, there was an urgent need to fill the blank spot within the history of the Oxford Calculators tradition in "mechanics" with the critical editions that are included in Part II of this work. We offer the critical editions from Latin manuscripts not only of the most famous Calculators' works, such as William Heytesbury's *De tribus praedicamentis*: *de motu locali* or John Dumbleton's Part III of the *Summa logicae et philosophiae naturalis*, but also of a hitherto unknown work by Richard Kilvington, i.e., his question on local motion and the question on local motion written by the anonymous author of the treatise *De sex inconvenientibus*.

In order to recount the history of the development of the theory of local motion, we have thoroughly examined the texts of all the Calculators from the beginning of the School, i.e., Richard Kilvington's questions (1326) until the very conclusion with Richard Swineshead's treatise *De motu locali* from his "Book of calculations" (1350). We have also compared our own conclusions resulting from these studies with those formulated by other historians of medieval science. We have mostly focused our attention on topics that were important to medieval thinkers and not those that could be most interesting from the modern point of view. Thus we have directed our research on the Oxford Calculators' tradition in science toward a prospecting of the innovative character of their learning, and here first of all against the background of Aristotelian theories, and then the subsequent search for possible innovations which could have inspired early modern scientists. Although all the Calculators dealt with four types of changes that Aristotle defined generally as motion, that is: generation, alteration, augmentation, and local motion, we decided to focus on their concepts of local motion, because some historians of science have claimed that Galileo took advantage of their solutions in this very respect.[1] It is beyond any doubt that the local motion, firstly described by Aristotle in his Book IV and VII of the *Physics*, was the core interest for physicists until the twentieth century. Thus far historians of science had been focusing on the most famous achievements of the Oxford Calculators, such as "the new rule of motion" or "Bradwardine's rule", as it is commonly known, and "the mean speed theorem". Our goal was rather to answer the main question of the evolutionary or revolutionary character of science on the basis of many more sources derived from the School itself.

1 See P. Damerow *et al.*, "Exploring the limits of preclasical mechanics", New York--London 1992, 2011 second ed.

Historians of medieval science dealing with the Oxford Calculators have described their theories from, as we see it, two different points of view: either from the perspective of a physicist, or that of a mathematician. Marshall Clagett, the author of *The Science of Mechanics in the Middle Ages*, has suggested with the very title of his book that the medieval philosophy of nature dealing with problems of motion should be called 'mechanics'. The respective parts of the book are devoted to static, kinematics and dynamics. Clagett has described these three disciplines of physics with modern terms and he has used the modern tool of equations of motion connecting speeds, times, distances, and so on. Additionally, he has offered "a dictionary of terms", with which he "translated" the Latin terms used by medieval authors into the modern terms of mechanics. Consequently, many followers of Clagett have described medieval "rules" of motion using modern connotations, and thus, in the secondary literature one can quite often see equations such as $s = vt$, which combine three, entirely different – as Aristotle says – species, namely distance traversed (s), time consumed (t) and speed of motion (v).

In modern on-line encyclopedias we find entries on the Oxford Calculators where we read, for example, that:

> It was proved by the Merton school that the quantity of motion in uniformly accelerated motion is equal to the quantity of an uniform motion at the speed achieved halfway through the accelerate motion; in modern formulation, $s = ½ at^2$ (Merton rule). Discussions like this certainly influenced Galileo indirectly and many have influenced the founding of coordinate geometry in the 17th century.[2]

The explanation of "Bradwardine's rule" of motion with the exponential or logarithmic function also suggests that the use of modern equations is perfectly correct. Thus, we are left with the conviction that already in the 14th century mathematicians and physicists were familiar with mathematical physics using properly mathematical functions. The most significant example of this attitude is John Murdoch's explanation in the on-line encyclopedia, where Bradwardine's rule is presented as follows:

2 www.britannica.com/science/mathematics/The-transmission-of-Greek-and--Arabic-learning#ref536236

If we generalize what we then discover, we can, in modern terms, say that his solution to his problem of the corresponding "ratios" of speeds, forces, and resistances is that speeds vary arithmetically while the ratios of forces to resistances determining these speeds vary geometrically. That is, to use symbols, for the series $V/n, \dots V/3, V/2, 2V, 3V, \dots nV$, we have the corresponding series $(F/R)^{1/n}, \dots (F/R)^{1/3}, (F/R)^{1/2}, F/R, (F/R)^2, (F/R)^3, \dots (F/R)^n$. Or, straying an even greater distance from Bradwardine himself, we can arrive at the now fairly traditional formulations of his so-called "dynamical law":

$$(F1/R1)^{v2/v1} = F2/R2 \text{ or } V = \log aF/R, \text{ where } a = F1/R1.$$

Furthermore, if we continue our modern way of putting Bradwardine's solution to his problem, we can more easily express the advantage it had over the medieval alternatives cited above. In essence, this advantage lay in the fact that Bradwardine's "function" allowed one to continue deriving "values" for V, since such values—the repeated halving of V, for example—never correspond to a case of $R > F$ (as was the case with $V \propto F/R$); they correspond, rather, to the repeated taking of roots of F/R, and if the initial $F1 > R1$ (as is always assumed), then for any such root $Fn/Rn = (F1/R1)^{1/n}$, Fn is always greater than Rn. With this in view, it would seem that Bradwardine's most notable accomplishment lay in discovering a mathematical relation governing speeds, forces, and resistances that fits more adequately than others the Aristotelian-Scholastic postulates of motion involved in the problem he set out to resolve. The fact that Bradwardine was thus able to state in its general form the medieval mathematics behind his function suggests that, although his expression of the function itself in mathematical terms was never general, this was due to his inability to formulate such a general mathematical statement. The best he could do was, perhaps, to give his function in the rather opaque, and certainly mathematically ambiguous, form we have quoted in *extenso* above, and then merely to express the mathematics of it all by way of example.[3]

3 J.E. Murdoch "Bradwardine Thomas", in Encyclopedia.com. See also Idem, *Mathesis in philosophiam scholasticam introducta*..., p. 226.

The second perspective adopted when presenting the development of the Oxford Calculators's science of local motion is the mathematical one. Edith Sylla, who from the very beginning of her research, i.e., since her Ph. D. dissertation (1970, published in 1991): "The Oxford Calculators and the Mathematics of Motion 1320–1350", up to the very last paper "Leibniz and the Calculatores", to be published in a volume dedicated to the history of the Oxford Calculators, consequently uses terminology suggesting that Calculators' "calculus of ratios", expresses the mathematical function: "upon which Bradwardine built a mathematical theory of the proportions of velocities in motions of an elegance still worthy of our appreciation."[4] She finds the source of Bradwardine's theory in a pre-Theonine version of Euclid's "Elements" and in the mathematical works of Archimedes and Apollonius.[5] In her elaborated paper, Sylla states that:

> Thomas Bradwardine worked entirely within the Campanus version of the "Elements", the version without Book VI, definition 5. Working with this tradition is what allowed him to put forth an understanding of compounding ratios or proportions that avoided the arithmetization involved in Book VI, and therefore made his rule for relations of proportions of force to resistance and velocities seem supremely natural and simple. Thus Bradwardine established the mathematical foundations for his theory of the proportions of velocities on a pure "pre-Theonine Euclid" [6]

She concludes this part of her paper as follows: "It appears, then, that Bradwardine's exclusion of the Theonine compounding of proportions by multiplication of denominations was a conscious strategy in "On the ratios of velocities in motions" and not inadvertent."[7] And in the next section of her paper, Sylla draws the following conclusions:

> Indeed, the mathematics of proportions found in Euclid's "Elements", Books V and VI, provided a ready-made tool for signifi-

4 E. Sylla, *The origin and fate of Thomas Bradwardine's "De proportionibus velocitatum in motibus" in relation to the history of mathematics*, [in:] "Mechanics and Natural Philosophy before the Scientific Revolution", Springer 2008, p. 69.

5 See ibidem.

6 Ibidem, p. 82.

7 See ibidem, p. 90.

> cantly raising the level of mathematics applied to earthly motion. What Bradwardine had to do in *De proportionibus* was to convince his readers of the gains to be had from following consistently the pre-Theonine theoretical approach. One advantage was that his rule apparently gave a measure of velocity *tamquam penes causam* at an instant. (…) Bradwardine's exclusively pre-Theonine approach also had, for better or worse, additional implications. It followed, for instance, that there is no comparison between proportion of greater inequality, proportions of equality, and proportions of lesser inequality.[8]

In the next part: "The reception of Bradwardine's rule …", Sylla discusses the relations between Bradwardine and Kilvington with regard to calculus of ratios and says:

> [A]lthough he [viz. Kilvington] knows about compounding proportions in the pre-Theonine way, this does not prove he has Bradwardine's law, since Euclid's definitions of a proportion *duplicate* and *triplicate* had been around long before Bradwardine. It is, however, impossible to deny that, within his questions on the *Physics*, in the question *Utrum in omni motu potentia motoris excedit potentiam rei motae*, Kilvington rejects the view that velocities are proportional to the excess of the force over the resistance and argues for the view that velocities are proportional to the proportion of force to resistance understood in the Bradwardinian sense (…) it is significant that Kilvington assumes without argument that proportions are compounded in the pre-Theonine or Bradwardinian sense (…).[9]

It seems that there is a much easier way to explain why Kilvington was convinced that the ratios should be compounded in the above-presented way. In his question on the *Physics* composed in 1326, that is at least two years before Bradwardine's *De proportionibus*, and in his commentary on the *De generatione et corruptione*, in the question *Utrum omne continuum sit divisibile in inifitum*, composed as early as in 1324, he stated that since the factors of motion, such as the intensity of acting power

8 See ibidem, pp. 90–91.

9 See ibidem, p. 96.

and resistance, and the results: speed, distance and time likewise, are continuous, as they can be infinitely divided, consequently proportions between them need to be the continuous geometrical proportion, as it is defined by Aristotle in Book V of the *Nichomachean Ethics*. The continuous proportion is defined by Euclid and interpreted by Campanus of Novara in a way that allows one to build the new calculus of ratios easily. Already in his commentary to the *De generatione et corruptione* Kilvington presented the new calculus of ratios and proportions. In his first question *Utrum generatio sit transmutatio distincta ab alteratione* in the first argument *quod non*, he referred to the same theorems of Averroes, as in his *Physics*, and he solved the problem with the help of Boethius' and Euclid's axioms.[10] In his question *Utrum omne continuum sit divisibile in infinitum* he made frequent use of different theorems and axioms from Euclid's' "Elements". In the fourth principal argument he shows that a duplicate (*duplicata*) proportion is not the same as a double (*dupla*) one, since they are like *superius* to *inferius*: a double proportion is always a *proportio duplicata* but not vice versa. He also clearly stated that Aristotle's rule of motion from Book VII – "if a given power moves a given mobile through a given distance in a given time, double that power will move a mobile twice as fast" – should be read as: "a given power will move a mobile sometimes twice as fast or more than twice as fast".[11] As we

10 Richard Kilvington, Ms. Seville Bibl. Colomb. 7–7–13, f. 1ra: "Prima consequentia patet quia proportio velocitatis est secundum proportionem motoris ad mota sicut dicit commentator IV *Physicorum* commento 7 et commento 71 et commento 35 et commento 39. Et probo antecedens quia proportio caliditatis B ad frigiditatem A est proportio sexquialtera et proportio caliditatis A ad frigiditatem B est sesquiatertia, quarum est proportio minor alia et patet per Boecium in *Arismetica*. (...) Sed contra: quia omnia duo aequalia habent ad tertium eandem proportionem sicut patet convenienter conclusione V cum 15 Euclidis, tunc arguo sic: latitudo caliditatis in summo sit tripla ad tertiam partem frigiditatis in summo, igitur medietas tertii latitudinis ad eandam tertiam partem est proportio sexquialtera, cum igitur illae sunt medietates latitudinis caliditatis in summo et in a tertia pars latitudinis frigiditatis, igitur proportio latitudinis caliditatis B ad frigiditatem A est proportio sexquialtera..."

11 See Ricardus Kilvington, *Utrum omne continuum sit divisibile in infinitum*, R. Podkoński (ed), [in:] "Mediaevalia Philosophica Polonorum" XXXVI(II), 2007, pp. 133–136, 164–165. P. 165: "Ad quartum principale concedo totum usque ibi: proportio A ad C est duplicata proportio respectu A ad B, et hoc concedo. Et nego consequentiam ulterius: igitur proportio A ad C est maior quam proportio A ad B, ad illum scilicet intellectum quod non est maior A ad C quam A ad

have already shown in Chapter III, Thomas Bradwardine made the best use of Kilvington's 26 arguments to formulate his famous "new rule of motion."[12]

To sum up, both Kilvington and Bradwardine claimed that every local motion is caused by a proportion of a greater inequality. Each of them maintained that his theory is only a new interpretation of Aristotle's and Averroes' statements, and in order to convince their readers, they both provided the same quotations which they analyzed and criticized afterwards. Kilvington surely was aware that the proper understanding of Euclid's definition of operations on proportions necessitates a new interpretation of Aristotle's and Averroes' rules of motion. On the one hand, Euclid's and Archimedes' theory of operations on proportions mean in fact that doubling a ratio corresponds to squaring the fraction which we form from the ratio. On the other hand, Aristotle's and

B. Unde dico quod licet proportio A ad C sit duplicata respectu proportionis A ad B, non sequitur quod illa proportio sit alia maior, quia proportio duplicata et proportio dupla non convertuntur, immo se habent sicut superius et inferius, quia omnis proportio dupla est duplicata et non econverso. Unde proportio dyametri ad costam est medietas duplae proportionis quæ est proportio nota et alia quae non est proportio nota, et dicitur proportio nota inter talia quae sic se habent ad invicem sicut numerus ad numerum – et talis non est dyametri ad costam. Unde capiatur aliquod quadratum cuius costa asignetur per 4, et linea dupla ad illam costam asignabitur per 8. Si esset certus aliquis numerus inter 4 et 8 qui sic se haberet ad 4 sicut 8 ad ipsum tunc linea asignata per numerum illum medium tunc esset æqualis dyametro quadrati cuius costa asignatur per 4. Sed quia non est aliquis talis numerus medius ideo non est talis proportio dyametri ad costam qualis est numeri ad numerum, quia si sic dyameter quadrati esset commensurabilis costæ, sicut patet per conclusionem 20 Euclidis, quae est ista: omnes quantitates habentes ad invicem talem proportionem qualis est numeris ad numerum sunt commensurabiles. (…) Nec sunt aliquae regulae positae ab Aristotele et Commentatore quae prohibeant velocitatem in moventibus præcise se habere sicut se habent duæ proportiones inter motores et inter mota. Ideo solum habetur quod unus motus erit velocior alio quando duo agentia movent per aliqua spatia, quod magis agens velocius movebit quam minus agens, ceteris paribus, et similiter intelliguntur regulæ in aliis motibus, unde in aliquo casu est verum quod duplum agens movet in duplo velocius per eadem resistentiam et aliquando plus quam in duplo velocius. Unde regula Aristotelis VII '*Physicorum*' intelligitur quod si aliquod movens moveat per aliquod spatium in aliquo tempore, duplum agens movebit in duplo velocius vel plus quam in duplo velocius."

12 See Chapter III, pp. 67–87.

Averroes' statements clearly point to the proportion between an active power and resistance, which is not squared but simply multiplied by two. Having noticed the contradiction of these two views, Kilvington first presented two main arguments against the Aristotelian proposition and finally concluded that while talking about a power moving one half of a mobile Aristotle meant precisely a double ratio between F and R; when talking about a power moving a mobile twice as heavy Aristotle meant taking the square root of the ratio of F : R. The general mathematical rules correspond to those of Aristotle's only in one case: if the ratio of the power of the mover to that of its mobile is two to one, the same power will move half the mobile with exactly twice the speed. Kilvington's calculus provided values of the ratio of F to R greater than 1 : 1 for any speed down to the state of rest, since any root of a ratio greater than 1 : 1 is always a ratio greater than 1 : 1. And, with the additional assumption, he accepted, that any excess, however small, of an acting power over resistance is sufficient to initiate motion and to continue it, he was able to describe a very slow motion with a speed greater than 0 and less than 1 ($0 < v < 1$). Hence, he avoided the most serious weakness of Aristotle's theory, which cannot explain the mathematical relationship of F and R in very slow motions, when speed is lesser than 1.

It is worth noting here, that both Kilvington and Bradwardine, while arguing against Aristotle's and Averroes' rules of motion from Book IV and VII of *Physics* introduced a lot of arguments based on everyday experience, like pushing a stone, pulling a barge on the river or rolling the clock face due to uneven suspended weights. Although Bradwardine in the Chapter I of his *De proportionibus velocitatum in motibus* presented the state of the art with respect to the theory of ratios, he nevertheless later employed it only in a very limited extent.

It is now widely accepted that in his *De proportionibus*, dated for 1328, Thomas Bradwardine advocated a new conception of the relations between ratios of motive powers to resistances and the resulting speeds, a conception that continued to be supported by Aristotelians until the early sixteenth century.[13] What has not been recognized until recent times is that the theory called "Bradwardine's rule" ("the new rule of

13 See e.g., E. Sylla, *The Origin and Fate of Thomas Bradwardine's…*, pp. 67–95; S. Rommevaux, *A treatise on proportion in the tradition of Thomas Bradwardine:* The De proportionibus libri duo *of Jean Fernel*, [in:] "Historia Mathematica" 2013, pp. 164–182.

motion") was based on the mathematical conception of compounding ratios familiar to Oxford scholars in the earlier 1320s, i.e., well before 1328. So here it seems to be a case of "Stigler's law of eponymy", published by Stephen Stigler in 1980, which states that no scientific discovery is named after its original discoverer.[14] In studying Kilvington's work, then, we find information about what was going on in Oxford natural philosophy before Bradwardine's *De proportionibus*, which previously had been recognized as the founding document of the Oxford Calculators' natural science. In attempting to trace the impact, spread, and decline of quantifying Aristotle, we should now realize that the activity of quantifying motion had a prehistory prior to 1328. In the opinion of Sylla and Murdoch, however, the tendency to remain close to Aristotelian "rules" of motion seems to be characteristic for all thirteenth- and early fourteenth-century commentators on the *Physics*. "The situation changed rather dramatically in 1328", when Thomas Bradwardine wrote his *Treatise on the Proportions of Velocities in Motion*. He removed the whole problem of relating velocities, forces and resistances from the context of an exposition of Aristotle's words, and investigated it in its own right.[15]

Modern historians of medieval science present "the new rule of motion", that is "Kilvingtonian/Bradwardinian rule" as follows:

> The velocity of motion will vary arithmetically when the proportions of force to resistance determining these velocities vary geometrically. Thus, if a given proportion of force to resistance produces a given velocity, then when that proportion is squared, the velocity will be doubled.[16]

A final remark needs to be made here: both Kilvington and Bradwardine claim that Aristotle and Averroes, Archimedes and Jordanus de Nemore, when talking about doubling a proportion mean "squaring" it and not multiplying it by 2; and both these Oxford thinkers consider only a double proportion, i.e., a proportion which is the result of multiplying the same ratios of force to resistance, so they do not, actually, present any function

14 See S.M. Stigler, *Stigler's law of eponymy.* "Transactions of the New York Academy of Sciences" 39 (1980), pp. 147–58.

15 See J.E. Murdoch, E. Sylla, *The Science of Motion*, pp. 224–225.

16 Ibidem, p. 225.

or general rule for motion which would describe different types of motion. Kilvington's and Bradwardine's originality consists in giving a description of the specific calculus that should be applied in the cases given by Aristotle to make them consistent. So, if we agree that to offer a specific calculus we can assume that Kilvington and Bradwardine give a new rule of motion; this rule is still, however, not any mathematical function as yet.

Nevertheless, Edith Sylla has been always convinced and stated that that "Bradwardine's rule" concerns, and properly describes, relations in natural phenomena, namely in actually occurring local motions, yet she claims that fourteenth-century natural philosophers were not interested in description of changes taking place in the real world but in the world of the imagination. In her opinion, Bradwardine provided a proper rule describing motion that was recognized by his contemporaries as no more than a speculative tool in the description of the natural world; and thus, for constructing more or less complicated imaginable cases. Consequently, the resulting "science of local motion" became a substantial basis only for logical exercises. At first glance, it seems that the contents of the chapter *De motu locali* from William Heytesbury's "Rules for solving sophisms" affirms perfectly Edith Sylla's above-mentioned conclusion.

As was shown in Chapter III of this book, William Heyetesbury was not interested in a description of local motion *tamquam penes causam*, that is with regard to the causes of motion. His interest was focused solely on the description of local motion *tamquam penes effectum*, i.e., concerning the distances traversed, times consumed and the speeds of motion. Therefore, he did not discuss "the new rule of motion" which relates the speed of motion to the ratio of force to resistance. As Sylla noticed, Heytesbury's reasoning has its sources in the concerns about instantaneous speed and overall speed and it is:

> closely related to concerns that were addressed at length in Richard Kilvington's *Sophismata* (…). In several of Kilvington's sophismata, he first exploited the confusions that might arise between instantaneous and overall velocities and then tried to sort them out. (…) Heytesbury, I propose, is adding mathematical tools to those of logic [in Kilvington's case] in the belief that they will be effective where logic alone gets bogged down.[17]

17 See E.D. Sylla, *The Oxford Calculators' Middle Degree Theorem in Context*, "Early Science and Medicine" vol. 15, No 4/5 (2010), pp. 347–349.

Heyetsbury begins with defining terms, first distinguishing uniform motion from non-uniform motion. Local motions are divided into two classes: the uniform and difform ones. A uniform motion is a motion in which equal spaces are traversed continually in equal parts of time. Difform motions can vary in infinitely many ways, both with respect to the magnitude or the subject moved, and with respect to time. Difform motion with respect to the subject moved is a motion in which different points of the body move with unequal speeds; for instance, a rolling wheel moves with difform motion since the speeds of the different points on the wheel vary with respect to the distance from the axis of rotation. Difform motion with respect to time is a motion in which unequal spaces are traversed in equal times. Motion can also be difform with respect to both time and the subject moved. Difform motions are subdivided into two classes: the uniformly difform motion and the difformly difform motion. Uniformly difform motion is motion in which the speed either increases or decreases uniformly, that is, a motion in which in any equal parts of time, equal latitudes of speed are either acquired or lost. A difformly difform motion is a motion in which a greater latitude of velocity is gained or lost in one part of time than in another equal to it.

The most gripping example of uniformly difform motion is a uniformly accelerated motion, like the motion of a body moving towards the earth. Heytesbury gave a general rule, named by historians of science as the "mean speed theorem," with which one might calculate the distance traversed on the basis of the value of the latitude of uniformly changing speed – either accelerating or decelerating. According to this rule, the distance traversed by a uniformly accelerated body in a given time is equal to the distance which would be traversed in the same time with an uniform motion where speed is equal to the mean speed of the accelerated/decelerated motion (half of the sum of the initial and final speed). A number of conclusions follow from this theorem:

A body which moves with a uniformly difform motion beginning from rest and terminating at some finite degree of speed traverses just half the distance traversed by a body which moves uniformly during the same time with a speed equal to the final speed in the uniformly difform motion.

The middle degree of a uniformly difform latitude of speed which begins at some degree and ends at another is greater than half the degree terminating the latitude in its more intense extreme; it follows that a body which moves with uniformly difform motion beginning at some degree of speed and terminating at another traverses more than half the distance which would be traversed by a body moving uniformly during the same time with a speed equal to the most intense speed of the uniformly difform motion.

In a uniformly difform motion beginning from rest and terminating at some finite degree of speed, the distance traversed in the first half of the time equals one third of the distance traversed in the second half. And conversely, in a motion in which the speed decreases uniformly from some degree to no-degree (i.e., to the state of rest), the distance traversed in the first half of the time equals three times the distance traversed in the second half.[18]

In Sylla's opinion, in his *Regulae*, Heytesbury only asserts "the mean speed theorem" or as she says: "the middle degree theorem" without proving it. In our opinion Heytesbury proves the theorem in his *Regulae*. Sylla maintains that the proof of the theorem is to be found in the work entitled *Probationes conclusionum tractatus regularum solvendi sophismata Guillelmi Heytesberi*. We are not sure if Heytesbury was the author of this work, where the theorem is proved as follows:

> Suppose that there is a latitude of motion from no-degree to eight degrees of velocity. Then consider three mobiles moving with a velocity of 4. Let a move for an hour with a uniform velocity of 4. Let b increase its motion uniformly from 4 up to 8 in half an hour, let c decrease its motion uniformly from 4 to no-degree in the same half an hour, and d uniformly gain the whole latitude of motion from no-degree to 8 in the same whole hour. d will traverse as much in the whole hour as b and c together traverse in a half of the hour, because c, while decreasing its motion will traverse as much as d traverses in the first half hour, going through the same degrees from no-degree to 4 or vice versa, and b, when increasing its motion from 4 to 8 will traverse exactly as much as d moving in the second half of the hour and increasing its motion from 4 to 8. But b and c in the half of the hour will traverse just as much as a traverses in the whole hour, because the degrees of

18 See Hanke, Miroslav and Jung, Elzbieta, "William Heytesbury", *The Stanford Encyclopedia of Philosophy* (Spring 2018 Edition), Edward N. Zalta (ed.), URL = <https://plato.stanford.edu/archives/spr2018/entries/heytesbury/>.

> b and c are always equal intervals above and below 4 degrees and together they add to 8. Therefore just as much distance is traversed by a moving uniformly with 4 degrees (the middle degree of the latitude), and by d uniformly increasing its velocity from no-degree to 8 degrees in the same time.[19]

According to Edith Sylla:

> the proof shows that the total degrees of velocity in a and d are the same by a pairing of the degrees of velocity in the various motions of a, b, c, and d. (...) If degrees of velocity are by definition identified by the distances that they cause to be traversed – more in longer times and less in shorter times – then if care is taken to keep times equal, finding that the total velocity is equal implies, but is not the same as, finding that the distance traversed will be equal.[20]

As Sylla rightly concludes: "the original Oxford middle degree theorem for accelerated motion (...) is a rule for finding what is "equally moved" (...). Of course, what is equally moved in some sense traverses equal distances, but this is a futher inference."[21]

Even though Heytesbury provided a simple and effective algorithm for calculating the ratio of such distances relative to any initial and final degrees of the latitudes of speed, it is symptomatic that he finally stated that: "such a calculation would be more troublesome than useful."[22] This remark, though a bit surprising at first glance, assures us in fact that for Heytesbury, as well as for all his contemporary Oxford natural philosophers, all considerations on the "rules" of local motion were conducted only on the theoretical level, with no reference to everyday or practical applications. On the one hand, such an attitude is of course a consequence of the acceptation of Aristotle's division of sciences,

19 See M. Clagett, "Science of Mechanics in the Middle Ages", pp. 247–75, 284–89.

20 See E. Sylla, *The Oxford Calculators' Middle*, p. 353.

21 See ibidem.

22 See William Heytesbury, *De motu locali,* § 238, p. 283: "Sed huiusmodi calculatio maiorem sollicitudinem ageret quam profectum".

where "physics" is purely a theoretical science.[23] On the other hand, as it seems, it is hard to find or imagine any reason for which fourteenth-century scientists would desire or need to know the "real" speed of anything expressed in units analogical to those we are used to.[24] This is further confirmed by the fact that all the Oxford Calculators, when pondering mathematically the relations between factors and speeds in local motions used only relative terms, like "faster" (*velocius*), "slower" (*tardius*) or "equally fast" (*eque velociter*).

It seems that the anonymous author of the treatise *De sex inconvenientibus* tried to construct a proof, similar to Heytesbury's one. As was shown in Chapter III, he surely was associated with the Oxford Calculators' school, and he was well acquainted with Kilvington's/Bradwardine's "new rule of motion", Heytesbury's "the mean speed theorem" as well as with Klivington's arguments for the increasement of speed in downward motion, i.e., free fall, from his question on local motion. In the fourth question of *De sex*…, the anonymous author discussed different opinions concerning the possible "measurement" of speed in uniformly accelerated motion, such as free fall, as well as in circular motion, and in uniformly difform, i.e., constantly accelerated or decelerated motion. As was shown above, the anonymous author had a pretty good knowledge of the procedures of argumentation used in sophisms, but, regrettably, only superficial knowledge of mathematics. The treatise is, however, good testimony to the rapid spread of the new theories of the Oxford Calculators and the interest in their conclusions that was aroused among the philosophers of nature.

23 See Aristotle, *Metaphysics*, 1025b19–22, Bk. E (VI), p. 778: "And since natural science, like other sciences, is in fact about one class of being, i.e. to that sort of substance which has the principle of its movement and rest present in itself, evidently it is neither practical nor productive." See also, E.D. Sylla, *The Oxford Calculators and the Mathematics of Motion*..., p. 42.

24 Such an interest, but again only on a purely theoretical level, arose among medieval natural philosophers with regard to the speed of light or other such "spiritual" qualities. Actually, the question was whether such qualities are propagated in any medium instantaneously, i.e., with infinite speed, or not. See e.g. Johannes Dumbleton, *Summa logicae et philosophiae naturalis*, ms. Cambridge, Gonville & Caius 499/268, f. 69rb: "Ulterius dubitatur in presenti utrum agentia spiritualia agunt succesive vel subito"; ibidem, 70ra: "Item, potest argui, quod aliquid potest infinite (ms.: infinitum) velociter moveri per tempus".

A much more interesting and clear explanation of "the new rule of motion" is to be found in Part III of John Dumbleton's *Summa logicae et philosophiae naturalis*. Dumbleton, as Sylla points out:

> excels in making use of latitudes of motion or velocity to quantify local motion. The keys points are that latitude of velocity is taken as homogeneous. Equal segments of the latitude of velocity, whether they occur at lower velocities or higher, correspond to equal differences of velocity. Equal segments of the latitude acquired in similar ways will always correspond to equal distances traversed.[25]

As was shown in Chapter III, John Dumbleton stated that the latitude of ratios of motive power to resistance and the latitude of speed are gained or lost concurrently. Thus, since the latitude of ratio is additive, so is the latitude of motion, i.e., speed, in a strict correspondence to the ratio. In our opinion, with respect to Dumbleton's theory, for the first time there can be found a mathematical function describing a motion *tamquam penes causam*, i.e., a ratio of motive power to resistance relative to the speed of motion. John Dumbleton agrees, in accordance to Aristotle and Averroes, that motion occurs only if the ratio of F : R is greater than the ratio of equality, i.e., the ratio that equals 1 : 1. Thus, as it seems, he accepted Kilvington's opinion that any excess, however small, of active power over resistance is sufficient for motion to occur or begin. On the Dumbleton's "gauge", the latitude of equality, when the power is equal to the resistance and thus a ratio of F : R = 1 : 1, it corresponds to "no-degree" of motion, that is to the state of rest (in our terms, in this case the speed equals zero). And since there cannot be determined the greatest ratio of F : R, the speed of motion can be, theoretically, faster and faster *in infinitum*. Thus, the possible range for the ratios of F : R is from 1 : 1, to ∞ : 1, and this also regards the range of speeds. With this assumption it is easy to pair ratios and speeds, because a one-to-one correspondence of speed and the ratio of F : R is uniquely assigned; only one, unique degree of speed corresponds to the unique ratio. Dumbleton is also the first one who clearly states that to "double" or "triple" the ratio of F : R means to "compound" i.e., to multiply it twice or three times with itself, respectively, and thus the corresponding speed of

25 See, E.D. Sylla, *The Oxford Calculators' Middle*…, p. 353.

motion will be twice or three times as fast as the previous one. It is worth reminding ourselves here that both Kilvington and Bradwardine wrote only about the doubled ratio (*proportion duplicata*), which, actually, does not allow us to determine or define these relations as a mathematical function.

The "mean speed theorem" proof, as provided by John Dumbleton, is rather short, and as Weisheipl notices:

> Dumbleton's proof differs from that of the *Probationes conclusionum* and from that of later Calculators in that his is indirect, a *reductio ad impossibile*, and not a satisfactory one at that. He assumes that if the grade were any other than the mid-point, a greater total distance would result or less total distance would result than the original hypothesis allows. Because of these absurdities Dumbleton concludes that every latitude of uniformly accelerated motion must correspond to its arithmetical mean velocity, that is, the same distance will be covered in a corresponding time by a body moving uniformly with a velocity equivalent to one-half the terminal velocity of the accelerated motion. Since every ratio contains within itself all its qualitative parts, the terminal velocity is composed of twice its mean degree; and its mean degree is one-half the terminal velocity.[26]

Edith Sylla did not find this proof unsatisfactory, and she provides a more detailed explanation.[27]

It is worth noting here that Dumbleton did not use the calculus of ratios as a mathematical tool to describe real phenomena. He surely was not following William Heytesbury in doing mathematical physics *secundum imaginationem*. John Dumbleton did not think, for instance, of approximating a continuous physical entity or process with a discontinuous mathematical function. Moreover, he even had objections with respect to calculations that involved products lacking a clear physical interpretation. In the examples of Dumbleton's mathematical physics to be found in his Part III: *De motu locali*, the strict adequacy between his mathematics and his physics is quite clear, as well as his Aristote-

26 See J.A. Weisheipl, *The Place of John Dumbleton in the Merton School*, "Isis", vol. 50 no. 4, p. 453.

27 See E.D. Sylla, *The Oxford Calculators' Middle…*, pp. 356–357.

lian and Ockhamist attitudes towards the status of mathematical entities. Given such attitudes, Dumbleton's mathematics, or quantification, plays an auxiliary role to his physics, or natural philosophy. As Sylla states: "His mathematics is further constrained, beyond his rather inadequate mathematical resources, by the extremely close tie he wants to preserve between the mathematical description and the underlying physical reality."[28] It seems that Dumbleton's main purpose was the same like Kilvington and Bradwardine's: to make the description of the Aristotelian world more precise and comprehensive through mathematics. This attitude is quite different from what occurs in modern physics, where the mathematical descriptions often seem to play the central role, with the physical interpretations of the mathematical entities in an auxiliary position.

Even if the Richard Swineshead's "Book of calculations" represents the most sophisticated stage in the development of natural philosophy within the circle of the Oxford Calculators, at the same time it is exemplary in the sense as to how strongly attached, or even deeply affected, to the Aristotelian worldview these otherwise ingenious thinkers were. At least in the context of his "science of local motion" Swineshead never crossed the boundaries of Aristotle's physics, even if he were to reach them in due course. In Chapter XIV of the *Liber calculationum*: *De motu locali*, dedicated from the outset to establishing the "rules" of local motion, he simply adopts the new, Kilvingtonian/Bradwardinian rule of motion and exploits it to its limits, dictated by logical and mathematical applicability and consistency. As was presented in detail earlier, the succesive cases he discussed there were formulated *a priori* by a consequent permutation of the imaginable changes in the factors of local motion, and the resulting changes of speed(s) were determined in a "geometrical" manner, on the basis of the already accepted or proven statements. The whole of his "science of local motion" is developed speculatively, Swineshead never referred to natural phenomena, either when formulating the "cases" or establishing the "rules". All his reasoning are mathematically and logically consistent and acceptable, even if – as Murdoch and Sylla have remarked – they are perfect examples of his mathematical ingenousness, not in that he did complex mathematics, but in that

28 See E. Sylla, *The Oxford Calculators and Mathematical Physics…*, p. 150.

he knew how to avoid complex mathematics.[29] In these respects, as we have remarked earlier, *De motu locali* should be recognized as Richard Swineshead's attempt to construct, and here employing the *calculationes* for this purpose, an exemplary and complete Aristotelian "science of local motion", analogous to Euclid's geometry, *mutatis mutandis*. It is worth noting that Swineshead was perfectly aware of the limitations and doubts formulated by his predecessors from the Oxford Calculators' circle, and tried hard to solve and overcome these. We mean here, of course, William Heytesbury's "restriction" that there can be no rules formulated with respect to difformly difform motions, and the problem of "multiple continua". As we have shown, Richard Swineshead managed to establish more or less general "rules" describing a special kind of difformly difform motions, and provided the detailed algorithms on how a given motive power should be "adjusted" in order to effectuate a uniformly decelerated motion in a uniformly difformly resistant medium. Still, these conclusions, however ingenious and mathematically correct, were formulated only *secundum imaginationem*. Even if in other parts of his *Liber calculationum* Swineshead seemed to touch upon natural phenomena, when describing, for example, the succesive deceleration of motion, the solutions he provided assure us that he was an Aristotelian thinker. This is further confirmed by the fact that even when he discussed elsewhere cases of motion beginning with no-degree of resistance, which could be interpreted as motion in a void – something impossible according to Aristotle – he introduced this no-degree only as an initial condition, or assumption, of an imaginary case in order to simplify the calculations that followed. Perhaps the most important evidence for Richard Swineshead's attachment to Aristotelian natural philosophy is the fact that he conducted all his analyses into different kinds of local motions only "with respect to cause" (*tamquam penes causam*) recognizing it as the only adequate description.[30] In this respect he observed, presumably, Aristotle's *dictum* that theoretical sciences in

29 See John Dumbleton, *Summa logicae et philosophiae naturalis*, pars III, *passim*. J.E. Murdoch, E.D. Sylla, "Swineshead", p. 204.

30 It is worth reminding here that in one of his short treatises on motion Richard Swineshead established the mutual equivalence between *penes causam* and *penes effectum* descriptions, yet his remark – formulated in the chapter "On maximum and minimum" of his *Liber calculationum* that "a distance does not resist' assures us that for him the description "with respect to cause" was the proper one.

general, and natural science in particular, deal with causes. Therefore, Richard Swineshead's account on local motion should be appreciated with respect to the range and complexity of the cases he considered and "solved". Yet, it must be stressed here, that in this Richard Swineshead was not striving to formulate any new, not to mention revolutionary, theory of local motion. His aim was rather to supplement and complete the "science of local motion" formulated by his predecessors within Aristotelian natural philosophy.

In fifteenth-century Italy, where *Liber calculationum* in general generated quite an interest among scholars, there appeared only one short text referring to the "science of local motion", namely Giovanni Marliani's *Probatio cuiusdam sententie Calculatoris de motu locali* (dated for 1460).[31] In this short treatise Marliani discussed and – in his own opinion – corrected one of the reasonings concerning the "mean speed theorem" included in Richard Swineshead's *De motu locali*.[32] Even if Marliani's treatise clearly shows his acquaintance with Swineshead's text and skill with respect to the calculus of ratios, it was clearly not intended to supplement the "science of local motion".

The only scholastic philosopher that managed to develop Richard Swineshead's conclusions a bit further was a Portuguese active at Paris University in the beginning of the sixteenth century, Alvaro Thomaz (Alvarus Thomas). In his *Liber de triplici motu proportionibus annexis (…) philosophicas Suiseth calculationes ex parte declarans*, published in 1509, Alvaro Thomaz referred not only to *Liber calculationum* but also to the works of Thomas Bradwardine, William Heytesbury, Albert of Saxony, Nicole Oresme, Paul of Venice, to mention the most recognizable fourteenth- and fifteenth-century natural philosophers dealing with *calculationes*.[33] He was surely well acquainted with the contents of Richard Swines-

31 See M. Clagett, *Giovanni Marliani and Late Medieval Physics*, New York 1941, p. 28. Fifteenth-century Italian thinkers were interested mainly in the problem of "reaction"; R. Podkoński, *Richard Swineshead's* Liber calculationum *in Italy. The Codex*, pp. 423–424.

32 Giovanni Marliani, *Probatio cuiusdam sententie Calculatoris de motu locali*, Venezia, Bibl. Naz. Marciana, lat. VI 105, f. 8r: "Quoniam Calculator in probando latitudinem motus uniformiter difformem suo gradui medio correspondere in tertia probatione format unam consequentiam (…) que fortassis multis esset dubia aut a multis negaretur," (after: M. Clagett, *Giovanni Marliani*, p. 103, footnote 5). For details, see M. Clagett, *Giovanni Marliani…*, pp. 103–124.

33 See E.D. Sylla, *Alvarus Thomas…*, p. 264.

head's treatise "On local motion", since he discussed some of the conclusions and reasonings included there, quoting a few of them *in extenso*.[34] Apart from the cases proposed by Swineshead, Alvaro introduced more complicated ones, namely discussing local motions which would be effectuated by its factors (i.e., motive power and resistance) changing in such a way that the ratio of these would be different in every proportional part of the duration of this motion. Interestingly enough, the pattern the ratio would change could be established more or less at random, which means that Alvaro was not referring here to a description of real phenomena at all. Even though he discerned some cases in which the mean speed could be calculated, these too were assumed *a priori*.[35] This is further confirmed by the context of his *calculationes*.

34 Alvarus Thomas, *Liber de triplici motu*, sig. p2ava–p3vb (after: E.D. Sylla, *Alvarus Thomas…*, p. 289): "Prima propositio. Si aliquis motus uniformiter continuo intendatur vel remittat a certo gradu usque ad certum gradum vel ad non gradum eius velocitas gradui medio correspondet (…). Secunda propositio. Omnis motus continuo velocius et velocius intensus correspondet quantum ad velocitatem gradui remissiori medio gradu inter extremem intensionis eius in principio motus et inter extremum intensionis in fine motus. Tertia propositio. Omnis motus velocius et velocius deperditus quantum ad transitionem spacii intensiori gradui gradu medio correspondet (…). Quarta propositio. Omnis motus tardius et tardius intensius quantum ad pertransitionem spacii gradui intensiori medio correspondet. Quinta propositio. Omnis motus tardius et tardius deperditus gradui remissiori medio correspondet. Sexta propositio. Omnis latitudo motus consimiliter omnino perdita et acquisita uni gradui omnino correspondet." See Ricardus Swineshead, *Tractatus de motu locali*, § 152, p. 334; § 157, p. 338; §§ 159, 161, 163, p. 339.

35 Alvaro Thomaz, *Liber de triplici motu*, q4va–q5rb (294): "Decima conclusio. Divisa hora per partes proportionales proportione dupla et a mobile in prima parte proportionali moveatur aliquantula velocitate et in secunda in sexquialtero maiori velocitate quam in prima et in tertia in sexquiquarto maiori velocitate quam in prima et in quarta in sexquisexdecimo maiori quam in prima et sic consequenter ascendendo per species proportionis superparticularis denominatas a numeris pariter paribus (…) spacium pertransitum in totali hora se habet ad spacium pertransitum in prima parte proportionali in proportione dupla sexquitertia; q6rb—vb (295—296): „Ex his satis facile apparet multa talia nobis incomprehensibilia esse. Nec tamen propterea hec ars reicienda est, quoniam et si infinita sint nobis incomprehensibilia, infinita etiam mathematica demonstratione valent a nobis infallibiliter demonstrari, puta ea que continuum ordinem alicuius proportionis observant ut superius dictum est. Cetera vero sicut nullum ordinem servant ita nullis regulis scientie astringi valent. Hic tamen

From his text it follows unambiguously that calculating the "mean degree" in such concocted examples was a kind of intellectual rivalry in the milieu of the Arts Faculties in his times, and his intention was to provide a participant *modi operandi* that guaranteed the victory. This very purpose of Alvarus's treatise is obvious, among others, from the passage where he advised on what to do when the case proposed by the opponent is too complicated for us to unravel. Then, we read, we should simply formulate an analogical case and ask the opponent to show the method of "solving" it.[36]

Obviously, the switch in the purpose for the "rules" of local motion as developed by Richard Swineshead and that occurred in the times of Alvaro Thomaz, from, Aristotelian in fact, advanced speculative science, to the "points of departure" for purely intellectual, abstract challenges, caused that in the following centuries *Liber calculationum* was

unum advertendum est quod plerunque homo arbitrabitur nullam esse seriem aut ordinem proportionum in aliquo casu sibi proposito, nihilominus maturius et diutius consideranti occurret talis ordo sicut in casu quarte conclusionis non apparet aliquis ordo alicuius proportionis continue, nihilominus ibi reperitur continuo equalitas velocitatum in partibus inequalibus".

36 Ibidem, q6vb (296): "Ubicumque occurit multiplicitas proportionum inter quas facile non reperitur proportio, censendum est multas earum irrationales esse ad invicem, quare et spacia pertransita irrationalia esse. Qua propter cum talis casus proponitur respondendum est spacium pertransitum in tota hora incommensurabile esse spacio pertransito in prima parte proportionali. Sed dices instabit tamen totis viribus illiberalis atque acerrimus calculator, grandiaque verba trutinando inflata bucca, supercilio elevato, rugataque fronte, atque ore tragico, rationem suam insolubilem personabit, multisque clamoribus respondentem vulgo superatum atque devictum nitetur ostendere. Respondeo quod in simili negocio duplici cautela utendum censeo. Prima pro delubrio et ridiculo habeatur argumentum eius tanquam inutile et intelligibile, petaturque calamus et atramentarium ut specie multiplicationis ceterisque algorismi speciebus calculari valeat velocitatis intensio in casu per eum posito. Secunda cautela. Dicatur breviter arguenti quod talis velocitas non potest infallibiliter et certitudinaliter calculari perinde atque multe alie difformes velocitates non valent naturaliter ad uniformitatem reduci. Et si clamoribus velit respondentem expugnare oppositum asseverendo, proponat ei respondens similem casum et dicat ei ut certificet illi de spacio pertransito adequato mediante tali velocitate difformi. Et si dixerit quod non est possibile naturaliter invenire velocitatem adequatam in tali casu, subiungat respondens quod nec in suo similiter pari ratione. Si autem dicat opponens se nolle tale spacium assignare quavis assignabile sit naturaliter, hoc idem dicat ei respondens."

counted rather among *libelli sophistarum* and recognized only as a set of difficult logico-mathematical exercises.[37]

As we stated earlier, the speculative "science of local motion" Richard Swineshead developed in his "Book of calculations" was intended to be possibly the most complete realization of this science within the background of scholastic natural philosophy with the help of the calculus of ratios with all the conditions formulated by Aristotle. Save for the few extremely complicated cases introduced later by Alvaro Thomaz, it seems simply impossible to find any other imaginable and at the same time logically consistent "configurations" of the factors of local motion for which Richard Swineshead did not provide a description in his treatise. With respect to his "science of local motion" we face the situation that any other, more complete theory of motion must have been founded on totally different assumptions. In this respect natural philosophy had to wait for Galileo and Newton, who began their considerations from common experience and simply started to "measure" observable phenomena, generally speaking.

The Novelty of Medieval Mechanics vis-á-vis Aristotelian and Galileian Theories

For our own purpose, we shall now summarize and review what appear, in the opinion of some historians of medieval science, to be the most important departures of fourteenth century mechanics from Aristotle's physics. First of all, here is a blend of the Aristotelian dynamics tradition and Archimedean statics and mathematical tradition. Secondly, there is a refutation of Aristotle's prohibition of *metabasis* and the use of mathematics as the proper method in natural philosophy. As we have emphasized, it was for the first time in the medieval period that mathematical strictness forced natural philosophers to invent a new rule describing motion. Thirdly, there is the separation of dynamics and kinematics, which led to the formulation of "the mean speed theorem" enabling one to compare the speed of a uniformly accelerated/decelerated motion

37 This opinion as to the nature and purpose of the Oxford Calculators' achievements in natural philosophy is still pursued by Edith Sylla (see E.D. Sylla, *The Oxford Calculators* …, p. 563).

with the speed of a uniform motion. Fourthly, there is the promotion of mental experiment.

Deeper insight into medieval mechanics, however, reveals the constant presence of the Aristotelian background. Even though Kilvington and Bradwardine had broken the Aristotelian prohibition of *metabasis*, they still remained within the framework of his physics, in which motion occurs because of the action of two necessary factors: moving power and resistance – acting as its direct causes. The speed of motion is determined by the ratio of moving power to resistance and "the new rule of motion" does not break this principle. Like Aristotle, Kilvington, Bradwardine, and their followers, maintained that constant motive power (and resistance likewise) causes a constant speed and not constant acceleration, something which was later only properly recognized by Galileo and formulated as the second law of motion by Newton in the seventeenth century.

Secondly, the notions 'uniform', 'uniformly difform' and 'difformly difform' motion were used not only to describe the distribution of changes in uniform, accelerated and decelerated motions. For when medieval natural philosophers considered the difformly difform speed, they had in mind not only non-uniform changes of speed, but also uniform changes of acceleration, i.e., a motion with equal increments/decrements of acceleration. Such motions do not occur as natural phenomena. Furthermore, such terms as 'uniformly difform' motion and 'uniform increasement of speed' were used in both contexts – of the motion of a free fall, i.e., downward motion, and of uniformly accelerated upward motion. This is a part of medieval mechanics to which we do not pay enough attention, since we look only for properly recognized problems.[38]

Thirdly, common as they were in the Middle Ages, mental experiments were rationalistic, only thought out, and not empirically rooted experiments, and these did not stimulate the development of an experimental science of motion.

38 Since the Aristotelian world was 'symmetric' with regard to gravity and levity, there was no inconsistency in the imaginative and "proper" description of accelerated upward motion. The very best example here is Kilvington's question on motion in a void from his commentary on the *Physics* (for details see E. Jung, *Motion in a Vacuum and in a Plenum in Richard Kilvington's Question*…, pp. 179–193).

Still, we agree with John Murdoch and Edith Sylla, who have pointed out that: "It would be an error to regard these new and distinctive 14th century efforts as moving very directly toward early modern science".[39] Galileo's familiarity with late medieval physics' departures from Aristotle, which even made him repeat some of their erroneous solutions, did not affect his general idea, since he used fragments of medieval mechanics for completely different purposes. Galileo, whom we want to make responsible for the beginnings of Newtonian dynamics, rejected or rather reformulated "the new rule of motion" while going back to the theory expressed by Avempace. Likewise, he read Archimedes' works in a different way and context than did the medievals, which allowed him to create mathematical physics while recognizing the distinction between statics and dynamics. It also permitted him to consider mechanics as a contemplative and mathematical science under geometry that could provide the mechanical arts with their principles and causes. With the two major achievements of Galileo's mechanics, namely the conception whereby the horizontal uniform motion of an unanimated body is held to be a state in which it remains until some external force causes it to change and the identification of free fall as a uniformly accelerated motion with the exposition of its role in nature, the new concepts in mechanics began a career that culminated in Newton's theory. In spite of this, Galileo was able to profit from the *secundum imaginationem* and *ceteris paribus* procedures, making broad use of mental experiments to convince his readers to accept Copernicus's heliocentric theory. Galileo's approach to the problem of a possibility of applying mathematical principles to physical phenomena was to view these principles not as pure mathematical abstractions but as laws that governed an experimentally rooted science of motion.

We would like to stress, however, that each step taken by new generations of fourteenth-century natural philosophers was a step forward, even though it was a step taken on the dead-end road of the Aristotelian science of motion. In our opinion, medieval mathematical physics was doomed, since even if it had succeeded in refuting the restrictive prohibition of *metabasis* associated with Aristotelian philosophy and accepted mathematics as its method, it did not develop empirical mathematics and experimental physics. This was because, ironically, the liberation of mathematics from the limitations of actual experience created a tool

39 J. Murdoch, E. Sylla, *The Science of Motion*…, p. 249.

of theoretical analysis that would make it impossible to cross over the threshold of an exact science. Even though a tradition in "mathematical physics" was to continually develop in England from Grosseteste to the middle of the fourteenth century and then was to be continued by French, Italian, and Spanish thinkers until the end of the sixteenth century, it never made the final step forward to abandon Aristotle. Paradoxically, Aristotelian physics appeared to be perfectly prone to accommodate all medieval attempts at providing it with mathematical precision. The fourteenth century revolution in mechanics was a revolutionary movement against the background of previous medieval theories, but not in relation to the seventeenth-century ones. The revolution was in the details. In its history medieval science, while taking an Aristotelian course, thoroughly explored that framework exposing its paradoxes and weakness yet reached the point where it was unable to overcome the lingering doubts. The big, decisive break was left to the successors of the medieval philosophers of nature.

After a deliberated study of the medieval science of motion and secondary literature we are forced to formulate the final conclusion: the fourteenth-century revolution in science should not be regarded as the first step towards the Scientific Revolution. In our opinion later medieval mathematical natural science should be treated only as a specific and fascinating phenomenon of medieval thought culture and evidence of the ingeniousness of the scholars that created it.

Editions

Introduction

In planning the contents and composition of the present volume, one of our main aims was to provide the reader with a selection of the most representative texts composed within the intellectual milieu of the Oxford Calculators' school that would illustrate clearly the development of the "science of local motion," being at the same time the ones that had previously not been published or which had not been presented in sufficient detail. Thus, the first of these texts is Richard Kilvington's question *Utrum potentia motoris excedit potentiam rei motae* from his commentary on Aristotle's *Physics*, dated for 1324 – 1326. Up to now it has been available for researchers only in the form of medieval manuscript copy, and we provide the very first printed, critical edition of this text in the present volume. The second text is the section *De motu locali* ("On local motion") of William Heytesbury's most famous treatise *Regulae solvendi sophismata* ("Rules for solving sophisms"), dated for 1335. The treatise was edited in print only once, in Venice in 1494. Therefore, the rare, preserved copies of this edition are not easily available for research now. What is more, for obvious reasons, it cannot be taken as critically edited version of the text. Hopefully, the first modern critical edition of this section included in the present volume will be a first step to an edition of the full text of this important and influential work. The third text we include in our selection is the question *Utrum in motu locali sit certa servanda velocitas?* from the anonymous treatise *De sex inconvenientibus* that was surely composed in the intellectual milieu of the Oxford Calculators' school between 1335–1338. Like other texts presented here, *De sex inconvenientibus* is still available only in medieval manuscript copies, therefore the present edition is the first modern, critical one of the section of this interesting treatise.[1] The last text provides the reader with a selection of substantial fragments of the Part III of John Dumbleton's *Summa logicae et philosophiae naturalis*: *De motu locali*. The exact date of composition of this monumental *Summa…* is unknown, yet it must have been written before 1349. There have never been any printed edition of John Dumbleton's treatise, but due to the reasons explained in detail below we decided to present only a transcription of these fragments from

1 Sabine Rommevaux-Tani – Director of the SPHERE, UMR, 7219, CNRS – has been working on the critical edition of the whole treatise *De sex inconvenientibus*.

the best manuscript copy of the text preserved, corrected against other copies, where it was absolutely necessary. Still, we are sure that the Latin text we provided reflects the author's autograph version.

In what follows short introductions to each of these editions are given first, wherein the preserved copies of respective works are enumerated, and reasons for choosing the base-copy of a specific text are explained in detail in each case. When preparing these editions, we introduced and observed the same rules for each Latin text we provided in the present volume. These rules are described next, likewise the abbreviations we employed within the *apparati critici* of these texts. The number of levels of *apparatus criticus* is, however, not the same for each of the texts. Therefore, the contents of the *apparatus criticus* in each case are explained separately. Finally, the editions of Latin texts are provided in the above-presented order, in each case the sigla of the manuscript copies referred to are given.

1. Richard Kilvington's Question *Utrum potentia motoris excedit potentiam rei motae* from His *Quaestiones super libros Physicorum*

The edition of Richard Kilvington's question: *Utrum potentia motoris excedit potentiam rei motae,* the fifth of eight that form his commentary on Aristotle's *Physics*, is based on the only complete copy included in the codex: Venezia, Biblioteca Nazionale Marciana, lat. VI, 72 (2810). A short fragment of this question can also be found in the Vatican, BAV, Vat. lat. 2148 manuscript.[2] The Venice manuscript belonged to the collection of Johannes Marchanova and was transcribed in 1439. All the remaining texts included in this codex are dated to the first half of the fifteenth century. The contents of this manuscript is as follows:

f. 11ra—76va, Caietanus de Thenis, *Recollectae in librum [octo libros] physicorum Aristotelis.*

f. 81ra—112vb, Richardus Kilvington *Quatuor quaestiones compilatae a reverendo viro magistro Ricardo super libro physicorum [Aristotelis].*

2 There are only two columns of this question preserved in this codex, viz. f. 77va-vb.

f. 81ra—89rb, *Utrum in omni motu potentia motoris excedit potentiam rei motae.*

f. 89rb—101ra, *Utrum qualitas suscipit magis et minus.*

f. 101ra—107vb, *Utrum aliquod corpus simplex possi aeque veolicter moveri in vacuo et in pleno.*

f. 107vb—112rb, *Utrum omne transmutatum in transmutationis inicio sit in eo ad quod primitus transmutatur.*

f. 113ra—116va, Hugo Senensis, *Utrum in vero augmento quaelibet pars corporis quod augetur augeatur.*

f. 123ra—132vb, <Anonimus>, *Utrum in coelo sit materia, vel si coelum sit corpus simplex, ut ponit commentator.*

f. 133ra—153rb, Magister de Sancta Sophia, *Utrum definitio elementorum sit bona, dicens: elementa sunt corpora et primae partes corpori humani, etc.*

f. 153rb—155ra, Hugo Senensis, *De modo generationis mixtorum ex elementis et elementorum permanentia in mixtis.*

f. 155rb—162vb, Marsilius de Sancta Sophia, *Utrum de sensationem requiratur productio specierum sensibilium ab obiecto in medium et sensitiva potentia.*

f. 163ra—165va, short, varied fragments of different texts on the motion of alteration, local motion and transmutation. Among which two sophisms are to be found:

f. 165va-167rb, *Qui fortiter variabitur alteratio uniformis.*

f. 167rb-165rb, *Non sic esse poterit qualiter esse poterit.* This sophism is interrupted, *expl* : *et qualiter talis est propositio de contingenti et cuicumque potentie appropriate est actus appropriatus sed actus huiusmodi non est absolute*

f. 165rb-169va, fragment of Richard Kilvington's question *Utrum omne transmutatum….*[3]

2. The Section *De motu locali* of Wiliam Heytesbury's *Regulae solvendi sphismata*

When preparing the critical edition of William Heytesbury's *De motu locali* we have consulted and collated all the available preserved manu-

3 For further details, see: *Biblioteca manuscripta ad S. Marci Venetiarum, digessit et commentarium addidit Joseph Valentinelli praefactus*, codices MSS. Latini, T.V, Venetiis MDCCCLXXI(1871), pp. 20–21; M.C. Vitali, *L'umanista padovano Giovanni Marcanova (1410/1418–1467) e la sua biblioteca*, "Ateneo Veneto", XXI(1983), pp. 127–161.

script copies of this section of his "Rules for solving sophisms," rejecting from the outset the incomplete ones. Thus, the following manuscripts were taken into account:

1. Bergamo, Biblioteca Civica Angelo Mai, MS 481 (IV 7) – dated 1442—1444;[4]
2. Brugge, Stedelijke Openbare Bibliothek 497 {= B} – dated roughly for 14th century;[5]
3. Cesena, Biblioteca Malatestiana, S. X. 5 – dated 1450;[6]
4. Cracow, Biblioteka Jagiellońska 621 {= K} – dated 1390;
5. Cracow, Bibl. Jagiell. 704 – dated roughly for 14th c.;
6. Erfurt, Wissenschaftliche Allgemeinebibliothek, Amploniana Cms 2o 135 {= E} – dated 1337;
7. Erfurt, Wissenschaftliche Allgemeinebibliothek, Amploniana Cms 4o 270 {= R} – before 1390;
8. Florence, Biblioteca Ricardiana 821 – dated for the years 1472–1473;
9. Leipzig, Universitätsbibliothek 529 – dated for the first half of the 14th c.;[7]
10. Leipzig, Universitätsbibliothek 1360 – composed between 1376—1400;[8]

4 Given that Paul Vincent Spade has already presented descriptions of almost all the manuscript copies of William Heytesbury's "Rules for solving sophisms" we shall not be providing any details. With respect to those copies Spade did not mention, viz. the codices preserved in Cesena (#3) and Leipzig (#9 and #11) detailed descriptions of these may to be found as indicated in the subsequent footnotes. See, P.V. Spade, *The Manuscripts of William Heytesbury's* Regulae solvendi sophismata: *Conclusions, Notes and Descriptions*, "Medioevo", 15 (1989), pp. 281–304.

5 Sigla refer to the manuscripts on which the present critical edition is based, for a detailed explanation, see below.

6 For in-depth information on this codex see the record in the Malatestiana library catalogue: URL=<http://catalogoaperto.malatestiana.it/ricerca/?oldform=-mostra_codice_completo.jsp?CODICE_ID=219>.

7 For in-depth information on this codex see, Peter Burkhart, *Die lateinischen und deutschen Handschriften der Universitätsbibliothek Leipzig*, Bd. 2: *Die theologischen Handschriften*, T.1: Ms. 501–625, Wiesbaden: Harrassowitz 1999, pp. 46–51.

8 The specific time of the composition of this manuscript is based on information to be found at: URL=<www.manuscripta-mediaevalia.de/?xdbdtn!%22obj%20 31581854%22&dmode=doc#|4>.

11. Leipzig, Universitätsbibliothek 1370 {= G} – dated for 1430—1440;[9]
12. London, Wellcome Historical Medical Library MS 350 – dated 1446;
13. Munich, Bayerische Staatsbibliothek Clm 23530 – dated roughly for $14^{th}/15^{th}$ centuries;
14. Oxford, Bodleian Library, Canonici misc. 221 – dated for 1422–1430;
15. 15. Oxford, Bodleian Library, Canonici misc. 409 {= C} – dated for late 14^{th} c.;
16. Oxford, Bodleian Library, Canonici misc. 456 – dated 1467;
17. Padua, Biblioteca Universitaria di Padova 1123 {= U} – dated for late 14^{th} c.;
18. Padua, Bibl. Univ. di Padova 1434 – dated 1448;
19. Prague, Statni Knihovna ČR 396 – dated roughly for 14^{th} c.;
20. San Giminiano, Biblioteca e Archivo comunale 25 – dated 1475;
21. Vatican, Biblioteca Apostolica Vaticana, vat.lat. 2136 {= A} – dated for the beginning of the 15^{th} c.;
22. Vatican, BAV, vat.lat. 2138 – dated for the beginning of the 15^{th} c.;
23. Venice, Biblioteca Nazionale Marciana, lat. VIII. 38 (3383) {= D} – dated 1391;
24. Verona, Biblioteca Civica 2881 – dated for 15^{th} c.

Interestingly enough, despite there being such a substantial number of preserved copies, the initial conclusion following the preliminary comparison was rather disappointing with respect to the preparation of a critical edition of the text. Each of the above-mentioned copies features its own, in some cases substantial, omissions and specific readings, what means that none could be recognized as the basis for any other surviving handwritten copy of this treatise. It seems that, since the "Rules for solving sophism" were presumably intended from the outset to be a handbook for first-year students, they were frequently copied by students themselves.[10] When we accept this hypothesis, it is

9 For in-depth information on this codex see: URL=<www.manuscripta-mediaevalia.de/?xdbdtn!%22obj%2031581862%22&dmode=doc#|4>.

10 In the Prologue to the "Rules…" it is explicitly stated that these were written for the use of *quibusdam juvenes studio logicalium agentes annum primum*. See Guilelmus Heytesbury, *Regulae solvendi sophismata*, Venetiis 1494, f. 1ra.

easy to explain why there are so many individual, and often wrong or fallacious, variant readings in the treatise's manuscript copies presently at our disposal. It is easy to imagine, that a first-year student had not yet been well enough acquainted with the system of abbreviations as well as the topic of the text, and that is why the author of one of the copies, e.g., wrote stubbornly '*pectus*' instead of '*punctus*' or introduced the inexistent numeral '*subquatriplum*' where '*subquadruplum*' should have been given.[11] These are the types of errors we encounter in the copy of *De motu locali* preserved in Padua, Bibl. Univ. 1123 codex (#17 on the above list, U), though in other respects still quite an acceptable copy. The above hypothesis also allows us to explain the presence of variant readings or supplementing fragments that were obviously intended as corrections to the basic text. Such "emendations" occur frequently within the more sophisticated argumentations in Heytesbury's treatise. Perhaps the most salient example of such a procedure is encountered within the reasoning aimed at determining the ratio of distances traversed in the first and second half of a duration of a uniformly accelerated motion, that is the direct corollary of William Heytesbury's famous Mean speed theorem.[12] The mathematically proper conclusion is here that the ratio of these distances is *subtripla* (i.e., the ratio between these distances is like 1 : 3), yet in many copies we read that the ratio is *subdupla* (1 : 2).[13]

Within the copies of Heytesbury's *De motu locali* analysed for the purpose of the present edition we can find certain instances – chiefly common omissions, especially *homoeoteleuta* – that allow us to establish prelusively a few families of the handwritten copies for this part of "Rules for solving sophisms". Yet, as we have already stated, there is no direct relationship between any two of them. For example, there are numerous noticeable variant readings common and characteristic for mss. Cracow, Bibl. Jagiell. 704 (#5); Leipzig, Universitätsbibl. 1360 (#10) and 1370 (#11, G), and Prague, Statni Knih. ČR 396 (#19), that may allow us to suppose that these all belong to the same family. At the same time the variant readings specific for each of the above-mentioned copies sug-

11 '*Subquadruplum*' is simply 'one fourth', while 'one third' in medieval Latin was '*subtriplum*', what number was represented by '*subquatriplum*' remains a mystery. See e.g., Guilelmus Heytesbury, *Regulae solvendi sophismata: De motu locali*, § 7–9; § 32 *et al.*

12 See above, pp. 89–91.

13 See Guilelmus Heytesbury, *De motu locali.*, § 35, p. 281.

gest that there must had existed also at least three, now lost, copies in this family, one of which served as the basis common for the Cracow (#5) and Prague (#19) versions of the text. What is more, even though three of the mentioned copies (*viz.* #5, #10 and #19) are dated for 14th century, the best – i.e., the one least corrupted – version of the text we can find in Leipzig, Universitätsbibl. 1370 (#11, G) as a codex dated for 15th century. The other family is presumably formed by the mss. Cesena, Bibl. Malatestiana S.X.5 (#3); Oxford, Bodleian Lib., Canon misc. 456 (#16); Padua, Bibl. Univ. 1434 (#18); and Verona, Bibl. Civ. 2881 (#24). Only in these four copies do we find the supplementing explanation within the reasoning concerning difformly difform motions (emphasized below in italics):

> Unde universaliter gradus terminans talem latitudinem secundum extremum intensius est remissimus citra aliud extremum eiusdem latitudinis cui non potest totus huiusmodi motus difformiter difformis correspondere. *Et gradus terminans illam latitudinem secundum extremum remissius est intensissimus citra aliud extremum eiusdem latitudinis cui non potest huiusmodi motus difformiter difformis correspondere.* Unde nec est possibile quod totus huiusmodi motus ita remisso gradui correspondeat, sicut idem motus poterit correspondere, nec ita intenso.[14]

At first sight it seems obvious that the supplementing fragment was lost in the remaining copies of Heytesbury's *De motu locali* due to *homoeoteleuton*, since the last word in the precedent sentence and in this very sentence are the same. However, we should not overlook the fact that all the above-mentioned copies to include this fragment originate from Italy and were transcribed around 1450. One could give a plausible explanation, of course, as to why all the older preserved copies of the text are corrupted in this respect, yet at present this would be just mere speculation with no firm foundations whatsoever.

We have noticed also some similarities that cannot be taken to be mere coincidencies between the Vatican, BAV, Vat.lat. 2136 (#21, A), Vat.lat. 2138 (#22) and San Giminiano, Bibl. e Archivo com. 25 (#20) copies of this text, and also between the Leipzig, Universitätbibl. 529 (#9) and

14 See Guilelmus Heytesbury, *De motu locali*, § 41, p. 284. There are a few minor differences in the fragment emphasized in each of the mentioned manuscript copies, but for clarity's sake we shall not mention them here.

London, Wellcome Lib. 350 (#12) versions. The instances mentioned notwithstanding, the dispersed tradition we have encountered with regard to the manuscript copies of "Rules for solving sophisms" makes practically impossible establishing the *stemma codicorum* at this stage of research. This fact was already noticed by Spade on the basis of his own analyses of a few other fragments of this work.[15]

With the idea in mind of preparing in the future a critical edition of the complete text of William Heytesbury *Regulae…* and here with respect to the following edition of the section *De motu locali*, we decided to establish and observe those editorial principles that, in our opinion, guarantee the critical character of this edition and provide the reader with a version of the text closest to the one intended and written by William Heytesbury himself.[16] Since including all the variant readings from each of the twenty four copies of the text as listed above would make the critical apparatus enormously big, and would be in fact pointless, we decided to limit the number of copies we have taken into account. Taking for granted the date of composition of the "Rules for solving sophisms" as 1335 we have assumed, somehow arbitrarily of course, that the copies dated for the 15th century in terms of the number of possible intermediary handwritten copies were more "distant" from the archetypic text than those transcribed in the 14th century.[17] Consequently, in the

15 See P.V. Spade, *The Manuscripts of William Heytesbury's…*, p. 275.

16 Here we should remember that there are a few more manuscript copies of the *Regulae…*, the ones that do not contain the last chapter of Heytesbury's treatise, traditionally entitled *De tribus predicamentis* – of which *De motu locali* is the first section, or which contain only fragments of this treatise. There was no point in scrutinizing these while preparing the edition of the *De motu locali* section, but – hopefully – a closer analysis of these remaining copies will provide us with a reliable grounds to eventually establish the *stemma codicorum*. The incomplete manuscript copies of Heytesbury's *Regulae…* are, according to Spade, contained in the following codices: (a) Brugge, Stedelijke Openbare Bibl. 500; (b) Erfurt, Amploniana Cms 2° 313; (c) Florence, Bibl. Riccardiana 790; (d) Milan, Bibl. Ambrosiana C 23; (e) Padua, Bibl. Antoniana XIX.407; (f) Padua, Bibl. Univ. 1570; (g) Vatican, BAV, Chigiani E.V.161; (h) Vatican, BAV, Chigiani E.VI.193; (i) Vatican, BAV, Ottobon. lat. 662; (k) Vatican, BAV, Vat.lat. 3144; (l) Venezia, Bibl. Nat. Marciana, Zanetti lat. 310; (m) Warsaw, Bibl. Narodowa MS III 8058. See P.V. Spade, *The Manuscripts of William Heytesbury's…*, pp. 282–304.

17 Surprisingly enough, in the catalogue record on the ms. Leipzig, Universitätsbibl. 529 (#9) the date for composition of this codex is estimated for 1334: quite obviously implausible. See footnote 4.

present edition we have included mainly those copies dated for the 14th century. However, we have accepted a few exceptions to this rule. We have included two copies dated for early 15th century, namely ms. Leipzig, Universitätsbibl. 1370 (#11, G) and ms. Vatican, BAV, Vat.lat. 2136 (# 21, A). The first was taken into consideration as the best representative of the already-mentioned family that consists of this copy and those included in the mss. Cracow, Bibl. Jagiell. 704 (#5), Leipzig, Universitätsbibl. 1360 (#10), and Prague, Statni Knih. ČR 396 (#19). Even though these three latter copies are roughly dated for the 14th century, in consequence we excluded them from the edition. We included the mentioned Vatican codex firstly because this copy of the text features some affinity to the other two 15th century copies, i.e., that preserved in the San Giminiano codex (#20), and in the Vatican, BAV, Vat.lat. 2138 (#22) codex, and thus can be assumed to be the representative of this family. Secondly, in the Vat.lat. 2136 (#21, A) copy of *De motu locali* we encounter evidence of it having been corrected against the other copy – for there are few indications of alternative readings (*alia lectio*) of specific terms on the margins of this copy. Unfortunately, none of the copies we have consulted for this edition fits perfectly these proposed alternative readings, yet these may be helpful when other, incomplete copies of this text, are to be analysed.

On the other hand, we have excluded from the present edition the copy of Heytesbury's "Rules…" preserved in the ms. Leipzig, Universitätsbibl. 529 (#9). In its catalogue record composition is estimated for 1334, which appears wrong, at least with respect to the part containing the copy of William Heytesbury's treatise.[18] What is more, the text of *De motu locali* included in this manuscript features so many individual, logically and/or contextually wrong readings and omissions, that it was either copied by a very negligent scribe, or from an already corrupted source, what leads to the inevitable conclusion of it being quite a "distant" copy of the "Rules for solving sophisms".[19] For similar reasons we rejected the copy included in the ms. Munich, Bayerische Staatsbibl. Clm 23530 (#13).

18 See footnote 4.

19 For example, in this copy we come across a substantial omission in the reasoning concerning the proof of the Mean speed theorem that spans from the second sentence of the § 35 to the middle of the § 36 in the present edition.

Finally, in the present edition of the *De motu locali* section of William Heytesbury's "Rules for solving sophisms" we refer to the following handwritten copies of the text:

1. Brugge, Stedelijke Openbare Bibl. 497, f. 56ra–57rb (B);
2. Cracow, Bibl. Jagiellońska 621, f. 40rb–43ra (K);
3. Erfurt, Wissenschaftliche Allgemeinebibl., Amploniana Cms 2o 135, f. 13rb–14vb (E);
4. Erfurt, Wissenschaftliche Allgemeinebibl., Amploniana Cms 4o 270, f. 27v–30v (R);
5. Leipzig, Universitätsbibl. 1370, f. 35v–39v (G);
6. Oxford, Bodleian Lib., Canon. misc. 409, f. 14rb–16ra (C);
7. Padua, Bibl. Univ. di Padova 1123, f. 60vb–62va (U);
8. Vatican, BAV, Vat.lat. 2136, f. 24va–27va (A);
9. Venezia, Bibl. Naz. Marciana, lat. VIII. 38 (3383), f. 66va–68vb (D).[20]

Despite Spade's statement that the Erfurt, Amploniana Cms 2o 135 (E) copy is not good enough to constitute the basic text for the edition of the whole *Regulae solvendi sophismata*, we decided to take it as the basic one for the present edition.[21] After establishing the logically and contextually consistent version of the text it turned out that this (E) copy of the text features the minimal number of individual readings compared to the remaining manuscript versions, and only a handful of these affect the actual reading of the text to a degree whereby it needs to be corrected according to other copies. Some of the marginal notes we have encountered in this manuscript, ones supplementing the text itself, assure us that it had been more or less diligently corrected following transcription. Surely, then, this is no archetypical text, something also confirmed by the correct readings common for other copies not present in this very version (E). On the other hand, (E) seems to be the copy that is the "closest" to the archetypical one in terms of the number of possible intermediary copies. For us supporting this hypothesis is also the fact that according to the note found in the codex

20 Sigla are not in alphabetical order for they were assigned arbitrarily during the preparatory stages of work on the critical edition and herein retained.

21 Here it is worth mentioning that for Spade none of the handwritten copies of Heytesbury's treatise are good enough to constitute a base text for the edition of the whole *Regulae*…. See P.V. Spade, *The Manuscripts of William Heytesbury's*…, p. 275.

Erfurt, Amploniana 2o 135 (E) by Wilhelm Schum, the author of the catalogue of Amplonian manuscripts, the codex (E) was composed in 1337.[22] What is more, Schum recognized all the texts included in this codex to have been written in an English hand.[23] Consequently, we can safely assume that this very copy of William Heytesbury's *Regulae solvendi sophismata* was copied in England no later than two years after the composition of the treatise itself, what makes it the earliest preserved copy of Heytesbury's work.[24] It is worth noting here, however, that only from the *explicit* of the very same copy of Heytesbury's "Rules…" included in the Erfurt, Amploniana 2o 135 (E) codex do we learn that the treatise itself was composed in 1335, consequently we have decided to take this copy as the basic one for the critical edition of the section *De motu locali* of William Heytesbury's "Rules for solving sophisms".[25]

3. The Question *Utrum in motu locali sit certa servanda velocitas* from the Anonymous *Treatise de sex inconvenientibus*

The question *Utrum in motu locali sit certa servanda velocitas?* is the fourth, final part of the anonymous treatise *De sex inconvenientibus*, the one dealing

22 See <W. Schum>, *Beschreibendes Verzeichnis der Amplonianischen Handschriften-Sammlung zu Erfurt*, Bearbeitet und herausgegeben mit einem Vorvorte über Amplonius und die Gesichte seiner Sammlung von Wilhelm Schum, Berlin, Weidmannsche Buchhandlung 1887 (Nachdruck, Weidmannsche Verlagsbuchhandlung, Hildesheim 2010), 88. We say "found by Wilhelm Schum" because at present it is impossible to find this note within the codex, something already observed by P.V. Spade (see P.V. Spade, *The Manuscripts of William Heytesbury's…*, p. 275.)

23 See ibidem, pp. 88–89.

24 As is argued above, we refute any possibility that the copy of William Heytesbury's text included in the codex Leipzig, Universitätsbibl. 529 (#9) was transcribed in 1334.

25 Ms. Erfurt, Wissenschaftliche Allgemeinebibliothek, Amploniana Cms 2o 135, f. 17rb: "Expl<icit> quidem tractatus optimus datus Oxonie a mag<istro> Wilhelmo de Hytthisbri a. D. M^{o}CCC^{o}XXXV^{o}."

specifically with the problem of local motion.[26] The complete text is preserved in the following manuscripts:[27]

1. Oxford, Bodleian Library, Canonici Miscellaneous 177, ff. 203rb–212va (O);[28]
2. Paris, Bibliothèque Nationale de France, fonds lat. 6527, ff. 156va–169vb (R);[29]
3. Paris, Bibl. Nat., fonds lat. 6559, ff. 28rb–42va (P);[30]

26 The remaining ones are as follows: Qu. I: *Utrum in generatione formarum sit certa ponenda velocitas?*; Qu. II: *Utrum in motu alterationis velocitas sit signanda vel tarditas?*; Qu. III: *Utrum augmentum continuum in augendo velocitet motum suum?* See J. Papiernik, "Zmiany jakościowe i ich miara…", pp. 18–20.

27 There are three more preserved copies of the treatise *De sex inconvenientibus*, viz. Cracow, Biblioteka Jagiellońska, ms. 739, Prague, Národní knihovna České Republiky, VIII. G.19, and Vatican, BAV, Vat.lat 3026, all of which are, however, incomplete. The Cracow copy ends within the first *inconvenientia* of the second article of question IV (f. 8vb: (*explicit*) et signatur C punctum ab A per radium procedentem a medio puncto A corporis luminosi in continuum et directum super C punctum), while the Prague copy ends with the final paragraph of the second article of the fourth question (f. 46v: (*explicit*)...et totum pertransitum ab a ante finem horae et sic non sequitur inconveniens adductum et probatio claret. Patet quia in eodem casu ad alia sic dicendum. Expliciunt quaestiones de motu Parisius disputatae.) In the Vatican codex varied initial fragments of *De sex inconvenientibus* in different parts of the manuscript are repeatedly transcribed. The first one, (f. 17r–20v) ends with the third *inconvenientia* of the second article of question I, the second is much shorter (f. 121va–124vb) and concludes with the second *inconveniens* of the first article of question I. For descriptions of these codices see, respectively: W. Wisłocki, *Katalog rękpisów Biblijoteki Uniwersytetu Jagiellońskiego*, cz. I, Kraków 1877–1881, 220 (https://jbc.bj.uj.edu.pl/dlibra/doccontent?id=285039); J. Truhlář. *Catalogus codicum manu scriptorum latinorum, qui in c. r. bibliotheca publica atque Universitatis Pragensis asservantur*; t. I. Pragae 1905, 594 (https://archive.org/details/cataloguscodicu03truhgoog/page/n625/mode/2up.); For description of Vatican ms see Vatican: https://digi.vatlib.it/mss/detail/Vat.lat.3026.

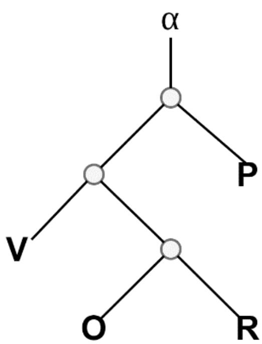
28 For detailed description of this codex see: https://medieval.bodleian.ox.ac.uk/images/ms/abz/abz0284.gif; https://medieval.bodleian.ox.ac.uk/images/ms/abz/abz0285.gif; https://medieval.bodleian.ox.ac.uk/images/ms/abz/abz0286.gif.

29 In the library's catalogue we read that the codex comprises : "1. *Alberti de Saxonia quaestiones in octo libros physicorum Aristotelis*; 2. *Tractatus de sexdecim (sic!) inconvenientibus: ibi de generatione, de motu locali, aliisque ad physicam pertinentibus*." See https://archivesetmanuscrits.bnf.fr/ark:/12148/cc656303.

30 For detailed description of this codex see, Z. Kaluza, *Nicolas d'Autrécourt*, pp. 195–198; G. Fernandez-Walker, *A New Source...*, p. 62. With respect to Richard Kilvington's works included in this codex, see E. Jung-Palczewska, *Works by Richard Kilvington*, "Archives d'Histoire Doctrinale et Littaeraire du Moyen

4. Venezia, Biblioteca Nazionale Marciana, lat. VIII. 19(3267), ff. 117v–145v (V).[31]

The first conclusion we arrived at after collating all four versions of the text is that none had served as the basis for the remaining ones. Ultimately we have selected the copy included in ms. Paris, Bibl. Nat., f. lat. 6559 (#2 on the above list = P) as the basic text for the present critical edition, since it presents the most consistent version, containing the least number of omissions and errors when compared to other copies. The Oxford (#1 = O) and Venetian (#4 = V) copies of the text feature over thirty common significant readings, (i.e., inversions, homoeoteleuta, errors, etc.) and the second Parisian copy (#2 = R) features almost thirty common instances with the Venetian copy. The Oxford and Paris (R) copies present the strongest mutual affinity, sharing more than a hundred instances. Consequently, one can safely conclude that they were transcribed from the same source. Taking into account all the specific or common significant instances the relations between all of these manuscript copies may be illustrated graphically with a *stemma codicorum* given below:[32]



Age", 67(2000), pp. 219–222; E. Jung, R. Podkoński, *Richard Kilvington on Continuity*, [in:] "Atomism in Late Medieval Philosophy and Theology," Ch. Grellard, A. Robert (eds), Leiden–Boston 2009, p. 65.

31 For detailed description of this codex, see: *Biblioteca manuscripta ad S. Marci Venetiarum, digessit et commentarium addidit Joseph Valentinelli praefactus*, codices MSS. Latini, T.VI, Venetiis MDCCCLXXI(1871), pp. 231–232. http://www.nuovabibliotecamanoscritta.it/Generale/ricerca/mostraImmagine.html?codice=60262&codiceMan=65406&codiceDigital=0&tipoRicerca=S&urlSearch=area1%3D3267.

32 In this graph "α" represents the autograph of the text while the empty circles represent its presumed, and now lost, intermediate copies. Of course it is possible that there existed other intermediate manuscript copies, but for now we have no firm grounds to substantiate such a hypothesis.

4. Selected Fragments of Part III: *De motu locali* of John Dumbelton's *Summa logicae et philosophiae naturalis*

As noted above, especially John Dumbleton's *Summa logicae et philosophiae naturalis* has been hitherto marginalised by editors and scholars.[33] One of the reasons being certainly the length of the whole work – *ca.* 400 thousand words![34] The other reason, no less decisive given the context, was that no early printed edition of Dumbleton's *Summa…* existed, since the work enjoyed much limited interest, if any, among thinkers active outside English universities in subsequent centuries, when compared, for example, with William Heytesbury's "Rules for solving sophisms" or Richard Swineshead's *Liber calculationum*.[35] This last circumstance meant that any researcher wanting to refer, read or prepare even a partial edition of Dumbleton's work was compelled to deal only with the

33 The only modern transcription of the full text of John Dumbleton's treatise, from a single manuscript copy, namely from the Vatican, Pal. lat 1056 codex, was included in J.A. Weisheipl's unpublished Ph.D. dissertation, *Early fourteenth century physics of the Merton 'school': with special reference to Dumbleton and Heytesbury*. Edith D. Sylla also provided short, selected passages from parts II to V of Dumbleton's *Summa…* in her Ph.D. dissertation, subsequently published (see E.D. Sylla, *The Oxford Calculators and the Mathematics of Motion 1320—1350…*, pp. 565–625). Now a project to prepare a critical edition of Parts I and II of John Dumbleton's *Summa logicae et philosophiae naturalis* is underway at St. Andrews University, led by Barbara Bartocci. For details, see: URL = <https://www.st-andrews.ac.uk/~slr/paradox.html.>.

34 Richard Swineshead's monumental *Liber calculationum* runs to *ca.* 200 thousand words.

35 The ratio of manuscript copies of Dumbleton's *Summa…* preserved in English libraries to those kept abroad is the reverse to respective numbers of, for example, Heytesbury's *Rules…* or Swineshead's *Liber calculationum*, with these being mostly preserved in libraries on the Continent. See below, *Introductory notes on the section "On local motion" (De motu locali) of William Heytesbury's "Rules for solving sophisms" (Regulae solvendi sophismata)*; R. Podkoński, *Richard Swineshead's "Liber calculationum" in Italy. Some Remarks…*, pp. 313–337. There may be many plausible explanations for this, the most obvious possibly being that when a library received the printed copy a given treatise, librarians simply binned any manuscript copies of the same text thereby making room for new books.

manuscript copy or copies of the text, what makes any such study considerably more difficult.

Compounding the difficulty is the existence of twenty-one known preserved codices with John Dumbleton's *Summa logicae et philosophiae naturalis* from which to choose:

1. Cambridge, Gonville & Caius, 499/268;
2. Cambridge, Peterhouse, 272;
3. Dubrovnik–Ragusa, Dominikanerbibliothek 32;
4. Klosterneuberg, SB 670;
5. London, B.L. Royal 10. B. XIV;
6. London, Lambeth Palace 79;
7. Oxford, Magdalen 32;
8. Oxford, Magdalen 195;
9. Oxford, Merton 279;
10. Oxford, Merton 306;
11. Padua, Bibl. Anton. XVII, 375;
12. Paris, Bibl. Nat. fonds lat. 16146;
13. Paris, Bibl. Nat. fonds lat. 16621;
14. Paris, Bibl. Universitaire 599;
15. Prague, Capit. Metropol. 1291 (L. XLVII);
16. Vatican, BAV, Pal. lat. 1056;
17. Vatican, BAV, Vat. lat. 954;
18. Vatican, BAV, Vat. lat. 6750;
19. Venezia, Bibl. Naz. Marcian VI. 79(2552);
20. Worcester, Bibl. Cathed., F. 6;
21. Worcester, Bibl. Cathed., F. 23.

All the above-listed manuscripts are dated roughly for the late fourteenth or early fifteenth century. None of the preserved copies contains the tenth part of this work, explicitly proclaimed by the author himself, and which presumably was never written. In some of the above-listed copies other sections of the *Summa*…are also missing.[36]

In preparing the present monograph we focussed only on Part III of John Dumbleton's work where the issue of different kinds of motion is discussed, and generally reflecting the scope of interest of the last parts of William Heytesbury's *Regulae solvendi sophismata* traditionally taken together as the section "On three predicaments" (*De tribus*

36 For details see: URL = <https://www.st-andrews.ac.uk/~slr/paradox.html.>.

predicamentis).[37] We read and collated the handwritten versions of this part of *Summa*… from each of the above-mentioned codices, save the one preserved in Dubrovnik as it proved impossible to gain access. Nevertheless, the initial conclusion resulting from this comparison was dissapointing. Each of the copies features unique substantial omissions and specific readings, what means that none could be recognized as a base text for any other of the preserved copies. Consequently, constructing the *stemma codicorum* on the basis of common instances (i.e., common *homoeoteleuta*, inversions, etc.) proved impossible at this stage of research. It is probable, of course, that comparing extensive passages from other parts of *Summa*... will be more instructive in this respect, but for the purpose of the present book we have ultimately decided to adopt another solution.

As we have just noted, the contents of Part III of John Dumbleton's treatise include not only the account on local motion and its mathematically formulated "rules," but also deals with the description of the motions of alteration and augmentation. Since in this book we focus on the Oxford Calculators' science of local motion, we have decided to select and present only those fragments of this part of the *Summa*… that concern this kind of motion, having in mind that a transcription of the whole of Part III would simply exceed the limitations of the present volume. Consequently, we have provided a fragment of the initial chapter of Part III of John Dumbleton's treatise and then chapters 5 to 12 in full length, all dealing specifically with *calculationes* in the context of local motion.[38] From amongst all the handwritten copies we have transcribed the one which – in our opinion – is most consistent in terms of the course of the logical and mathematical argumentations included, as well as with respect to the notation of the general terms used in specific reasonings and their compatibility with the line drawings included on the margins of this copy which were intended to illustrate these reasonings. The copy we have chosen is the one included in the codex Oxford,

37 See Chapter I above.

38 It is worth noting here that the division into chapters, most probably, was not introduced by the author himself, as some appear to be arbitrary, i.e., they do not reflect the logical division of the specific lines of reasoning (see Johannes Dumbleton, *De motu locali*, e.g., §§ 16–17; pp. 53–54). Nevertheless, this division must have been introduced quite early on, for in almost every handwritten copy of the text we encounter the same division into chapters.

Magdalen 32. The salient feature of this copy are numerous *marginalia* besides the already mentioned line drawings. These *marginalia*, however, are not the reader's notes or comments, but fragments of the actual reasonings, ones that in other copies of the same text are incorporated into the main text, though not always and neither in all of them. It seems, therefore, that the copy included in the Oxford, Magdalen 32 codex was transcribed first by an inattentive scribe and next corrected by much more diligent one against the source-copy of the text. The close scrutiny of this copy reveals that the author of the *marginalia* took special attention to substitute the missing passages. The consistency and completeness of this version might suggest that this copy was transcribed from a source that was either author's autograph or a copy that not very "distant" from an autograph in the terms of number of intermediate copies.[39]

5. Presentations of the Texts – editorial rules, the contents of *Apparati critici*, and *Abbreviations* used

All the Latin texts included in the present volume are classicized, i.e., the diphthongs "*ae*", "*oe*" are introduced. Consequently, "*sicut*" is given instead of "*sicud*", and "*sed*" instead of "*set*". The typically later medieval "*pertransietur*" (etc.) with the classical "*pertransiretur*" are substituted. The modern use of capital letters is adopted and the standard abbreviations for "*Socrates*" and "*Plato*" are expanded. For the sake of clarity, we have differentiated between "v," as a consonant, and "u," as a vowel. The division into sentences, paragraphs and parts, as well as the punctuation itself are all ours. Each of the texts was amended, where the actual reading had no sense in the context, in most cases due to the most obviously

39 According to Ralph Hanna, the author of the draft catalogue of the manuscripts preserved in the library of Magdalen College, Oxford, frequent marginal corrections characteristic for this codex show that the scribes were checking each other's work against their prototype. We would like to thank Dr David Rundle, Centre for Medieval and Early Modern Studies, Kent University, who is currently preparing a revised and updated version of the said catalogue, for kindly sharing with us the above information on this codex.

flawed interpretation of the abbreviation by the scribe. Also, for the sake of clarity some missing words and passages are introduced in "sharp" brackets: "<…>". Where there are more than one preserved copies of a specific text these substituted fragments and words are based on a most consistent version to be found in these. Also, in a few instances we have excluded the words we found superfluous with "square" brackets: "[…]".

In what follows the contents of the *apparati critici* are explained in detail.

5.1. Richard Kilvington, *Utrum in omni motu potentia motoris excedit potentiam rei motae*

Since there exists only a single copy of the text of this very question, it is transcribed below, according to the general rules presented above. The division of the text into the sections according to the notes that are to be found on the margins of the manuscript copy is supplemented, where necessary, and "sharp" brackets are employed in these cases. In a few places the numbers for successively listed opinions have been introduced too.

The transcription is supplemented with three levels of *apparatus criticus*. In the first level all the corrected, mistakenly expanded or interpreted words and formulas are indicated. In the second level the exact addresses to the works of Aristotle's, Averroes's, Euclid's implicitly referred to in the text are provided. Also, in this level the reader finds the addresses and fragments of those works of contemporary and later thinkers that clearly were based on Kilvington's text, or that have been transcribed from it *in extenso*, usually with no indication of the source; namely, to the works of Thomas Bradwardine, John Buridan, Julius Scaliger and others. A few fragments of the earlier works by Richard Kilvington, i.e., his *Sophismata* and *Commentary on De generatione et corruptione*, where he discussed the same issues are also transcribed here. In the third level the division of the text into sections as provided in the margins of this copy is indicated.

5.2. William Heytesbury, *De motu locali*

For the reasons presented above we have decided to take copy Erfurt, Amploniana 2o 135 (E) as the basic one for the present critical edition of "On local motion" (*De motu locali*) section of William Heytesbury's "Rules for solving sophisms". Only in those instances where the actual reading is obviously wrong have we corrected the text, relying on readings from the remaining copies, indicating the variant reading in the *apparatus criticus*. Here it is worth noting that there are fewer than ten such instances in the whole text and in most cases we are dealing with simply the wrong grammatical form of the same word, such as "*equaliter*" for "*equali*", or an obviously erroneous interpretation of an abbreviation, like "*proximos*" instead of "*primos*".[40] Similarly, there are almost the same number of omitted words, either accidentally or purposedly – in one case the proper word was transcribed but (later?) deleted (expuncted).[41]

The first level of the *apparatus criticus* presents all the variant readings from the remaining handwritten copies of the text included in this edition. For the sake of brevity we have omitted only those deemed irrelevant, such as: *ergo/igitur*, *ille/ipse/iste et al.*, *sive/seu/vel*. In the second level of the *apparatus criticus* the few marginal notes and comments that are to be found in the manuscript copies are given. These are mostly traces of attempts to indicate the succesive sections of the text. In the third level of the apparatus two fragments of Aristotle's "Physics" implicitly referred to in the opening paragraphs of *De motu locali* are provided *in extenso*. These show clearly how deeply embedded Heytesbury's work was in the tradition of Aristotelian natural philosophy.

40 See Guilelmus Heytesbury, *Regulae solvendi sophismata: De motu locali*, § 4, p. 270; § 31, pp. 279–280.

41 See ibidem, § 18, pp. 273–274. By accidental omissions we mean those later substituted on the margins of this copy (see e.g., § 12, p. 272; § 28, pp. 277–278; § 31, pp. 279–280).

5.3. Anonymous, *Utrum in motu locali sit certa servanda velocitas*

The first level of the *apparatus criticus* presents all the variant readings from the remaining handwritten copies of the text included in this edition. For the sake of brevity we have omitted only the irrelevant ones, such as: *ergo/igitur*, *ille/ipse/iste et al.*, *sive/seu/vel,* as well as simple inversions: *sua loca/loca sua*, and different verbal forms: *intendetur/intenderetur/intenditur*. In the second level of the *apparatus criticus* bibliographical references for the sources cited in the text are provided.

5.4. John Dumbleton, *De motu locali*

Since, as it is explained in detail above, we decided to transcribe the text from the manuscript copy Oxford, Magdalen 32, correcting and supplementing the text against the Cambridge, Gonville & Caius 248/499 version only where absolutely necessary, in the first level of the *apparatus criticus* we have indicated all the passages or single words that appear as the *marginalia* in the basic copy of the text as well as the ones we decided to correct. In the second level we have provided the exact addresses of the earlier discussed or proven theses the author refers to in the course of his reasonings. In the third level of the *apparatus* we have provided *in extenso* the fragments of Aristotle's, Averroes's, Avicenna's and Euclid's texts implicitly referred to in the text.

Abbreviations

< >	*includunt verba quae addenda sunt*
[ ]	*includunt verba quae delenda sunt*
?	*lectio incerta*
a. m.	*alia manu*
add.	*addidit/addiderunt*
corr.	*correctum/correxit*
del.	*delevit*
exp.	*expunxit(delevit)*
hom.	*homoeoteleuton*
inv.	*invertit/inverterunt*
iter.	*iteravit*
lin.	*scriptum supra lineam*
marg.	*in margine*
ms.	*imanuscripto*
om.	*omisit/omiserunt*

Ricardus Kilvington

Utrum in omni motu potentia motoris excedit potentiam rei motae

Quaestiones super libros Physicorum

Ms. Venezia, Biblioteca Nazionale Marciana, lat VI, 72 (2810), ff. 81ra–89rb

RICARDUS KILVINGTON

<QUAESTIONES SUPER LIBROS PHYSICORUM>

UTRUM IN OMNI MOTU POTENTIA MOTORIS EXCEDIT POTENTIAM REI MOTAE

<ARTICULUS PRIMUS DE EXCESSU SUFFICIENTE AD MOTUM>

1 Et probo quod non, quia tunc vel esset dare: <1.> minimum excessum sufficientem ad motum, ut ponunt quidam; <2.> vel maximum excessum non sufficientem ad motum, ut ponunt alii; <3.> vel quiscumque excessus sufficeret ad continuandum motum non tamen ad incohandum, ut ponunt tertii; <4.> vel quiscumque excessus sufficeret ad incohandum motum sicut ad continuandum, ut ponunt quarti.

2 Non potest poni altera duarum opinionum primarum. Quod probo sic, quia tunc tota terra, si foret extra suum locum naturalem non moveretur quousque medium eius fieret medium mundi, cuius oppositum vult Aristoteles II *De coelo* textu commenti 107 et Commentator in eodem commento. Consequentiam probo quia aliter maior pars terrae ex una parte centri non haberet excessum nec quiscumque excessus maioris partis terrae supra partem minoris sufficeret ad continuandum talem motum.

3 Item, aliter: non omne maximum mixtum movetur ad motum dominantis elementi, quia aliquando dominatur unum elementum super aliud per excessum sufficientem ad motum, quod est contra Aristotelem I *De coelo* textu commenti 7 et contra Commentatorem eodem commento.

4 Item, aliter: elementum motum de potentia elemetali non haberet tantum naturaliter de loco quantum acquireret de forma, quod est contra Aristotelem et Commentatorem VIII *Physicorum* textu commenti 4, et textu commenti 32, et illis commentis; II *De coelo*

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11 Aristoteles…12 commento] Cf. Arist., *De coelo*, II, 297b10—12; Aver., *In De celo et mundo*, II, com. 107, 473, 57—65. | 19 Aristotelem…20 commento] Cf. Arist., *De coelo*, I, 268b31—269a2; Aver., *In De celo*, I, com. 7, 17, 26—28. | 23 Aristotelem…24 4] Cf. Arist., *Phys.*, VIII, 252a15—17, Aver., *In Phys.*, VIII, com. 4, 341ra. | 24 textu…commentis] Cf. Aver., *In Phys.*, VIII, com. 32, 370ra. | II…4,1 28] *Locus non inventus*.

textu commenti 28 et IV *De coelo* textu commenti 22, et illis commentis. Et consequentiam probo, quia pono quod ignis incipiat agere in terram puram, cuius centrum est centrum mundi. Tunc quacumque levitate inducta in illa terra erit terra mixta, ergo appetit locum superiorem quam prius quando fuit pura, quia illud quod minus grave est, intensive appetit locum superiorem. Igitur si non impediatur ista terra, sequitur quod ascendet quacumque levitate inducta, quam terram aer acquiret et illa <acquiret> aliquid de forma <ignis> nihil de loco superiori, quod est contra Aristotelem et Commentatorem. Et sic ergo quilibet excessus sufficit ad motum, quia si illa terra ascendet propter inductionem cuiuslibet levitatis, ergo ascenderet antequam levitas dominatur supra gravitatem. Ergo a multo fortiori quilibet excessus levitatis supra gravitatem causabit motum. Ideo dicitur a quosdam, quod terra mota de potentia essentiali non acquireret tantum de loco quantum de forma propter argumenta subsequentia, quia ex isto sequitur oppositum conclusionis, scilicet quod esset motus | talis terrae versus sursum 81rb
naturaliter antequam levitas excedat gravitatem.

5 Item, si ista terra acquireret de loco sicut de forma, tunc sequitur quod aliquis ignis aequalis virtutis cum primo igne applicatus alteri medietati terrae illius faceret illam medietatem aeque cito acquirere concavum orbis lunae sicut alius ignis faceret aliam medietatem, si uterque ignis aequaliter generans tribuat suo generato de forma ignis et de loco.

6 Primo probo quod hoc sit falsum, quia pono quod altera illarum medietatum ascendat continue per partem aeris subtiliorem, tunc sequitur quod altera habebit formam in summo antequam locum in summo.

7 Item, si prima terra tantum acquireret de forma sicut de loco, tunc cum habebit medietatem formae et fuerit aequaliter gravis et levis, locatur naturaliter supra convexitatem aeris et supra magnam partem ignis ad punctum medium inter concavum orbis lunae et centrum terrae quod est supra valde magnam partem ignis, ut sequitur ex

1 IV…2 commentis] Cf. Arist., *De coelo*, IV, 310b1; Aver., *In De celo*, IV, com. 22, 694, 99—101: Illud quod movetur ad suum locum non movetur nisi secundum quod aliquod movetur ad suam formam: quod enim movetur ad formam est illud quod est in potentia formae.

14 Ideo] Responsio secundum quosdam *marg.*

dictis II *De generatione* ubi supponit <Aristoteles> quodlibet elementum superius esse decuplum ad elementum inferius sibi immediatum. Signetur ergo terra per unum, aqua per 10, aer <per> 100, et ignis per 1000. Tunc sequitur quod ex dicto Philosophi quod ignis in sua sphaera excedit alia omnia tria elementa residua in maiori proportione quam in millecupla qualis est proportio 9 ad 1; ergo punctus medius inter centrum et concavum orbis lunae est supra valde magnam partem ignis. Ergo si ad illum punctum locabitur generatum quando erit aequaliter grave et leve, quando erit magis grave quam leve locabitur naturaliter in sphaera ignis supra magnam partem ignis. Consequens est falsum, quia omne mixtum movetur naturaliter ad locum elementi dominantis et in loco illo naturali locabitur, ut patet I *De coelo* commento 7.

8 Pro istis posset responderi, quod Philosophus intelligit quod generatum tantum acquirit de loco quantum de forma si non impediatur a medio; immo si moveretur in vacuo ita esset.

9 Contra istud arguo sic: pono quod esset tale mixtum ex igne et terra ubi ignis sufficienter dominaretur ad corrumpendam terram sibi admixtam in die naturali, et quod illud mixtum contingat centrum mundi. Suppono vacuum infinitum super illud mixtum inter ipsum et suum locum naturalem. Tunc corrupta medietate gravitatis terrae in fine primae medietatis <diei> praecise aliquid pertransitum de isto vacuo per istud mixtum erit sicut dupla proportio levitatis ad gravitatem ad istam proportionem quae fuit in principio, ergo in fine secundae partis proportionalis diei erit iterum tantum de isto vacuo pertransitum. Consequentiam probo, quia si mansisset eadem proportio praecise per totam secundam partem diei quae fuit in principio secundae partis proportionalis, tunc fuit tantum pertransitum in secunda parte proportionali quantum

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1 II…3 immediatum] Cf. Arist., *De gen. et corr.*, II, 333a32; Idem, *Meteorologica*, 339a—341b. | 3 Signetur…4 1000] Cf. Ricardus Kilvington, *Comm. in De gen. et corr.*, ms. Brugge, Stedelijke Openbare Bibl. 503, 37rb: totus ignis est decuplus ad totum aerem; 37va: una pars aeris est aequalis potentiae cum igne decuplo ad ipsum, quia ex illo aere potest generari ignis decuplus ad ipsum; f. 38va: ex una parte terrae potest generari ignis decuplus ad ignem qui potest generari ex tanta parte aquae. | 13 I…7] Cf. Aver., *In De celo*, I, com. 7, 16, 21—24. | 14 Philosophus…16 esset] Cf. Arist., *De gen. et corr.*, II, 333a32; Idem, *Meteorologica*, 339a—341b.

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17 Contra] Argumentum contra responsionem *marg.*

pertransivit in prima parte proportionali. Sed iam maioratur ista proportio in secunda parte porportionali diei, igitur pertransibit iterum tantum vel plus quam pertransiebatur in prima | parte 81va proportionali diei, quoniam in fine partis tertiae proportionalis potentia motiva illius mixti in duplo plus excedit suam resistentiam, quam excessit in fine secundae partis proportionalis; ergo motus erit in duplo velocior in tertia parte proportionali quam in secunda. Et per consequens pertransibitur aequale spatium in subduplo tempore, scilicet in tertia parte proportionali quae subdupla est respectu secundae partis proportionalis, sicut pertransibitur in tota secunda parte proportionali. Consequentia patet ex definitione velocioris motus et tardioris VI *Physicorum* commento 41. Ergo in infinitis partibus proportionalibus diei infinita spatia aequalia, quorum nulla pars unius est pars alterius, sint pertransita, si semper maneat proportio potentiae motivae ad suam resistentiam dupla praecise per totam partem proportionalem posterioris ad proportionem quae fuerit in parte proportionali immediate praecedenti. Vel si proportio dupletur et plus quam dupletur in parte posteriori proportionali respectu partis proportionalis prioris, sequitur <quod>, in quacumque parte proportionali posteriori prima parte proportionali erit, plus quam iterum tantum pertransit, quantum pertransiebatur in prima parte proportionali. Ergo in fine diei erunt infinita aequalia pertransita quorum nulla pars unius est pars alterius, vel infinita inaequalia quorum nulla pars unius est pars alterius; et quodlibet pertransitum in posteriori parte proportionali erit magis quam fuerit pertransitum in prima parte proportionali. Et tunc in fine diei erit infinitum spatium vacuum pertransitum per illud mixtum motum solo modo in tempore finito, quod est falsum, quia in fine diei erit illud mixtum alicubi, ubi terminabitur illud vacuum pertransitum in isto extremo, et illud vacuum terminatum est ad aliud extremum ubi incepit mixtum moveri; ergo totum vacuum pertransitum terminatum est ex utroque extremo. Ergo illa propositio est impossibilis apud imaginantem: ‘vacuum infinitum pertransitum est tempore finito’, quia ex illa sequitur quod <vacuum> sit finitum, ut

13 nulla] *corr. ad sensum, ms.* minima 23 pertransita] *corr. ad sensum, ms.* pertransitum 32 propositio] *corr. ad sensum, ms.* proportio

12 VI…41] Cf. Aver., *In Phys.*, VI, com. 11, 253ra.

est prius dictum. Et tamen totus casus est possibilis apud imaginationem. Ergo ex possibili secundum hanc imaginationem sequitur impossibile etc. Quod videtur esse contra Commentatorem IV *Physicorum* capitulo de vacuo commento 72.

10 Hic potest adduci argumentum de mixto habente aequaliter de gravitate et levitate quale oportet ponere propter omnem deductionem de transmutato de gravitate summa ad levitatem summam etc. In medio transmutationis erit aequaliter grave et leve et stabit ubicumque ponetur vel stabit in loco medio inter centrum terrae et concavum orbis lunae, et movebitur de ceteris locis, quod habetur pro inconvenienti I *De coelo* commento 7, quod tale mixtum non recedet a centro terrae, si ibi ponitur. Primo, quia non a forma terrae quia ista est in loco suo naturali nec a forma ignis quia ignis aequali virtute resistit. Nec potest dici quod utraque forma appetat
81vb locum medium secundum quod sunt sic mixtae, quia tunc | mixtum non esset de se, quod est contra Commentatorem X *Metaphysicae* commento 23. Nam per Commentatorem II *De coelo* commento 37 hoc est causa corruptionis mixti, formae in quo appetunt loca diversa, et aliter non moveretur mixtum in vacuo quia non haberet resistentiam.

11 Praeterea mixtum praedictum tantum acquirens de loco quantum de forma semper est in loco suo naturali proportionato illi mixto sub illo gradu, ergo in non instanti appetit locum superiorem per formam acquisitam, nec per formam acquirendam appetit alium locum, quia ista nondum est; ergo in nullo instanti appetit motum ad locum superiorem, immo, ut videtur, in quolibet instanti temporis praedicti appetit quietem, ita quod non intendunt Philosophus et Commentator deducere impedimentum medii ad hoc quod motum de potentia essentiali acquirat tantum de loco quantum de forma <sicut est> de oleo inflammato, quia sicut partes inflammantur ita proportionaliter ascendunt. Et istud experimentum manifeste patet in aere etiam in medio resistente, ergo etc.

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1 dictum] *corr. ad sensum, ms.* debetur

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3 Commentatorem…4 72] Cf. Aver., *In Phys.*, IV, com. 72, 163rb. | 11 I…7] Cf. Aver., *In De celo*, I, com. 7, 17, 28—31. | 16 Commentatorem…17 23] Cf. Aver., *In Metaph.*, X, com. 23, 274vb—275ra. | 17 Commentatorem…37] Cf. Aver., *In De celo*, II, com. 37, 339, 47—53. | 27 Philosophus…31 ascendunt] Cf. Aver., *In De celo*, IV, com. 22, 694—695, 114—121.

12 Iam redeo ad illud unde sermo venit primus et probo, quod quiscumque excessus sufficiat ad motum, quia descendit lapis continue in medio densiori et grossiori versus inferius quousque lapis ille quiescat in medio propter densitatem illius medii. Tunc in illo casu non est dare minimum excessum sufficientem ad motum, quia in quolibet instanti provenit iste motus ex aliquo excessu et post illud instans provenit motus ex minori excessu. Et cum non sit dare ultimum intrinsecum motus et medium illius sit continue magis spissum et magis resistens, nec potest dici quod in aliquo casu aliquis sit maximus excessus non sufficiens ad motum, quia sit excessus iste A, adhuc minor excessus A sufficit ad motum, ergo A excessus sufficit ad motum. Argumentum probo, quia in circumvolutione sphaerae vel rotae provenit motus alicuius partis propinquioris centro ex minori excessu quam sit A excessus, quia quanto partes sphaerae vel rotae centro fuerint propinquiores tanto tardius moventur, ut patet VI *Physicorum* commento 41. Capiatur ergo aliqua pars talis in rota cuius motus provenit ex minore excessu quam sit A, ad quam figatur aliqua corda per clavum, ad cuius cordae extremitatem sit aliquis lapis colligatus. Tunc sic: volvatur rota ut prius et ita tarde movebitur corda fixa ad praedictam rotam sicut et ista pars et ita tarde ergo movebitur lapis colligatus ad extremum cordae sicut corda et pars rotae praedicta. Ergo si motus partis praedictae proveniat ex minori excessu quam sit A excessus, sequitur quod motus lapidis colligati, qui est motus per se et in actu, provenit ex minori excessu | 82ra qui sit <A>, ergo A sufficit ad motum, quia quiscumque excessus in motu rotae sufficit ad motum, quia aliqua pars rotae movetur in duplo tardius quam suprema superficies et aliqua pars in quadruplo tardius et sic in infinitum, ut apparet capitulo de vacuo commento 71; ergo quiscumque excessus sufficit ad motum.

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16 VI…41] Cf. Aver., *In Phys.*, com. 85, 301ra. | 26 quiscumque…28 infinitum] Cf. Thomas Bradwardinus, *Tractatus proportionum seu de proportionibus velocitatum in motibus* (in: *Thomas of Bradwardine, his Tractatus de Proportionibus; its significance for the development of mathematical physics,* H. Lamar Crosby Jr., ed., University of Wisconsin Press, Madison(WI) 1955, 98, 265—285. | 29 capitulo…71] Cf. Aver., *In Phys.*, com. 72, 163va.

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1 Iam] Secundum argumentum principale quaestionis *marg.*

13 Pro istis argumentis consimilibus dicitur 'quod quilibet excessus sufficit ad motum continuandum sed non ad incohandum', et haec est tertia opinio de numero praedictarum. Contra quam arguo sic: si non quilibet excessus sufficit ad incohandum motum vel – ut prius – erit dare minimum excessum sufficientem ad motus incohationem vel maximum non sufficientem. Non primo modo, quia sit iste A, tunc sequitur quod motus proveniens ex A excessu sit tardius qui potest incohare, quia ex minori minimo excessu posset motus incohare. Si iste sit minimus excessus qui sufficit ad incohationem motus, non est dare maximum excessum non sufficientem ad incohationem motum, quia aliquando debilius incohat actionem in fortius, sicut posito quod ignis fortior et aqua debilior approximentur adinvicem ad agendum. Tunc est dare instans in quo aqua non agit in ignem et sine medio post aget in eundem ignem, ergo tunc aqua, quae est debilior, incipit agere, et per consequens incipit actionem in ignem fortiorem se, quem tamen ignem non excedit ista aqua. Ergo, a multo fortiori, quiscumque excessus sufficit ad actionem.

14 Item, elevato altero bracchio aequilibrae descendet alterum, bracchium elevatum elevando reliquum bracchium ad aequalitatem. Et etiam in aequilibra aequalibus appensis ponderibus <bracchium> ex parte graviori descendere cogetur, quantumcumque modicum sit unum ponderosius alio, ut patet per secundam conclusionem *De ponderibus*. Ergo quiscumque excessus sufficit ad incohandum motum.

15 Praeterea ad eandem conclusionem: si agens A posset continuare actionem in aliquo passo, in quo non posset incohare actionem, sit B passum, et tunc arguo sic: A in principio motus habuit eandem proportionem ad B sicut in medio motus, quia nec A vigoratur nec B minoratur, ut suppono. Ergo si A potest continuare actionem in B, potest consimilem actionem incipere. Consequentia patet capitulo

27 B] *corr. ad sensum, ms.* D

17 quiscumque…18 actionem] Cf. Bradwardinus, *op. cit.*, 122, 279—292. | 23 secundam…24 ponderibus] Cf. Jordanus Nemorarius, *De ratione ponderis*, (in: *Liber Jordani de Nemore 'De ratione ponderis'*, E. Moody, ed., M. Clagett, E. Moody, *The Medieval Science of Weights*, Madison 1952, R1.02, 176). | 31 capitulo…10,1 71] Cf. Aver., *In Phys.*, IV, com. 68, 156vb.

1 Pro] Tertium argumentum principale *marg.*

de vacuo commento 71. Istud confirmo sic: si A potest continuare actionem in B, continuabit consimilem actionem aliqua velocitate, <sed incohare> certam actionem in eodem passo aequali velocitate est difficilius sicut volunt omnes sustinentes hanc opinionem. Cum ergo sit facilius in duplo incohare actionem in duplo tardiorem in B, et eadem ratione in quadruplo <sit> facilius incohare actionem in quadruplo tardius in B, et sic in infinitum; ergo si A potest continuare actionem in B sic aliqua certa velocitate, potest etiam incohare actionem in B aequalis virtutis, sicut velocius | continuat 82rb
actionem in B. Ergo si potest continuare actionem in B, potest etiam incohare licet tardius.

16 Propter ista argumenta et consimilia dicunt quidam quartam opinionem, scilicet quod quiscumque excessus sufficit ad motum tam incohandum quam continuandum.

17 Contra quam opinionem arguo sic: ex ista opinione sequitur, quod quaecumque virtus pulsiva quantumcumque debilis potest

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3 certam actionem] *corr. ad sensum, ms.* certa actione 9 continuat] *corr. ad sensum, ms.* continuatur

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1 si…11 tardius] Cf. Ricardus Kilvington, *Sophismata*, Sophisma 33[34], in: *The Sophismata of Richard Kilvington*, B.E. Kretzmann, N. Kretzmann, eds., Oxford 1991, 86.; <Pseudo-Duns Scotus>, *Ioannis Duns Scoti in octo libros Physicorum quaestiones et expositio*, Venetiis 1617, lib. I, q. 14, 91b-92a: Secunda suppositio, quod omnis excessus est divisibilis. Tertia quod quilibet excessus sufficit ad motum. probatur: quia actio, vel motus provenit secundum proportionem potentiae motoris ad resistentiam moti, et ideo, quantum agens plus potest in agendo quam passum sit in resistendo, sequitur, quod ab agente potest produci motus, vel actio. Ad istud dicunt aliqui concedendo, quod quilibet excessus sufficit ad continuandum motum; sed non quilibet sufficit ad incohandum. Sed contra, supposito, quod non sit in infinitum difficilius incohare motum, quam continuare. Tunc probo; quod si quilibet sufficit ad continuandum, quod etiam quilibet sufficit ad incohandum, et sit A unus excessus, qui non sufficit incohare, sed sufficit continuare, et sit in centuplum difficilius incohare motum, quam continuare, tunc capiatur excessus minor A, scilicet B, ad quem se habet in centupla proportione. Tunc si B sufficiat continuare motum, quia quilibet excessus sufficit continuare, igitur A sufficit incohare motum; cuius tamen oppositum ponebatur. Et probatur consequentia: quia praecise in centuplo difficilius est incohare motum; quam continuare, et A se habet in centupla proportione ad B, igitur cum B sufficit continuare, sequitur quod A sufficit incohare, et ita sicut arguitur de A potest argui de quolibet excessu. | 4 Cum…7 infinitum] Cf. Bradwardinus, *op. cit.*, 98, 254—259.

movere totam terram a suo loco, quia si apponatur maior pars ex una parte centri quam ex alia, tunc maior pars terrae expellit minorem quousque centrum totius terrae fiat medium mundi, ut patet II *De coelo* in textu commenti 107 et isto commento. Sed quaelibet virtus pulsiva quantumcumque debilis est maioris virtutis vel aequalis ad pellendum sicut parva pars terrae addita. Ergo etiam nec potest dici, quod propter additionem parvae partis terrae ex una parte centri magis quam ex alia non movetur terra, sed propter additionem magnae movetur tota terra, quia tunc numquam moveretur terra quousque medium eius fieret medium mundi, quia aliquando prius haberet eandem proportionem maior pars ad resistentiam partis <minoris> quam proportionem habuit compositum ex medietate terrae et modica terra addita ad partem residuam terrae, ergo non moveretur tota terra, si esset extra suum locum naturalem quousque eius medium fieret medium mundi, nisi quiscumque excessus sufficeret ad motum; etiam et ubi est. Etiam quodcumque grave habens potentiam pulsivam posset pellere terram extra suum locum naturalem, cum iuvamento eius medietatis terrae quae est ex ista parte centri ex qua parte est illud pellens, posito quod una medietas terrae sit ex una parte centri et alia medietas ex alia.

18 Istud confirmo sic: terra se habet in medio sicut pondus aequale in aequilibra, quia ex omni parte centri est pondus aequale, ut vult Philosophus supposita sphericitate terrae. Ergo ex quacumque parte terrae fiat addicio ibi fiet motus, sicut est de ponderibus inaequalibus, ut patet per secundam conclusionem *De ponderibus*.

19 Probatur contra: ergo sequitur quod musca posset movere totam terram et quaecumque maior gravitas ex una parte centri quam ex alia faceret terram motam, sicut videtur esse concedendum iuxta sententiam Philosophi ubi prius.

20 Contra, tunc sequitur quod sol continue magis calefaciat aliquam partem terrae, magis quam aliam, et per consequens magis desiccat et levificat unam quam aliam, sequitur, ut videtur, quod terra plus levefacit ex una parte quam ex alia; et si sic, ergo continue pars

22 aequale] altius *add. sed. exp.*

4 II…commento] Cf. Arist., *De coelo*, II, 297b12; Aver., *In De celo*, II, com. 107, 493, 66—67. | 26 secundam…ponderibus] Archimedes, *On the Equilibrium of Planes*, 31.

gravior continue propellit ergo levior<em> quousque tota terra sit spherica, et aequalis virtutis gravitatis ex una parte centri sicut | ex 82va alia. Ergo tota terra esset in continuo motu et non naturaliter quiesceret in suo loco, quod est contra Philosophum II *De coelo,* commento 27 et est contra Philosophum *De motu animalium*, ubi dicit, quod omne motum in suo motu necessario indiget aliquo fixo, ideo centrum indiget, terra quiescente, ut super eam sustentetur. Et Commentator II *De coelo* commento 66 dicit quod tria elementa acquirunt suas perfectiones per motum circularem et terra per quietem.

21 Probo tamen quod non et arguo sic: utraque eius medietas quiescit violente, quia ablata una eius medietate alia descendit naturaliter quousque medium eius fieret medium mundi, ergo nunc utraque violentat aliam. Nec potest dici, quod partes, quando inexistunt toti, maxime appetunt salvare suum totum, quia II *De anima* dicit Aristoteles in textu commenti 30, quod unumquodque aeternum appetit salvare se ipsum, sicut experimento notum est, ut si teneatur magna globa certa, per extremum frangitur et recedit pars a parte. Idem patet de gutta aquae pendente super digito, quod una pars guttae recedit ab alia, ergo quaelibet pars habet inclinationem propriam et non quaelibet nititur continuari cum toto. Et per consequens utraque medietas terrae appetit suum medium proprium esse medium mundi, ergo cum non sit in utraque medietate quiescit violente. Et cum omne totum sit suae partes, sicut patet I *Physicorum* commento 7, et II *De anima* commento 10, et IV *De coelo* commento 43, ergo tota terra in loco suo maxime naturali quiescit violente, quod est contra Philosophum locis praeelectis.

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3 tota…4 loco] Cf. Johannes Buridanus, *Quaestiones in Aristotelis 'De coelo'*, qu. 22: *Utrum terra semper quiescat in medio mundi*, 507, 17—30. | 4 Philosophum…5 27] Cf. Arist., *De coelo*, II, 296b26—27. | 5 Philosophum…animalium] Cf. Arist., *De motibus animalium*, 698a14—16, J. Hamesse, vol. I, 61. | 8 Commentator…10 quietem] Cf. Aver., *In De celo*, II, com. 66, 402—403, 28—42. | 15 II…17 ipsum] Cf. Arist., *De anima*, II, 413a1—3. | 24 I…25 7] *Locus non inventus.* | 25 II…10] *Locus non inventus.* | IV…26 43] *Locus non inventus.*

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11 Probo] Argumentum quod terra non quiescit naturaliter *marg.*

22 Similiter, si terra movetur spheriter <motu> naturali, tunc sphaeritas esset sibi naturalis, ut patet II *De coelo* commento 100, 105, 107, et 108, ergo non sphaeritas esset violenta in terra. Consequens falsum, quia aeternaliter est montuosa et valosa et per consequens aeternaliter non sphaerica, ergo non-sphaeritas est sibi naturalis, quia nullum motum est aeternum, ut patet II *De coelo,* commentis 17, 38 et 96.

23 Praeterea probo, quod non quilibet excessus sufficit ad motum, quia Aristoteles II *De coelo* textu commenti 126 et 127 et Commentator illis dicit quod quaelibet potentia passiva terminatur per minimum a quo potest pati, quaelibet potentia activa terminatur per maximum in quod potest agere. Et si sic, igitur non quilibet excessus sufficit ad motum, quia si A agens potest agere in B quod est maximum in quod A potest agere, adhuc A excedit B plus quam per minorem excessum, et talis excessus non sufficit ad motum, quia tunc posset agens datum in passum magis passo dato, et tunc primum passum non fuit passum maximum in quod potuit agens datum.

24 Ideo pro istis auctoritatibus de terminatione potentiae activae et passivae adversus diversimode dicitur.

<Articulus secundus> de terminatione potentiae activae

25 Primo de terminatione potentiae activae dicitur quadrupliciter. Nam a quibusdam dicitur quod <1.> potentia activa quaelibet terminari debet per maximum simpliciter in quod potest, et haec est
82vb opinio prima. Alii dicunt quod <2.> terminari | debet per maximum in quod potest cum ceteris circumstantiis et non per maximum simpliciter in quod potest, et haec est opinio secunda. Tertio dicunt quod <3.> terminari debet per maximum in quod potest sensibiliter ita quod in maius illo non potest sensibiliter, et

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2 II…3 108] Cf. Arist., *De coelo*, II, 297a25—56; Aver., *In De celo*, II, com. 105, 471, 3—33; com. 107, 473, 58—65. | 4 aeternaliter…5 sphaerica] Cf. Buridanus, *op. cit.*, qu. 23: *Utrum terra sit sphaerica*, 509, 5—7. | 6 II…7 96] Cf. Aver., *De celo*, II, com. 17, 301, 133—134; com. 38, 240; com. 96, 456. | 9 Aristoteles…12 agere] Cf. Arist., *De coelo*, I, 281a10—15, 281a18—19; Aver., *In De celo*, I, com. 116, 221, 18—28; 223, 35—36. *Com. 126 et 127 erronee indicati.*

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21 de…activae] *marg.*

haec <est> tertia opinio. Quarta opinio est quod <4.> non debet potentia activa terminari aliquo praedictorum modorum, sed terminari debet per minimum in quod non potest, et haec est opinio quarta.

Argumentum contra primam opinionem

26 Contra primam opinionem arguo sic: si A agat in B passum maximum in quod A potest agere, A tunc excedit B passum per excessum divisibilem, ergo aliquod est passum maius B sicut C, quod C passum A excedit minus quam A excedit B, <exempli gratia> per medietatem tanti excessus per quantum excessum A excedit B, et quilibet excessus sufficit ad motum (ut est prius argutum); ergo A potest agere in C quod est maius quam B. Ergo B non est maximum in quod potest A agere, quod est contra casum.

27 Item, in motu gravis in medio non est dare maximum motum localiter per illud grave, per Commentatorem IV *Physicorum* capitulo de vacuo commento 72, quia si esset dare maximum aeris motum localiter, sit ille aer A. Tunc A se totum movetur descendendo per descensum gravis, et movebitur aliquis alius aer extra A per pulsum ipsius A; ergo A non fuit maximus aer motus quem potest grave movere localiter; <ergo> aliqua potentia motiva ipsius gravis <non> terminatur per maximum simpliciter in quod potest.

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1 non…4 quarta] Anon., *Probationes conclusionum*, Venetiis 1494, 194ra: Probatur quod non sit dare maximum quod Socrates sufficit portare, quia si sic sit illud A, et arguitur sic. Socrates sufficit portare A, igitur per aliquem excessum potentia Socratis excedit resistentiam A ponderis. Consequentia patet, quia ex proportione aequalitatis non provenit actio nec motus. (…) Tunc signato isto excessu, et arguitur sic. Per medietatem illius excessus potest Socrates portare A, igitur per medietatem cum residuo potest Socrates portate A et ultra. Igitur A non est maximum quod Socrates potest portare. Et sicut arguitur de A ita potest argui de quocumque maiore A quod Socrates sufficit portare. Igitur etc.; <Pseudo-Duns Scotus>, *op. cit.*, lib. I, q. 14: *Utrum potentia activa terminetur per maximum in quod posset*, 91ra: Oppositum arguitur: Quia si potentia activa agit in aliquod: igitur excedit illud, et cum excessus ille sit divisibilis, sequitur, quod minori excessu possit in unum magis resistens agere, et per consequens illud datum non erat maximum, et ita argueretur de quodlibet; J. Buridanus, *Quaestiones super libris de caelo et mundo*, E.A. Moody, ed., Cambridge Mass., 109—110. | 15 Commentatorem…16 72] Cf. Aver., *In Phys.*, IV, com. 74, 164.

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5 Argumentum…opinionem] *marg.* | 14 Item] secundum *marg.*

28 Item, quaelibet potentia motiva quantumcumque debilis potest movere quamcumque sphaeram gravem tangentem planum in puncto quantaecumque gravitatis vel quantitatis fuerit ista sphaera, quia quaelibet potentia pulsiva potest facere quod gravitas ex una parte est a iuvamento illius gravis pellentis, <et> excedit gravitatem alterius partis sicut in aequilibra, ergo potest facere sphaeram quantumcumque gravem tangere planum in puncto; ergo non terminatur talis potentia per maximum in quod potest.

Argumentum contra secundam opinionem

29 Contra secundam opinionem arguitur sic: si potentia activa terminatur per maximum in quod potest cum ceteris circumstantiis, tunc non magis terminaretur per magnum quam per parvum, quia non est maioris virtutis movere magnum per parvum spatium in aliquo tempore quam movere parvum per maius spatium proportionabiliter in eodem tempore. Consequens falsum et contra Philosophum I *De coelo* textu commenti 116. Et videtur contra rationem, quia si quaeramus quam fortis est Socrates, determinamus eius potentiam vel fortitudinem non per parvum circumstans debiliter sed per maximum absolute quod potest levare vel quod potest portare. Item, patet per Philosophum ubi prius.

Argumentum contra tertiam opinionem

30 Contra tertiam opinionem arguo sic et probo, quod quaelibet potentia activa debet terminari per maximum simpliciter in quod potest et absque conditionibus aliquibus, sive sensibiliter sive insensibiliter [quia] agat potentia irrationalis, sicut ignis, secundum ultimum suae potentiae per tantum tempus per quantum potest durare (nam quaelibet res mensuratur certa periodo ultra quam non
83ra potest | durare, ut patet I *De coelo* commentis 128, 133, 117 et 118), tunc causatur maximum passum ab igne; illud est maximum, in quod potest ignis datus, quod non potest durare per maius tempus.

31 Item, si potentia activa potuit ad aliquid et in maius illo et sic in infinitum, tunc posset agere in corpus infinitum, si esset, cuius

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16 Philosophum…116] Cf. Arist., *De coelo*, I, 281a12—14; Aver., *In De celo*, I, com. 116, 221, 19—22. | 28 I…118] Cf. Aver., *In De celo*, I, com. 117, 223, 34—38; com. 118, 224—225, 25—30; com. 128, 247, 47—49; com. 133, 253, 8—10.

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9 Argumentum…opinionem] *marg.* | 21 Argumentum…opinionem] *marg.*

oppositum vult Philosophus I *De coelo* textu commenti 36. Et consequentia patet per modum arguendi Aristotelis et Commentatoris I *De coelo* textu commenti 118, et isto commento ubi probant, quod quaelibet res naturalis habet certam periodum et certum tempus ultra quod non potest durare, quia si res corruptibilis posset durare per aliquod tempus et maius illo et sic in infinitum, tunc posset durare per tempus infinitum. Ita arguo in proposito per simile argumentum.

Argumentum contra quartam opinionem

32 Contra quartam opinionem arguo sic: impotentia terminatur per minimum in quod non potest, ut patet I *De coelo* commento 116 et commento 117, ergo potentia non terminatur per eundem terminum. Probo etiam quod dicta Aristotelis repugnant, nam ipse dicit quod potentia activa terminatur per maximum in quod potest et impotentia per minimum in quod non potest. Sit ergo A maximum in quod potest potentia data et B minimum in quod non potest eadem potentia. Tunc A excedit B per divisibile, quia omnis excessus sit per divisibile, ut patet capitulo de vacuo commento 71, ubi dicitur quod omne excedens dividitur in excessum et in illud quod excedit, ergo contingit dare passum medium inter A et B, quod est maius A et minus B, quod sit C. Aut ergo potest in C potentia data aut non. Si sic, ergo A non est maximum in quod potest. Si non potest in C, ergo B non est minimum in quod non potest. Consequentia est necessaria et demonstrativa, ergo impossibile est determinare aliquam potentiam activam per maximum in quod potest et impotentiam per minimum in quod non potest. Et hoc est quod volui determinare.

33 Item arguo aliter contra istam opinionem sic: si potentia activa, quae est A, posset terminari per minimum in quod non potest, sit illud B. Tunc A in qualibet parte B non potest agere, et cum quolibet

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1 Philosophus…36] Cf. Arist., *De coelo*, I, 272a20. | 2 Aristotelis…3 118] Cf. Arist., *De coelo*, I, 281a29—b2; Aver., *In De celo*, I, com. 118, 225, 31—36. | 11 I…12 117] Cf. Aver., *In De celo*, I, com. 116, 221—222, 32—36; com. 117, 223, 34—38. | 18 capitulo…71] Cf. Aver., *In Phys.*, IV, com. 71, 160ra.

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9 Argumentum…opinionem] *marg.* | 13 Probo] Argumentum quod dicta Aristotelis implicant primo De coelo *marg.* | 28 Item] Secundum argumentum contra quartam opinionem *marg.*

minus B potest agere. Ergo si A incipiat agere in aliquam partem B non cessabit actio quousque totum fuerit, secundum se et quamlibet eius partem, passum ab A <applicatum>, ergo B non est minimum in quod A non potest, quia in qualibet parte B potest A.

34 Item probo quod B non potest esse minimum in quod non potest A, quia tunc sequitur quod A sit maximum quod non potest in B, quia quantumcumque modicum A intenditur, tunc poterit in B, ergo quodlibet maius et fortius potest in B, ergo B potest impedire A, ne agat in ipsum et non potest impedire aliquod maius et fortius A; ergo virtus impeditiva vel resistiva post B terminatur per maximum in quod potest sicut parva quae non potest resistere maiori et fortiori A. Ergo, eadem ratione, potentia motiva vel activa debet terminari per maximum in quod potest sicut potentia resistiva vel impeditiva terminatur per maximum in quod potest.

83rb **35** Item, descendat aliquid dividens | super terram dividendo de terra tantum quantum potest dividere, tunc capiatur totum divisum per illud dividens, tunc illud divisum <est> maximum in quod illud dividens potest, quia ultra extremum illius divisi non potest.

<Articulus tertius> de terminatione potentiae passivae

36 Quantum ad determinationem potentiae passivae per ultimum suum dicitur a quibusdam quod <1.> quaelibet potentia passiva debet terminari per minimum simpliciter a quo potest pati, et haec est prima opinio. Alii dicunt quod <2.> potentia passiva debet terminari per minimum a quo potest pati non simpliciter, sed cum certis circumstantiis, et haec est secunda opinio. Tertii dicunt quod <3.> potentia passiva terminari debet per maximum a quo non potest pati, ita quod ab isto non potest pati et a quolibet maiori potest pati, et haec est tertia opinio. Quarti dicunt quod <4.> potentia passiva terminari debet per minimum a quo potest pati convenientissime et illud est suum obiectum maxime proportionatum, sicut visus determinari debet per visibile sibi maxime proportionatum a quo convenientissime patitur, et haec est quarta opinio. Sed quinta opinio et ultima est quod <5.> non quaelibet potentia passiva terminari debet per minimum quod potest ipsum dividere vel per maximum a quo non potest pati. Nam potentia passiva in aliquo dividendo non debet terminari per

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5 Item] tertium argumentum *marg.* | 15 Item] quartum argumentum *marg.* | 19 de…passivae] *marg.*

minimum quod potest ipsum dividendum dividere nec per maximum quod potest ipsum non dividere secundum aliquam sui partem, eo quod quaelibet potentia divisiva potest aliquam partem cuiuscumque divisibilis dividere quantumcumque fuerit difficilis divisionis, ut ponit haec opinio ultima.

Contra primam, secundam, tertiam opinionem

37 Sed contra tres primas opiniones istius articuli arguitur per argumenta prius adducta contra similes opiniones de terminatione potentiae activae.

Contra quartam opinionem

38 Contra quartam opinionem, quia contra illam non fuit prius argutum: si potentia passiva terminari debet per minimum a quo potest pati convenientissime scilicet per suum obiectum convenientissimum et sibi maxime proportionatum, tunc non deberet terminari ab isto a quo patitur cum magna difficultate. Consequens est falsum, quia non determinamus potentiam visivam et bonitatem visus per minimum simpliciter quod visus potest videre et istos imputamus melioris visus, quia minora possunt videre licet cum difficultate.

Primum argumentum pro quinta opinione

39 Quinta opinio dicit propter istas, quia si stramen caderet super lapidem durissimum aliquam partem posset dividere licet insensibilem. Nam si stramen potest istam partem lapidis, supra quam cadit, dividere, capiatur aliud agens quod partem aliam illius lapidis posset dividere, sit illud agens B, et partem lapidis divisam C. Tunc qualis est proportio B ad stramen praedictum in potentia divisiva talis est proportio C ad aliquam maiorem partem lapidis,

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13 suum] subiectum *add. sed exp.*

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16 quia…19 difficultate] Cf. <Pseudo-Duns Scotus>, *op. cit.*, qu. 15, 96a: sed potentia passiva si patiatur ab aliqua distantia non oportet, quod patitur a qualibet minori: Nam visus dato quod videat visibile a remotius, tamen non oportet quod videat a qualibet distantia minori: nam visibile potest tantum applicari visui, quod non videatur.

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6 Contra…opinionem] *marg.* | 10 Contra…opinionem] *marg.* | 20 Primum…opinione] *marg.*

quae sit D; ergo permutatim sicut B ad C ita stramen ad D. Consequentia patet I *De coelo* commento 62 et 63. Ergo si B potest dividere C, sequitur quod stramen possit dividere D. Et assumptum probo, nam medietas praedicti lapidis est subduplae difficultatis ad dividendum quam totus lapis, et eadem ratione quarta pars
83va subquadruplae difficultatis | et sic in infinitum, ut pars inexistit <toti>; ergo supra aliquam partem lapidis, ut inexistit, dominatur stramen sufficienter ad dividendum. Antecedens probo, quia si agentia aequalia econtrario dividant lapidem praedictum quousque sibi obviaverunt in medio, tunc facerent actiones aequalis difficultatis; ergo istae duae actiones simul sumptae sine difficultate <altera> ad alteram sunt, ergo tota resistentia lapidis est in duplo maior quam resistentia alterius medietatis, quatenus inexistit toti. Et sic de omnibus partibus proportionalibus illius lapidis fieri potest simile argumentum. Ergo aliquam partem lapidis potest stramen dividere.

40 Eadem opinio potest aliter confirmari: si stramen potest aliquam partem aeris dividere, quae sit A, teneantur ergo duo alia agentia aequalia quorum utrumque potest dividere lapidem praecedentem, quae sint B C. Descendat ergo B super lapidem praedictum in eodem tempore in quo C descendet super A. Tunc in eodem tempore in quo C dividit A, B dividit aliquam partem lapidis praedicti, quae pars sit E. Tunc arguo sic: A et E dividuntur ab agentibus aequalibus in aequali tempore, ergo A et E sunt resistentiae aequales, prout inexistant suis totis, ergo quidquid excedit A excedit E. Sed stramen excedit A, ergo potest dividere E; et per consequens quodcumque agens potest dividere aliquam partem aeris vel ferri durissimi quod potest aliquam partem lapidis praedicti dividere.

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3 D] *corr. ad sensum, ms.* B

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1 permutatim...D] Cf. Campanus de Novara, *Elementa*, def. XII, 171: Quantitates, quae sunt in proportione una, antecedens ad consequentem et antecedens ad consequentem; dicetur econtrario sicut consequens ad antecedentem, sic consequens ad antecedentem. Itemque permutatim, sicut antecedens ad antecedentem, sic etiam consequens ad consequentem. | 2 I…63] Cf. Aver., *In De celo*, I, com. 63, 122—123, 40—52; com. 64, 123, 23—28.

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17 Eadem] secundum argumentum *marg.*

41 Quod confirmo sic, quia aliter esset dare minimum agens aut debilissimum quod potest aliquam partem lapidis praedicti dividere aut maximum et fortissimum quod non potest aliquam partem lapidis praedicti dividere. Non primo modo, quia minus isto agente dato posset minorem partem lapidis dividere. Nec secundo modo, quia, si esset dare maximum quod non posset aliquam partem lapidis dividere ex quo illud agens potest in aliquam aliam resistentiam quae sit B sequitur, quod quaelibet pars istius lapidis esset maioris resistentiae quam B, ergo iste lapis esset resistentiae infinitae. Consequentia patet, quia si esset resistentiae finitae tantum, tunc in aliqua proportione excedit B et in eadem proportione excedit B aliquam sui partem quantum ad modum resistendi. Consequentia patet, ut prius.

42 Propter istas evidentias videtur ista opinio esse vera.

Contra quintam opinionem

43 Contra quam tamen arguo sic: ex ista sequitur quod quaelibet gutta cadens supra lapidem aliquam eius partem auferat, quod est contra Philosophum VIII *Physicorum* textu commenti 23 et contra Commentatorem illo commento, ubi dicunt quod ultima gutta cavat lapidem et non praecedentes.

44 Item, si stramen cadens super lapidem durrissimum posset aliquam eius partem dividere, tunc, cum tota resistentia sit minor divisa aliqua parte lapidis quam praefuit et potentia dividentis aequalis, posito quod non debilitetur dividens, sequitur quod quaelibet potentia non debilitata | quae potest dividere partem, 83vb
potest dividere totum. Consequens falsum, quia non quodcumque excedens partem excedit totum. Videmus enim, quod trunci medius

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10 finitae] *corr. ad sensum, ms.* infinitae

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18 Philosophum…20 praecedentes] Cf. Arist., *Phys.*, VIII, 253b14—19; Aver., *In Phys.*, VIII, com. 23, 359va; Thomas Aquinas, *Comm. Phys.*, VIII, lec. 5, n. 1008; Johannes Dumbleton, *Summa logicae et philosophiae naturalis*, VI.21, ms. Cambridge, Peterhouse 272, 56v: Praeterea quod iuxta processu Philosophi debet minimum in actu intelligi minimum sensibile, apparet per dicta Commentatoris ubi ponit, quod passio guttarum descendentium non sit a quocumque parte, sicut aestimari potest, immo quod sit in eo actu subito.

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1 Quod confirmo] confirmatio *marg.* | 15 Contra…opinionem] *marg.* | 21 Item] secundum argumentum *marg.*

divisi latera reclinant ut se iuvent, ergo agent aliquod nisum ad concernendum finitum <...> est aliter accipere grave dividens minoris inclinationis versus deorsum quam sit ille in suis <partibus> ad concernendum, quia si ponitur inter tales partes non descenderet nec divideret aliquam partem.

45 Item, si stramen descendens supra lapidem posset aliquam partem lapidis dividere et non totum, tunc vel esset dare minimam partem illius lapidis quam non posset dividere vel maximam partem eiusdem quam posset dividere. Consequentia patet, quia accepto toto tempore per quod dividit aliquid de lapide praedicto in quarti medio, vel est dare maximam partem quam potest dividere aut minimam quam non potest dividere, ut patet per deductionem convenientem. Non potest poni maxima quam potest dividere, quia adhuc maiorem partem excedit per minorem excessum, et quilibet excessus sufficit ad motum, ergo etc. Nec poni potest minima pars quam non potest dividere stramen praedictum, quia sit ista A; tunc quamlibet partem potest stramen illud dividere, igitur potest A dividere.

46 Item, si incipiat stramen dividere aliquid de lapide, tunc ex quo in qualibet divisione aliquid corrumpitur, sequitur quod immediate post hoc alia pars lapidis illius erit corrupta; et cum corrupta parte rei inanimatae non maneat illa res, sequitur quod immediate post hoc non manebit iste lapis, et per consequens est dare ultimum instans sui esse. Igitur rei permanentis esset dare ultimum instans sui esse, quod est contra Philosophum VIII *Physicorum* et in multis aliis locis.

47 Item, si immediate post hoc erit iste corruptus, ergo immediate post hoc nihil aget in ipsam divisionem.

<Articulus quartus de velocitate motus>

48 Item, si quaestio esset vera, tunc velocitas motus vel sequeretur excessum potentiae moventis super potentiam rei motae vel proportionem potentiae moventis super potentiam rei motae.

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25 Philosophum…Physicorum] Cf. Arist., *Phys.*, VIII, 260a13—19. | 29 velocitas…30 motae] Cf. Aver., *In Phys.*, VII, com. 36, 335ra; Bradwardinus, *op. cit.*, 86, 4—6. | 31 proportionem…motae] Cf. Bradwardinus, *op. cit.*, 110, 2—5.

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6 Item] tertium argumentum *marg.* | 19 Item] quartum argumentum *marg.* | 29 Item] argumentum principale contra quaestionem *marg.*

49 Prima opinio videtur fundari super dicto Commentatoris, capitulo de vacuo commento 71, ubi quaestio habet locum, dicit quod "omnis motus est secundum excessum potentiae moventis super rem motam". Et VII *Physicorum* commento 39 et ultimo dicit, quod: "secundum excessum potentiae alterantis super potentiam alterati erit velocitas motus alterationis in quantitate temporis".

<Contra primam opinionem>

50 Sed contra istam opinionem potest sic argui multipliciter, nam haec opinio videtur destrui omnes regulas Philosophi in VII *Physicorum* versus finem, quoniam si haec esset vera, tunc falsificaretur regula Aristotelis et Commentatoris VII *Physicorum* <textu> commenti 38, et isto commento 38, quod moventia aequalia aggregata aeque velociter praecise moventur sicut unum motum praedictorum separatorum moveret unum mobilium separatorum. Nam ex | ista opinione sequitur, quod duo aequalia 84ra
coniuncta mobilia in duplo velocius <moverentur> quam unum movens divisum moveret unum mobilium praedictorum per se. Quia per duplum excessum excedunt duo moventia aequalia duo mota aequalia quam unum movens de numero istorum moventium excedit unum illorum mobilium, <ut> manifeste patet per exemplum in numero. Nam unus binarius excedit unam unitatem per excessum qui est unitas, et alius binarius separatus excedit aliam

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18 moventium] *corr. ad sensum, ms.* mobilium

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1 Commentatoris…4 motam] Cf. Aver., *In Phys.*, IV, com. 71, 161rb—va; Bradwardinus, *op. cit.*, 86, 9—11. | 4 VII…6 temporis] Cf. Aver., *In Phys.*, VII, com. 39, 337va: secundum excessum potentiae alterantis supra potentiam alterati erit velocitas alterationis et quantitas temporis, in quo est alteratio; Bradwardinus, *op. cit.*, 86, 14—15: secundum excessum potentiae alterantis supra potentiam alterati erit velocitas alterationis et quantitas temporis. | 10 Aristotelis…14 separatorum] Cf. Arist., *Phys.*, VII, 250a25—28; Aver., *In Phys.*, VII, com. 38, 336vb: Duo moventia diversa utrumque movet aliquod pondus per aequalem spatium in aequali tempore, dunc duae potentiae motivae congregatae movebunt congregatum ex duobus ponderibus per illud spatium in illo tempore; Bradwardinus, *op. cit.*, 88, 43—47: Si duae potentiae divisim moveant duo mobilia per aequalia spatia in aequali tempore, illae duae potentiae coniunctae movebunt illa duo mobilia coniuncta per aequale spatium in aequali tempore cum priori. | 20 Nam…23,4 separatis] Cf. Bradwardinus, *op. cit.*, 86—88, 37—41.

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7 Sed] argumentum contra primam opinionem *marg.*

unitatem per aequalem excessum; si congregantur illi duo binarii excedunt duas unitates congregatas per duplum excessum qui per binarium est duplus ad unitatem, quae fuit primus excessus in binariis separatis.

51 Item, ex ista opinione sequitur quod alia regula Aristotelis sit falsa, quae ponitur in textu commenti 36 in VII *Physicorum*, quae est: Si aliquis motor moveat aliquid mobile per aliquod spatium in aliquo tempore, subduplus motor movebit medietatem mobilis tanti per tantum spatium in aequali tempore et aeque velociter, quia unitas est eadem proportio medietatis ad medietatem sicut totius ad totum. "Sic declarant geometrae" et dicit Commentator VII *Physicorum* commento 36. Sed illud foret falsum, si velocitas motus sequitur excessum, nam totus motor excedit totum mobile per certum excessum, et medietas motoris excedit medietatem moti per medietatem illius excessus tantum, ut patet in exemplo priori, sicut quaternarius excedit binarium <per binarium>, ita medietas eius scilicet binarius excedit medietatem binarii per unitatem quae est medietas prioris excessus.

52 Item, probo, quod velocitas motus non semper sequitur excessum, quia videmus quod si aliquis unus homo moveat aliquid ponderosum motu valde tardo, quod ponderosum potest vix ille homo movere; si alius homo sibi coniungitur, illi duo homines movebunt illud ponderosum multo plus quam in duplo velocius, et tamen excessus non duplicatur nisi <duplo> praecise, ergo velocitas non sequitur excessum.

53 Istud confirmo: si in horilogio per certum tempus moveretur rota certa velocitate, moto addito – licet non dupletur pondus – dupla velocitate movebitur, ut experimento notum est. Illud apparet manifeste de pondere suspenso ad aliquod circumvolubile, quod per suum descensum movetur insensibiliter et movet rota quodam motu

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5 Aristotelis…10 totum] Cf. Arist., *Phys.*, VII, 250a25—28; Bradwardinus, *op. cit.*, 86, 25—30; 88, 61—72. | 11 Sic…12 36] Cf. Aver., *In Phys.*, VII, com. 36, 335va: ut declaraverunt Geometrae; Bradwardinus, *op. cit.*, 88: sicut universaliter demonstrant geometri. | 15 sicut…18 excessus] Cf. Bradwardinus, *op. cit.*, 86, 20—23. | 20 si…23 velocius] Cf. Aver., *In Phys.*, VII, com. 38, 337ra; Bradwardinus, *op. cit.*, 98, 287—291.

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5 Item] secundum argumentum *marg.* | 19 Item] tertium argumentum *marg.* | 26 Istud] quartum argumentum *marg.*

insensibili. Sic accidit in horilogio: ad pondus si suspendatur iterum tantum pondus totum descendet cum impetu et circumvolvet rotam multo plus quam in duplo velocius, ut patet manifestae sensationi.

54 Item, posito quod aliquis homo trahat unam fabam per unam cordam currendo ita velociter sicut potest, tunc si alius homo tantae potentiae ad currendum sibi iniungatur ad trahendum illam fabam praedictam, | illi duo homines non trahent velociter quam unus 84rb illorum per se. Ergo velocitas motus non sequitur excessum.

55 Item, posito quod tres homines forte nitantur trahere navem quam non possunt movere nisi girando huc illuc, tunc isti tres homines unam actionem in navem faciunt. Si addatur quartus homo aequalis potentiae alicui praedictorum, illi quattuor homines movebunt navem directe per magnum spatium, ut experimento notum est. Ex quo sequitur, quod minus quam agens duplum facit plus quam duplam actionem, quia si primi tres homines agant in navem prius directe per medietatem tanti spatii, per quantum movent isti quattuor homines, fecissent medietatem actionis praecise quam faciant isti quattuor homines; sed sic non fecerunt secundam medietatem tantae actionis, et tamen agens fuit minus quam subduplum. Ergo velocitas actionis non sequitur excessum.

56 Item, exemplum potest adduci de homine portante aliquod pondus cum quo vix potest ire lento passu. Tunc minori addito quam aequali pondere movebitur in duplo tardius et vix aliquo modo vadit. Ergo, ut prius, velocitas motus non sequitur excessum.

57 Item, si velocitas motus sequeretur excessum, tunc ex similitudine motus ad mota et ex proportione istorum geometrica non sequitur aequalitas velocitatis motuum, quia non sequitur ex aequalitate proportionum aequalitas excessuum, ut manifeste patet. Nam aequalis est proportio quattuor ad duo et duorum ad unum, et tamen excessus sunt inaequales sicut binarius et unitas. Et per consequens ad quod deducitur est contra Aristotelem et

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19 minus] *corr. ad sensum, ms.* unus

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1 ad…3 sensationi] Cf. Bradwardinus, *op. cit.*, 98, 292—297. | 14 minus…20 subduplum] Cf. Arist., *Phys.*, VIII, 253b15—254a1; Aver., *In Phys.*, VIII, com. 23, 359ra. | 31 Aristotelem…25,2 commentis] Cf. Aver., *In Phys.*, VII, com. 31, 311va; com. 36, 335vb; com. 38, 336vb—337ra.

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4 Item] quintum argumentum *marg.* | 9 Item] sextum argumentum *marg.* | 25 Item] septimum argumentum *marg.*

Commentatorem VII *Physicorum* commento 31, 36 et illis commentis, capitulo de vacuo commento 71 et multis aliis locis, ubi semper ex aequalitate proportionis moventium ad mota arguunt aequalem velocitatem motuum illorum mobilium.

58 Pro istis et consimilibus dicitur secunda opinio, quod velocitas motuum sequitur proportionem moventis ad motum et non excessum, quia si sequeretur excessum, tunc sequeretur quod si aliquod mobile simplex quod in dupla proportione excedit quod non posset moveri in aliquo medio subtiliori mundi quam in duplo velocius, quia nullum medium potest excedere per plus quam per duplum excessum. Tunc per subtiliationem medii in infinitum non potest motus istius mobilis velocitari in infinitum, quod est contra Aristotelem et Commentatorem capitulo de vacuo commento 71, ubi [per definitionem praedictam proportionis, sequitur quod proportio 9 ad 4 sit dupla ad proportionem 6 ad 4 nam] quaestio habet locum. Ubi dicit Commentator sic uniformiter: "cum ergo sic
84va fuerint duo motores et duo mota, et proportio | unius motoris ad suum motum fuerit sicut proportio reliqui motoris ad suum reliquum motum, tunc illi motus erunt aeque veloces, et cum diversificabitur proportio diversificatur et motus". Et in fine in eodem dicit sic: "diversitas motuum in velocitate et tarditate est secundum proportionem hanc, quae est inter duas potentias" motivam scilicet et resistivam. Et II *De coelo* commento 36 dicit sic Commentator: "velocitas motus et tarditas non fiunt nisi secundum proportionem potentiae motoris ad potentiam rei motae; quanto fuerit potentia motoris ad motum maior tanto motus erit velocior, et quanto minor tanto motus erit tardior". Et VII *Physicorum* commento 35 ex duplicatione proportionis motoris ad motum arguit

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10 nullum] *corr. ad sensum, ms.* minimum

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2 capitulo…71] Cf. Aver., *In Phys.*, IV, com. 71, 160vb. | 3 semper…4 mobilium] Cf. Bradwardinus, *op. cit.*, 88, 50—58. | 11 per…12 infinitum] Cf. Bradwardinus, *op. cit.*, 92, 126—132. | 16 cum…20 motus] Cf. Aver., *In Phys.*, IV, com. 71, 160vb; Bradwardinus, *op. cit.*, 110, 7—13. | 21 diversitas…22 potentias] Cf. Aver., *In Phys.*, IV, com. 71, 162va; Bradwardinus, *op. cit.*, 110, 14—16. | 24 velocitas…27 tardior] Cf. Aver., *In De celo*, II, com. 36, 334, 21—25; Bradwardinus, *op. cit.*, 110, 17—21.

Commentator duplam velocitatem motus et dicit sic: "Cum diviserimus motum contingit necessario quod proportio" illius motoris ad medietatem moti sit dupla ad proportionem quae praefuit inter motorem et totum motum, et ideo movebit idem motor medietatem moti in duplo velocior quam totum motum istum vel mobile.

59 Ex quibus concluditur haec opinio secunda.

<Contra secundam opinionem>

60 Contra quam opinionem potest argui multipliciter.

<Prima conclusio>

61 Primo modo ex ista positione sequitur ista conclusio, quod duplum grave praecise non movebitur praecise in duplo velocius in eodem medio quam grave subduplum. Quia posito quod grave simplex excedat resistentiam medii sicut tria excedunt unum, tunc duplata sua gravitate se habebit ad eandem resistentiam sicut sex ad unum, quae proportio non est dupla ad primam proportionem, nam proportio nonupla est praecise dupla ad proportionem triplam, ut potest demonstrari per definitionem proportionis duplicatae positam V Euclidis: nam sicut unum ad tria ita tria ad novem. Ergo proportio novem ad unum est praecise dupla ad proportionem <trium> ad unum, per definitionem proportionis duplicatae quae est, quod si fuerint tria continue proportionabilia proportione inaequalitatis, tunc proportio tertii ad primum est proportio secundi ad primum duplicata. Ex quo sequitur, quod proportio nonupla est praecise dupla ad proportionem triplam. Et per consequens proportio sextupla est minor quam dupla ad triplam. Et sequitur

22 nonupla] *corr. ad sensum, ms.* quadrupla

1 Cum…6 mobile] Cf. Aver., *In Phys.*, VII, com. 35, 335ra: velocitas propria unicuique motui sequeretur excessum potentiae motoris super potentiam moti. Et ideo, cum diviserimus motum contingit necessario ut proportio potentiae motoris ad motum sit dupla illius proportionis: et sic velocitas erit dupla ad illam velocitatem; Bradwardinus, *op. cit.*, 110, 22—26. | 16 definitionem…17 Euclidis] Cf. Johannes Campanus de Novara, *Elementa*, V, def. X, 168: Cum fuerint tres quantitates continue proportionales, dicetur proportio primae ad tertiam proportio primae ad secundam duplicata.

8 Contra] primum argumentum contra secundam opinionem *marg.*

quod si grave excedit resistentiam medii in quo movetur in tripla proportione praecise, quod duplum grave non movebitur in illo medio in duplo velocius immo minus quam in duplo velocius, cum velocitas motus sequatur proportionem, ut ponit haec opinio.

62 Item, posito quod aliquod grave excedat resistentiam medii in quo movetur sicut sex excedunt quattuor. Tunc duplata sua gravitate se habebit ad suam resistentiam eandem sicut duodecim ad quattuor quae est maior quam dupla proportio ad proportionem sex ad quattuor. Nam sicut sex ad quattuor ita novem ad sex, quia utraque proportio est sexquialtera, ergo proportio novem ad quattuor est
84vb dupla ad proportionem | sex ad quattuor, quoniam sexquialtera proportio duplicata est dupla ad unam sexquialteram. Et proportio novem ad quattuor est sexquialtera duplicata, quia quandocumque sunt tria proportionabilia continue proportio primi ad ultimum componitur ex proportionibus tertii ad secundum, secundi ad primum, secundum modum loquendi mathematicorum. Ideo, secundum eos sequitur, quod proportio novem ad quattuor erit sexquialtera duplicata, eo quod consistit ex duabus sexquialteris scilicet proportione novem ad sex et sex ad quattuor. Sequitur ergo, quod in isto casu ubi grave excedit resistentiam medii in sexquialtera

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1 resistentiam] *corr. ad sensum, ms.* regulam 20 resistentiam] *corr. ad sensum, ms.* regulam

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13 quandocumque…16 mathematicorum] Cf. Campanus de Novara, *Elementa*, V, def. X, 168; def. IX, 109: Quando vero tres quantitates proportionales fuerint, prima ad tertiam duplicem proportionem habere dicitur quam ad secundam; Cf. etiam: *The First Latin Translation of Euclid 'Elements' commonly ascribed to Adelard of Bath*, 146: Cum fuerint tres quantitates proportionales, erit proportio primae ad tertiam sicut proportio primae ad secundam repetita; Cf. etiam: *Robert of Chester's (?) Redaction of Euclid's 'Elements', the So-called Adelard II Version*, Vol. I, 161: Cum fuerint tres quantitates proportionales, dicetur proportio primae ad tertiam proportio primae ad secundam duplicata; Cf. etiam: *Johannes de Tinemue's Redaction of Euclid's 'Elements', the So-called Adelard III Version*, 129: Cum fuerint tres quantitates proportionales, erit proportio primae ad tertiam, primae ad secundam geminata; Bradwardinus, *op. cit.*, 78, 297—302: Si fuerit proportio maioris inaequalitatis primi ad secundam ut secundi ad tertium, erit proportio primi ad tertium dupla ad proportionem primi ad secundum et secundi ad tertium. | 17 proportio…19 quattuor] Cf. *Johannes de Tinemue's Redaction*, 129: Verbi gratia: proportio 9 ad 4 est quasi proportio 9 ad 6 dupla quia constat quasi totaliter ex ea quae est inter 9 ad 6 et 6 ad 4 cuius aequaliter sit, utrobique sit sexquialtera.

proportione, quod grave duplum non movebitur in eodem medio praecise in duplo velocius immo plus quam in duplo velocius. <Grave duplum non movebitur in duplo velocius> in eodem medio praecise nisi quando grave subduplum excedit resistentiam medii in proportione dupla praecise, ut per praedicta poterit demonstrari faciliter. Ergo non sequitur: grave duplum praecise moveri in duplo velocius in eodem medio, quod videtur contra Philosophum IV *Physicorum* capitulo de vacuo textu commenti 71 et 74 et contra Commentatorem illis commentis, et I *De coelo,* commento 33, 51, 63 et 65, et III *De coelo*, commento 26 et 27, et III *Physicorum* capitulo de infinito commento 42, et VIII Physicorum commento 80. Et idem videtur contra primam conclusionem Archimedis *De ponderibus*, quae est quod: inter quaelibet gravia est velocitatis in descendendo et ponderis eodem ordine sumpta proportio. Et contra primam conclusionem Iordani *De ponderibus*, quae est: inter quaelibet gravia <est velocitatis> in descendendo [et ponderis velocitas] eodem ordine sumpta proportio. Et contra secundam conclusionem Euclidis *De ponderibus* quae est quod si duorum corporum gravium eiusdem generis fuerit unum in quarto <maius> respectu alterius, tunc virtutem illius necesse est esse multiplicem ad virtutem alterius descendentis. Istae conclusiones tantum significant quod proportio velocitatis motus alterius gravis <ad alterum> erit secundum

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13 velocitatis] *corr. ad sensum, ms.* virtus

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7 Philosophum…9 commentis] Cf. Aver., *In Phys.*, com. 71, 160vb; com. 74, 164vb. | 9 I…10 65] Cf. Aver., *In De celo*, I, com. 33, 101—102; com. 51, 102, 11—15; com. 63, 121; com. 65, 125, 14—15. | 10 III^{1}…27] Cf. Aver. *In De celo*, III, com. 26, 546—547; com. 27, 550, 75—82. | III^{2}…11 80] *Loci erronee indicati — non inventi.* | 12 Archimedis…17 proportio] Cf. Jordanus de Nemore, *Liber de ponderibus*, P.01, 154: Inter quaelibet duo gravia est velocitatis in descendendo proprie, et ponderis eodem ordine sumpta proportio; Jordanus de Nemore, *Elementa super demonstrationem ponderum*, E.1, 128: Inter quaelibet gravia est velocitatis in descendendo eodem ordine sumpta proportio, decensus autem et contrarii motus proportio eadem sed permutata; Bradwardinus, *op. cit.*, 96, 236—238: Idem patet per primam conclusionem *De ponderibus*, quae sic dicit: 'Inter quaelibet gravia est velocitatis in descendendo et ponderis eodem ordine sumpta proportio; cf. etiam 100—102, 338—343. | 18 si…21 descendentis] Cf. Euclides, *Liber de ponderoso et levi et de comparatione corporum ad invicem*, II, 28: Si duorum corporum eiusdem generis fuerit unum multiplex alterius, et virtutem illius virtutis alterius similiter esse. | 21 proportio…29,3 aequalitatis] Cf. Bradwardinus, *op. cit.*, 102—104, 364—385.

proportionem gravitatis ad gravitatem et ponderis ad pondus, quod non oportet semper in eodem medio, ut prius dictum est, si velocitas motus sequatur proportionem aequalitatis.

<Secunda conclusio>

63 Secundo ex ista opinione sequitur ista conclusio, quod idem grave simplex motum in aqua alicuius densitatis non movebitur praecise in duplo velocius in aere duplae subtilitatis. Haec autem conclusio sequitur per similes determinationes prioribus. Nam si illud grave excedat resistentiam aquae in proportione maiori quam dupla, tunc excedit aerem duplae subtilitatis in proportione minori quam dupla ad illam proportionem, ut patet per deductionem priorem, ergo illud grave movebitur in aere duplae subtilitatis minus quam in duplo velocius quam movebitur in ista aqua. Et si illud grave simplex excedat resistentiam aquae in proportione minori quam in dupla, tunc excedet resistentiam duplae subtilitatis in proportione maiori quam dupla ad proportionem primam, sicut demonstrari potest ut prius per definitionem proportionis
85ra duplicatae. | Ergo tunc movebitur tale grave in aere duplae subtilitatis praecise plus quam in duplo velocius quam movebatur in aqua. Ergo non est verum quod semper illud grave movebitur praecise in duplo velocius in aere duplae subtilitatis ad aquam quam moveretur in aqua. Quod apparet contra Philosophum capitulo de vacuo textu commenti 71, 72 et contra Commentatorem illis commentis. Item etiam videtur esse contra tertiam conclusionem Archimedis *De ponderibus*, quae est quod: quanto ponderosius est per quod fit transitus tanto difficilius fit transitus in descendendo.

13 resistentiam] *corr. ad sensum, ms.* regulam 14 resistentiam] *corr. ad sensum, ms.* regulam

19 semper…21 aqua] Cf. Bradwardinus, *op. cit.*, 94, 187—204. | 21 Philosophum…23 commentis] Cf. Arist., *Phys.*, IV, 215b1—12, 161, 8—19; Aver., *In Phys.*, IV, com. 71, 159ra; Bradwardinus, *op. cit.*, 94, 186—200.

4 Secundo] secundum argumentum *marg.*

<Tertia conclusio>

64 Tertio ex ista opinione sequitur ista conclusio, quod istae regulae Philosophi in VII *Physicorum* sint falsae. Quarum una ponitur in textu commenti 37 et isto commento quae est illa: si "aliquis motor moveat mobile per aliquod spatium in aliquo tempore", eadem potentia motoris movebit medietatem illius mobilis per duplum spatium in aequali tempore et per aequale spatium in medietate temporis, ut dicitur commento 36. Et alia regula ponitur textu commenti ultimi quae est quod: "si aliqua potentia moveat aliquod mobile per aliquod spatium in aliquo tempore, dupla potentia movebit idem mobile per duplum spatium in aequali tempore" – quod est ex praecedentibus notum esse falsum. Nam ponatur proportionem moventis ad suum mobile triplam praecise, tunc potentia dupla non movebit subiectum mobile in duplo velocius immo minus quam in duplo velocius, quia si potentia duplicetur resultat proportio sextupla motoris ad eandem resistentiam, quae proportio non est dupla ad triplam immo minus quam dupla, ut ex praecedentibus satis constat.

65 Item, ponendo quod motus excedat resistentiam sui mobilis in proportione sexquialtera, tunc duplex motus praecise moveret idem mobile plus quam in duplo velocius, eo quod tunc proportio secundi motoris ad idem mobile erit maior quam dupla ad proportionem primi motoris ad idem mobile, ut ex praedictis per exemplum in numeris satis patet.

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6 medietate temporis] *corr. ad sensum, ms.* immediate transivit 15 sextupla] *corr. ad sensum, ms.* quadrupla

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2 Philosophi…7 36] Cf. Arist., *Phys.*, VII, 250a, 1—4, 274, 7—10; Aver., *In Phys.*, com. 36, 335va; Bradwardinus, *op. cit.*, 96, 205—209. | 7 textu…10 tempore] Cf. Arist., *Phys.*, VII, 250a25—28; Aver., *In Phys.*, VII, com. 39, 337va; Bradwardinus, *op. cit.*, 96, 216—219.

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1 Tertio] tertium argumentum *marg.*

66 Per similes deductiones arguitur contra regulas alias multas in septimo.

<Quarta conclusio>

67 Quarto ex ista opinione sequitur ista conclusio, quod si essent aliqua duo gravia simplicia aequalia in omnibus circumstantiis requisitis quorum unum movetur in aqua pedalis quantitatis et aliud descendat in aere bipedalis quantitatis duplae tamen subtilitatis, quod isti duo motus descensus sunt aeque veloces, si fuerint in eodem tempore praecise, quia utrobique in utroque motu est aequalis proportio motoris ad suam resistentiam; ergo motus sunt aeque veloces. Consequentia patet commento 71 ubi quaestio habet locum. Consequens est falsum, quia per unum motum pertransitur duplum spatium in aequali tempore per positum, ergo unus motus est in duplo velocior alio. Consequentia patet per definitionem 'velocioris' datam VI *Physicorum* commento 71. Et primo per assumptum probo videlicet, quod aer et aqua aequaliter resistant suis moventibus in isto casu, quia medietas aeris est subduplae
85rb resistentiae | praecise ad totam aquam, quia sunt aequalis quantitatis medietas moti aeris dati et tota aqua data, et cum hoc medietas aeris est duplae subtilitatis ad istam aquam; ergo medietas istius aeris est praecise subduplae resistentiae ad illam aquam et eadem medietas aeris est praecise subduplae resistentiae ad totam aquam. Datus ergo totus iste aer et tota illa aqua comparati ad medietatem illius aeris habent eandem proportionem ad istam medietatem aeris quantum ad actum resistentiae; ergo totus aer datus et tota aqua data sunt aequalis resistentiae inter se. Consequentia patet per unam conclusionem V Euclidis. Et assumptum probo, scilicet quod medietas aeris quatenus inexistentis suo toto sit subduplae

9 resistentiam] *corr. ad sensum, ms.* regulam 11 motum] *corr. ad sensum, ms.* medium 18 medietas[1]] *corr. ad sensum, ms.* medietatis

10 commento 71] Cf. Aver., *In Phys.*, IV, com. 71, 159rb—160va. | 13 per…14 71] Cf. Aver., *In Phys.*, IV, com. 71, 159ra. | 26 conclusionem…Euclidis] Campanus de Novara, *Elementa*, Vol. I, V, concl. (V.)9, 183: Si fuerit aliquarum quantitatum ad unam quantitatem proportio una, ipsas esse aequales. Si vero unius ad eas proportio una, equales esse necesse est. | 27 medietas…32,5 aeris] Cf. Bradwardinus, *op. cit.*, 116—120.

3 Quarto] quartum argumentum *marg.*

resistentiae praecise ad resistentiam totius, quia si aliqua duo agentia aequalia dividerent istum aerem quousque sibi invicem obviarent in medio ita quod inciperent dividere in extremo oppositorum, tunc agent duas actiones aequalis difficultatis supposita uniformitate istius aeris.

<Quinta conclusio>

68 Quinto ex ista opinione sequitur quod aliquod corpus movetur velocius alio et hoc in medio denso. Et subtiliando isto ad minimum mixtum <A>, cuius gravitas excedat levitatem in proportione tripla, enim tantum resistat illud medium praecise quantum resistit sibi sua resistentia intrinseca, adhuc potest illud mixtum moveri in hoc medio quia excedit suam resistentiam intrinsecam ut tria unum et in eadem proportione suam extrinsecam; ergo excedit istas duas resistentias congregatas in sexquialtera proportione sicut 3 <excedunt> 2; ergo potest moveri in hoc medio aliqua certa velocitate. Accipiatur ergo aliqua terra simplex parva, quae propter suam parvitatem vix sufficit dividere hoc medium, quod simplex sit B, ita quod B moveatur in hoc medio tardius quam A. Subtiliatur ergo illud medium quousque illud <B> possit moveri in illo duplo velocius, quod potest esse possibile per Philosophum et Commentatorem capitulo de vacuo, commento 72.

69 Et primo probatur esse possibile, quia medium potest subtiliari quousque potentia mota ipsius B excedit resistentiam illius medii in duplo plus quam nunc excedit, et tunc movebitur in isto duplo velocius; et non potest moveri in duplo velocius quam movetur in isto, [quam in A], et tunc non movetur B velocius quam A. Ergo sequitur conclusio proposita, quod A nunc movetur velocius B cum magna resistentia huius medii, et minorata ista resistentia ex ista proportione non sufficit potentia mota A ad movendum ipsum velocius B cum minori resistentia medii ut prius sufficeret cum maiori.

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25 B] *corr. ad sensum, ms.* A

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19 Philosophum…20 72] Cf. Arist., *Phys.*, IV, 215b20—216a3; Aver., *In Phys.*, IV, com. 72, 163ra. | 26 A...maiori] Cf. Bradwardinus, *op. cit.*, 90, 96—116; 114, 98—107.

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6 Quinto] quintum argumentum *marg.*

70 Quod autem A non possit moveri in alio medio in duplo velocius quam nunc movetur probo sic, quia quantumcumque medium subtilietur non durabit tota sua resistentia intrinseca et extrinseca. 85va Aliqua tota resistentia extrinseca corrumpetur, | supposito quod tota sua resistentia intrinseca maneat ut prius, ergo quantumcumque medium subtilietur non movetur A in duplo velocius in aliquo medio subtiliori – manente sua resistentia intrinseca non minorata – quia si posset moveri in aliquo medio in duplo velocius, cum in vacuo non movetur plusquam in duplo velocius, sequitur quod non posset moveri in vacuo velocius quam in pleno. Consequens est falsum, quia semper in pleno haberet resistentiam totam quam haberet in vacuo duplicatam, ultra scilicet resistentiam medii.

71 Quod tamen A posset moveri in duplo velocius quam nunc movetur in medio dato – quod sit C, probo sic quod fortius potentia <rei> motae <maioratur> vel maioratur sua quantitas quousque mediam aequat velocitatem A in C medio. Tunc arguo sic: motus A et B in velocitate sunt aequales, ergo in istis mobilibus aequalibus, si dematur unum, quae sint remanentia, erunt aequalia per communem enim conceptionem; ergo ab aliqua alia parte resistentiae medii quae est similis utrique motus <erunt aequales> quousque B moveatur in duplo; sequitur quod tunc motus A erit aequalis motui B. Consequens falsum, ut est prius probatum, cum A in minimo medio potest moveri in duplo velocius quam nunc movetur in C.

72 Immo quod plus est: si A poneretur in vacuo ubi haberet resistentiam subduplam praecise, non movetur in duplo velocius quam nunc movetur immo plus quam in duplo velocius. Cuius demonstratio talis est: A nunc habet proportionem sexquialteram ad suam totam resistentiam, sicut tria habent se ad duo, ergo dempta medietate resistentiae suae sit ablata tota resistentia medii, et posito A in vacuo se habebit ad suam resistentiam in proportione tripla sicut tres ad unum, quae proportio maior est quam sexquialtera duplicata, <quae> dupla est praecise ad unam sexquialteram, et proportio tripla maior est quam proportio sexquialtera duplicata. Quod apparet exemplum in numeris, nam proportio sex ad quattuor est sexquialtera, proportio novem ad quattuor est sexquialtera duplicata, ut per praecedentia satis constat, et proportio tripla est duodecima ad quattuor, quae maior est quam proportio novem ad quattuor, sicut in vacuo A mobile <movetur> plusquam in duplo velocius quam movetur in C medio.

73 Et si accipiatur aliquid grave mixtum excedens totam resistentiam intrinsecam et extrinsecam C medii in proportione tripla, ita quod resistentia intrinseca et extrinseca sunt aequales, et movetur D mixtum in medio, quod mixtum excedit totam suam resistentiam congregatam sicut sex excedunt duo, tunc si movetur illud mixtum, quod sit D in vacuo movebitur minus quam in duplo velocius, quoniam in vacuo se habebit ad suam | resistentiam in proportione 85vb sextupla, sicut sex ad unum, quae proportio non est dupla ad triplam immo minus quam dupla ad istam, nam proportio nonupla est praecise dupla ad triplam, ut per praecedentia demonstratum est.

<Sexta conclusio>

74 Similiter sequitur haec conclusio, ut videtur quod aliquid movetur in pleno et in vacuo aeque velociter. Quia recento casu praedicto de A quod movetur in C medio, ubi excedit totam suam resistentiam congregatam in sextupla proportione, tunc si accipiatur aliquod mixtum habens gravitatem duplicem ad gravitatem A et similiter levitatem duplicem ad levitatem A, quod mixtum sit B, sequitur quod B movebitur velocius in C medio quam A, quia potentia sua motiva ad suam resistentiam est eadem, quoniam eadem est proportio dupli ad duplum et subdupli ad subduplum. Et similiter potentia motiva B magis excedit resistentiam C medii quam potentia motiva A, quia potentia motiva B dupla est ad potentiam motivam A per totum, ergo potentia motiva B plus excedit suam resistentiam totam extrinsecam et intrinsecam quam potentia motiva A suam resistentiam congregatam excedit; ergo B movebitur velocius in C medio quam A, sit, gratia exempli, quod in sexquialtera proportione. Accipiatur igitur D mixtum habentem consimilem proportionem gravitatis ad levitatem sicut habet A vel B, quod mixtum consimiliter se habebit ad B et in tantum excedet B sicut B <excedet> A; tunc sequitur quod in C medio D movebitur velocius B in eadem proportione in qua B movetur velocius A in sexquialtera proportione duplicata.

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16 sequitur] *corr. ad sensum, ms.* similiter 17 motiva] *corr. ad sensum, ms.* movetur

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1 si…10 est] Cf. Bradwardinus, *op. cit.*, 114, 98—126.

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11 Similiter] sextum argumentum *marg.*

75 Item, eadem ratione, si esset quartum mixtum sic se habens ad D sicut D ad B movetur in duplo velocius B in eadem proportione in qua D velocius B; ergo per multiplicationem talium mixtorum convenerit devenire ad aliquod mixtum habens aequalem proportionem gravitatis ad levitatem sicut A, quod movetur in triplo velocius in C medio quam A, quod mixtum sit G. Tunc exempla ista mixta A B D G moverentur aeque velociter in vacuo quia haberent eandem proportionem gravitatis ad levitatem sicut A quod moveretur in triplo velocius in vacuo quam faciat in C medio, quia haberet in vacuo praecise subduplam resistentiam. Ergo sequitur quod G non possit moveri in triplo velocius quam facit A in C pleno, ut est prius probatum, <et> G movetur in triplo velocius quam A in eodem, sequitur ergo quod G moveretur velocius in C pleno quam faceret in vacuo, quod est prius intentum, et ex quo sequitur intentum scilicet quod aliquis posset ita moveri aequaliter in pleno et in vacuo.

<Septima conclusio>

76 Septimo sequitur haec conclusio, quod aliquid movebitur infinita
86ra | velocitate, si velocitas motus sequatur proportionem. Quoniam si descendat grave in aliquo medio pertransita medietate suae resistentiae movebitur in duplo velocius quam fecit in principio, quia habebit tunc subduplam resistentiam. Per consequens habebit tunc proportionem duplam praecise ad suam resistentiam. Sed in principio motus se habuit ad totam suam resistentiam in proportione dupla praecise; ergo, eadem ratione, pertransita secunda parte proportionali sui spatii movebitur in quadruplo velocius, quoniam tunc habebit subquadruplam resistentiam, ergo, eadem ratione, sic deinceps in infinitum: aliquando movebitur dupla velocitate ad velocitatem qua movebatur in principio, et aliquando tripla, et aliquando quadrupla, et sic in infinitum; ergo infinita velocitate movebitur antequam attingat suum locum.

77 Similiter concedo istam opinionem, si ly 'infinita' accipitur syncategorematice. Sed ex hoc non sequitur, quod tandem movebitur infinita velocitate, quia ly 'infinita' a parte praedicati tenetur semper categorematice.

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1 si…16 vacuo] Cf. Bradwardinus, *op. cit.*, 116, 127—140.

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17 Septimo] septimum argumentum *marg.*

78 Contra, probo istam consequentiam per modum arguendi Aristotelis III *Physicorum* textu commenti 60 et Commentatoris illo commento ubi sic arguit: si aliquod corpus posset habere aliquam quantitatem duplam ad istam et triplam ad istam, et quadruplam, et sic in infinitum, sequitur quod tamdiu posset esse infinitum in actu. Ita arguo in proposito: iste motus gravis simplicis descendentis potest habere duplam velocitatem ad istam quam nunc habet, et triplam, et quadruplam, et sic in infinitum, et sic habebit de facto, igitur tandem illa velocitas est infinita in actu.

79 Consimiliter etiam arguit Philosophus et Commentator I *De coelo* textu commenti 87 et 88 et illis commentis, quia si terra moveretur per distantiam infinitam ad suum locum, quod tandem moveretur motu infinitae velocitatis in actu, quia quanto est propinquior loco suo tanto moveretur velocius.

80 Illud ergo argumentum confirmo per illam rationem: si accipiatur tota velocitas qua movebitur ista terra pura antequam attingat suum locum, posito quod moveretur ad istum locum solum per distantiam finitam, adhuc tota velocitas quae terminabitur ad ultimum instans illius motus non habebit certam proportionem nec finitam ad aliquam velocitatem uniformem, quoniam velocitate data non erit dupla quoniam post erit tripla et quadrupla et sic in infinitum, ergo in infinitum excedit quamlibet velocitatem uniformem; et per consequens sequitur quod sit infinitum in actu.

<Instantia> de velocitatione motus gravis descendentis

81 De illa velocitate motus gravis descendentis diversi diversas causas assignant. Quidam dicunt quod velocitas in suo descensu est

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2 Aristotelis…5 actu] Cf. Arist., *Phys.*, III, 207b1—6, 130, 3—8; Aver., *In Phys.*, III, com. 60, 113vb. | 6 iste…9 actu] Cf. Kilvington, *Comm. in De gen. et corr.*, ms. Brugge, Stedelijke Openbare Bibl. 503, 31rb: Concedo quod haec propositio erit possibilis: aliquid movetur infinita tarditate et infinita velocitate. | 10 Philosophus…11 commentis] Cf. Arist., *De coelo*, I, 277a26—33; Aver., *In De celo*, I, com. 88, 57vb. | 25 velocitas…37,1 resistentiae] Cf. Arist., *De coelo*, II.7, 289a22—28; Simplicius, *In Aristotelis De coelo libros commentaria*, vol. 7, 264—267; Buridanus, *Quaestiones in Aristotelis 'De coelo'*, Lib. II, qu. 12, 441.

propter minoritatem suae resistentiae, et alii propter propinquitatem ad suum locum, et tertii propter continuationem sui motus, et quarti propter gravitatem accidentalem acquisitam, et quinti propter pulsum medii.

86rb **82** Contra tres primas opiniones argui | potest coniunctim per argumentum ultimo adductum. Ergo si duo gravia aequalis virtutis descendant in eodem medio, et unum incipiat a loco superiori et aliud a loco inferiori adhuc cum fuerint inaequaliter distantia a terra non aeque cito attingent ipsam, ut experimento notum est. Et ideo dicunt quidam, quod velocius movetur ista terra quae superius incipit moveri, quando istae duae terrae aequaliter distent a centro mundi. Et dicitur quod hoc est propter continuationem motus. Ideo dicunt, quod non oportet talem motum velocitari in duplo, et in triplo, et in infinitum, quia continue medium sub ipso descendente plus condensatur, ideo plus resistit aequali quantitati medii inferius quam superius.

83 Contra: ex ista opinione sequitur quod motus coeli et horilogii continue velocitatur propter continuationem motus.

84 Item, propter continuationem motus non sequitur quod sit maior proportio motoris ad motum, ergo non sequitur quod motus sit velocior per talem continuationem.

85 Nec erit velocitas motus gravis propter gravitatem accidentalem, ut ponit quarta opinio, quia omnis motus est causa caloris, ut II *De*

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3 accidentalem] *corr. ad sensum, ms.* actualem 8 inaequaliter] *corr. ad sensum, ms.* in quasi 18 continuationem] *corr. ad sensum, ms.* continentiam 22 accidentalem] *corr. ad sensum, ms.* actualem

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1 propter[2]…2 locum] Cf. Arist., *De coelo*, III, 301b18—30; I.8, 277a27—b8; Aver., *In De celo*, I, com. 88, 160, 10—12; II, com. 35, 333, 93—95; Aquinas, *In libros De coelo et mundo*, 050CM, 1b1, 1c17, vol. 4, 14; Buridanus, *Quaestiones in...'De coelo'*, Lib. II, qu. 12, 440. | 3 propter[1]…4 medii] Cf. Gualterus Burlaeus, *Super Aristotelis libros de physica auscultatione...commentaria*, Venetiis 1589, Lib. VIII, 1099—1100: Intelligendum quod illud quod communiter dicitur, scilicet quod motus naturalis rectus intenditur in fine propter approximationem ad terminum motus, non est verum, quoniam grave non moveret velocius propter solam approximationem ad centrum (...) dicunt quidam quod grave in descendendo continue acquirit novam gravitatem accidentalem (...) medium continue gravius et gravius subsequitur ipsum continue fortius pellens ipsum. | 23 omnis…38,1 82] Cf. Arist., *De coelo*, II.7, 289a22—28; Buridanus, *Quaestiones in ...'De coelo'*, Lib. II, qu. 12, 439.

coelo commento 82, ergo terra descendens continue acquirit levitatem accidentalem, quia continue sicut calefit sic levefit.

86 Nec videtur propter pulsum medii, ut ponit quinta opinio, quia cum post terram descendentem continue relinquitur vacuum et medium sequens, secundum Ieronum, continue corrumpitur in illis, adhuc velocitatur motus gravis propter valescere resistentiae et non propter pulsum medii.

87 Item, velocior pulsus medii praesupponit velociorem motum descendentem, et velocior motus descendens praesupponit velociorem insecutionem medii, ergo idem praesupponit se.

88 Item, tunc terra magis violenter <descendit> in loco inferiori, si illi demoveretur, quam in loco superiori, quia in loco inferiori movetur velocius et plus inclinat ad suum <locum naturalem>. Consequens falsum, quia locus superior magis distat a suo loco naturali.

89 Quod autem sic aliqua velocitas sit infinita in actu, loquendo de infinito categorematice, probo. Quia suppono quod aliquod grave motum descendat continue in medio densiori versus inferius sed continue uniformiter densitante, quousque quiescat in medio propter eius densitatem. Capio totum medium pertransitum ab illo gravi antequam quiescat, et volo quod illud medium sic disponitur quam superius eius medietas sit alicuius densitatis, et secunda pars proportionalis <sit> in duplo subtiliori densitate, et tertia pars in triplo subtiliori, et sic in infinitum, et quod medium sic dispositum sit B, et grave quiescens | sub illo sit A. Et pono quod aliquid aliud 86va grave in duplo gravius A, quod sit C, descendat per B medium quousque tangat A. Tunc infinita velocitate movebitur antequam tangat A et velocius movebitur cum tetigerit A quam umquam prius movebatur; ergo cum C tetigerit A, movebitur infinita velocitate in actu. Maior patet per casum. Minor patet, quia quodcumque pellens quantumcumque debile potest pellere A versus inferius, quia quantumcumque modicum fortificetur A posset descendere ad aliquam partem medii, ergo <partem medii> suppositi penetrare, quia A quando quievit, fuit fortissimum grave quod non potuit aliquam partem medii sibi suppositi dividere, ut suppono et ut sequitur ex casu; ergo quantumcumque grave potest manere cum A

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2 accidentalem] *corr. ad sensum, ms.* actualem

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16 Quod] argumentum quod aliqua velocitas sit actu infinita *marg.*

resistentia et cum minima resistentia B medii poterit quodcumque grave movere, ergo C cum tetigerit A, movebitur velocius quam umquam movebatur pertranseundo B, et per consequens infinita velocitate movebitur C cum tetigerit A, loquendo de ‘infinito’ categorematice, quod est intentum.

Ad oppositum quaestionis

90 Ad oppositum huius quaestionis, secundum Aristotelem et Commentatorem IV *Physicorum* capitulo de vacuo textu commenti 71, VII *Physicorum* textu commenti 33 et 36 illis commentis, et II *De coelo* commento 6, introducatur illud argumentum contra quartam opinionem quae ponit quemlibet excessum sufficere ad motum <dictam> in principio quaestionis.

<Determinatio quaestionis>

91 Ad illam quaestionem quando quaeritur “utrum in omni motu etc.” dico quod accipiendo motum indifferenter pro quacumque transmutatione reperta in qualitate, quantitate, ubi, et accipiendo potentiam motoris pro potentia activa et potentiam rei motae pro potentia passiva, et loquendo de excessione prout se extendit ad quemcumque excessum in quantitate vel qualiter repertum, secundum quod omne excedens dividitur in excessum et in illud per quod excedit, (sicut loquitur Commentator capitulo de vacuo commento 71), et ad excedens non prout aliquod agens dicitur excedere passum resistens, quia potest capere motum ubi resistens non potest impedire necesse motus secundum quod acceleratur, et in mobilibus aliis ubi movens non excedit mobile divisibiliter, sed iste excessus et ista proportio est indivisibilis, (ut patet VII *Physicorum* commento 39); suppositis ergo quibusdam terminis communibus de motu et de potentia activa et passiva, ut est prius dictum et de excessu communiter disiunctivo, tamen prout se extendit ad utrumque membrum prius positum, quaestio est vera, sic
86vb intendo quod in omni | motu potentia activa motoris excedit

7 Aristotelem…9 71] Cf. Aver., *In Phys.*, IV, com. 71. | 9 VII…commentis] Cf. Aver., *In Phys.*, VII, com. 33, 333rb. | II…10 6] *Locus erronee indicatus, non inventus.* | 20 Commentator…21 71] Cf. Aver., *In Phys.*, IV, com. 71. | 25 VII…26 39] Cf. Aver., *In Phys.*, VII, com. 39. | 30 in…40,1 proprie] Cf. Bradwardinus, *op. cit.*, 108, 472—476, 493—511.

6 Ad…quaestionis] *marg.* | 13 Ad] Decisio(*sic!*) quaestionis *infra columnam*

communiter vel proprie, divisibiliter vel indivisibiliter, iuxta intellectum prius dictum, potentiam resistivam seu passivam rei motae.

92 In ista quaestione dissolvenda sunt quattuor articuli principaliter declarandi. Primus est de excessu sufficiente ad motum numquid quilibet excessus sufficiat ad motum. Secundus est de potentia activa qualiter debet terminari per suum finem. Tertius de potentia passiva qualiter ipsa habet terminari. Quartus est, numquid velocitas motus sequatur proportionem motoris ad rem motam vel excessum.

Pro primo articulo

93 Pro primo articulo dico, quod quilibet excessus sufficit ad motum. Et haec opinio sufficienter probari potest per argumenta adducta in primo et principali articulo quaestionis.

94 Ad primum argumentum quod fuerit argumentum contra istam opinionem – respondeo concendendo conclusionem ad quam deducitur, videlicet quod "quaelibet potentia pulsiva, quantumcumque debilis fuerit, potest movere totam terram cum toto iuvamento quod habebit ex una parte terrae, supposito quod medietates sint aeque graves et supposita sphaeritate terrae." Unde dico quod sic est ibi sicut est in aequilibra aequalibus appensis ponderibus: quaelibet potentia gravis pulsiva potest alterum bracchium deprimere, ita in terra medietas sic se habet ut ponderosa in aequilibra. Unde si medietati aliquid gravitatis addatur plus quam alteri, ex parte graviori fiet descensus sicut in bracchiis aequilibrae. Istud videtur satis consonum processui Philosophi II *De coelo* locis praeallegatis, ubi vult quod maior gravitas semper compellit minorem quousque tanta gravitas sit ex una parte centri, scilicet quanta ex alia.

95 Et ultra ad aliam formam quando arguebatur: "ex ista sequitur quod terra esset in continuo motu propter hoc, quod sol continue magis calefacit et exsiccat unam partem magis quam aliam" – non reputo, inconveniens conclusive concedere quod terra movetur sic millies in die. Verum tamen non est necessarium quod semper ita moveatur, quod potest esse quod cum sunt impedientia quod aliquando impeditur terra – alio motu non obstante – maiore

16 quaelibet…19 terrae] Cf. § 28. | 29 ex…31 aliam] Cf. § 20.

10 Pro…articulo] *marg.* | 14 Ad…argumentum[1]] *marg.*

allevacione terrae ex una parte quam ex alia vel propter maiorem densitatem aeris vel eius difformitatem vel aliquorum aliorum impedientium. Sed dico, omnibus aliis paribus concedi potest conclusio et iuxta sententiam Philosophi, quod terra est centrum in continuo motu. Et quando dicitur, quod Philosophus vult in *De motu animalium*, quod omne motum in suo motu necessario indiget fixo, ideo centrum indiget terra quiescente, dico quod intelligit quod terra quiescit a tali motu circulari quali movetur coelum. Non enim volo ponere, quod terra moveatur motu circulari aeternaliter circa suum centrum, sicut posuit Plato, de terra consimiliter sicut de igne probat, recitatur sua opinio II *De coelo,* commento 96. Sed dico,
87ra quod ceteris <paribus> terra | movetur aeternaliter dicendo alterius motibus secundum partem graviorem, verum tamen motu valde tardo et insensibili. Et quiescit terra a motu sensibili, non obstante quod insensibiliter moveatur. Et haec opinio satis videtur sequi ex sententia et processu Philosophi II *De coelo* versus finem illis locis allegatis. Dico etiam ultra, quod si terra naturaliter debet esse sphaerica est tam vallosa et in quibusdam partibus plana, propter convenientiorem habitudinem animalium, sicut aqua congregata in certis locis non obstante quoddam terram undique circumdare, ita quod ista sunt quoddammodo violenta et aliqualiter propter

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5 Philosophus…7 quiescente] Cf. Arist., *De motu animalium*, 698a14—16. | 7 intelligit…8 coelum] Cf. Buridanus, *Quaestiones in ...'De coelo'*, Lib. II, qu. 22: *Utrum terra semper quiescat in medio mundi*, 507, 31—35: Tunc est ultima dubitatio (...) scilicet 'coelum semper movetur circulariter, ergo terra debet semper quiescere in medio'. Dico ergo quod sic debet quiescere quia non debet moveri circulariter, nec etiam tali motu recto quin semper medium gravitatis debeat manere medium mundi. | 11 II…96] Cf. Aver., *In De celo*, II, com. 96, 453, 25—28. | 12 terra…14 insensibili] Cf. Buridanus, *Quaestiones in ...'De coelo'*, Lib. II, qu. 22: *Utrum terra semper quiescat in medio mundi*, 507, 16—25: Et per hoc solvitur alia dubitatio, scilicet utrum terra aliquando moveatur secundum se totam motu recto. Et possumus dicere quod sic, quia continue de ista terra altiore cum fluviis fluunt multae partes terrae ad profundum maris, et sic augetur terra in parte cooperta et diminuitur in parte discooperta; et per consequens non remanet idem medium gravitatis sicut ante fuit. Modo ergo, mutato medio gravitatis, illud quod de novo factum est medium gravitatis , movetur ut sit medium mundi, et illud quod ante erat medium gravitatis, ascendit et recedit; et sic elevatur tota terra versus partem discoopertam, ut semper medium gravitatis fiat medium mundi.

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20 circumdare] nota quod aliquid violentum potest esse perpetuum *marg.*

naturam, non tamen violenta propter peius sed propter melius, et talia violenta sunt sive possunt esse aeterna. Sed alia violenta corrumpentia vel destruentia, quae sunt propter peius et non propter melius, non sunt aeterna. Et sic intelligit Aristoteles II *De coelo* textu commenti 17, 18 et 19 et Commentator sic intelligit illis commentis.

96 Et ad argumentum quando arguebatur quod ex ista opinione sequitur "quod aliqua moventia essent infinita in actu" – dico quod hoc non sequitur ex supposito casu. De A B C – dico quod C non movebitur infinita velocitate cum tetigerit A nec movetur velocius quam umquam movebatur in B.

97 Et ultra quando dicebatur quod "quodlibet grave cum tanto iuvamento supposito ipsi A potest pellere ipsum A versus deorsum" – dico quod verum est, quod potest ipsum movere per accidens. Nam non quodlibet erit per se movens sed partibile, ita quod ipsum et A coniuncti possunt unam partem dividere medii sibi suppositi. Dico quod quodlibet grave, cum tanto iuvamento quantum iuvat, posset aliquam partem proportionalem medii suppositi dividere. Unde non dico quod quodlibet grave additum ipsi A moveatur infinita velocitate. Nam aequalibus appensis in aequilibra potest quodcumque grave additum alteri appenso deprimere ipsum; ex hoc tamen non sequitur quod duplum grave ad grave appensum deprimeret ipsum infinita velocitate.

98 Sed fortius posset argui ad eandem conclusionem supposito de B ut prius, et quod C sit unum grave simplex descendens per ipsum B, cui supponitur unum aliud medium, quod sit A, consimilis dispositionis sicut est B, ita tamen quod subtilior pars A superior sit, subtilior pars B sibi opposita est inferior, et distant ista duo media A et B per vacuum interceptum aequale ipsi C. Isto supposito: C potest moveri quousque pertransierit tantum B eo quod semper aliqualem resistentiam habebit ab ipso B quamdiu aliqua pars ipsius C erit in ipso B, et cum fuerit directe in medio inter B et A, ita quod una eius superficies continguetur cum B et alia cum A, tunc C movebitur

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4 Aristoteles…6 commentis] Cf. Arist., *De coelo*, II, 286a18—20; 286a31—33, 296a24—34; Aver., *In De celo*, II, com. 17, 301, 136—142; com. 19, 303.

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7 Et…argumentum] responsio quando arguebatur quod aliqua sit velocitas infinita *marg.*

infinita velocitate. Quod probo sic, quia tunc habebit minorem resistentiam quam numquam prius habuit; sed infinita velocitate 87rb prius movebatur et nunc velocius quam umquam | prius movebatur cum habeat minorem resistentiam nunc; ergo sequitur quod nunc movetur infinita velocitate in actu et infinita velocitate movebatur ante hoc et infinita movebitur immediate post hoc, ergo nunc movetur velocius quam immediate ante hoc movebatur vel immediate post hoc movebitur.

99 Si respondeatur ad hoc concendendo conclusionem ad quam deducitur quod nunc movetur infinita velocitate, sed ista velocitas solummodo durat per instans, nec potest durare per tempus quia si duraret per tempus per ipsum transiretur spatium infinitum in tempore finito, quod est impossibile – contra istam responsionem arguo sic: nunc est aliqua proportio inter C et suam resistentiam. Possibile est quod continue maioratur sua potentia motiva ipsius C sicut continue maiorem resistentiam habebit, ita quod continue per aliquod tempus maneat eadem proportio ipsius C ad suam resistentiam qualis fuit proportio qua C incepit dividere ipsum A. Vel aliter potest dici quod C movetur, cum fuerit in medio inter A et B, quia tunc totaliter est in vacuo; sed immediate ante fuit aliqua pars eius in pleno et immediate post hoc erit aliqua pars eius in pleno, et ideo immediate movebatur, et tunc non movetur, sed immediate post hoc movebitur infinita velocitate non categorematice sed sincategorematice, quia non tanta quam maiori.

100 Praeter istud argumentum fuit una difficultas in principio quaestionis inducta incidenter scilicet: “numquid generans tantum tribuat de loco quantum de forma”. Et dico quod non oportet nec in pleno nec in vacuo, quia generatum potest habere formam ignis in summo antequam habuerit locum naturalem ipsius, et hoc propter densitatem ipsius medii, quia densitas potest impedire velocitatem motus localis, etsi non tantum impediat velocitatem generationis nec in vacuo. Quia si esset vacuum inter centrum et concavum orbis lunae, ignis appropinquaretur terrae purae iuxta centrum inter terram <et concavum orbis lunae>, terra ista – quacumque levitate inducta – immo non ascenderet antequam levitas dominaretur supra gravitatem, sicut accidit de oleo inflammato ascendendo.

25 Praeter] numquid generans tantum tribuat de loco quantum de forma *marg.*

101 Ad auctoritatem Commentatoris, quae videtur esse satis expresse contra istam opinionem, nam VIII *Physicorum* commento 8 dicit sic Commentator: "quando ignis generatur secundum totum, statim habet ubi superius secundum totum, et cum generatur pars singula habet statim singulam partem illius ubi, verbi gratia, in ligno combusto aut oleo inflammato". Et in eodem VIII commento 32 dicit sic: "generans est illud, quod dat corpori simplici generato formam suam et omnia agentia continuanda formam, quorum unus motus est in forma, et ideo cum forma fuerit completa in eo quod implebitur ubi suum determinatum et accidentia alia requisita, nisi aliquid impediat." Et III *De coelo* | commento 38 dicit sic: aequalia 87va
mota in loco a generante <moventur> et cum in eis fiat aliqua forma a generante constituta in illa parte esse quae constituitur ex loco primo et aliis accidentibus propriis nisi aliquid impediat, "verbi gratia, cum oleum fit ignis statim in prima parte eius in qua consistit igneitas existunt omnia accidentia existentia in igne, secundum quod est ignis et unum illorum accidentium est motus ad superius". Et IV *De coelo* commento 22 dicit Commentator, quod "cum aliqua pars olei fiat ignis statim, cum forma ignea advenerit, acquiret de locali motu secundum quod acquirit de accidentibus, et cum completa fuerint accidentia alia, complebitur in eo motus in loco nisi aliquid impediat a motu".

102 Pro omnibus istis dico, quod si intelligantur quod si generatum aliquid acquirat de forma, tunc ista forma acquisita naturaliter appeteret locum sibi proportionabilem et ad illud moveretur nisi impediretur. Sed aliquando impeditur in pleno propter densitatem medii, et in vacuo posset impediri propter formam contrariam, nam tamdiu impeditur forma ignis ab ascensu quamdiu forma terrae secum coniuncta est fortior et intensior. Et quam cito dominabitur forma ignis super formam terrae, tunc ascendet totum vel pars deductis aliis impedimentis, ut patet de combustione ligni et inflammatione olei, locis praeallegatis per exempla Commentatoris.

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16 existunt…existentia] *corr. ad sensum, ms.* exeunt intra existentiam

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2 VIII…6 inflammato] Cf. Aver., *In Phys.*, VIII, com. 4, 341ra. | 6 VIII…11 impediat] Cf. Aver., *In Phys.*, VIII, com. 32, 370vb. | 11 III…17 superius] Cf. Aver., *In De celo*, III, com. 28, 555, 135—144. | 17 IV…22 motu] Cf. Aver., *In De celo*, IV, com. 22, 694—695, 111—118.

103 Ad aliam formam quando supponebatur, quod “mixtum ex terra et igne, in quo sufficienter dominatur ignis ad corrumpendum terram secum admixtam in die naturali, ascendat in vacuo infinito, ita quod corrumpantur partes proportionales terrae et <maioretur> velocitas motus ipsius mixti secundum proportionem partium propotionalium terrae” – dico quod hoc est impossibile, quia nullus terminus signabilis partibus proportionalibus potest motus velocitari quadam proportione istarum partium, potest tamen velocitari secundum minorem proportionem partium terrae proportionalium. Et similiter potest tardari in partibus proportionalibus temporis et etiam super partes proportionales spatii quacumque proportione. Et si tardetur motus super partes proportionales spatii secundum proportionem istarum partium, spatium numquam erit pertransitum. Sed si tardetur motus in partibus proportionalibus temporis secundum proportionem istarum, pertransitur aliquod spatium. Unde etsi motus non poterit velocitari secundum proportionem partium proportionalium temporis in singulis partibus proportionabilibus ipsius temporis, potest tamen motus velocitari super omnes partes proportionales spatii secundum proportionem istarum partium. Omnes istae propositiones patent diligenter quaerentibus.

104 Ad aliam formam de mixto aequaliter de terra et de igne – dico quod tale quiesceret ex se ubicumque poneretur. Et quando Commentator dicit I *De coelo* commento 7, quod “impossibile est <enim in corporibus compositis> aliqua componi aequaliter ex contrariis” quia “si esset” possibile “contingeret quod aliquod corpus non moveretur omnino sed staret in quocumque loco poneretur, <sed staret aut superius aut inferius> aut staret in medio
87vb istorum | contrariorum et moveretur in ceteris, quod non invenitur” – dico quod Commentator intelligit de mixto perfecto ex quattuor elementis, quod impossibile est tale compositum ex contrariis aequaliter, nec invenitur de mixto corpore imperfecto quod componitur ex duobus contrariis, ita quod dua moveant sub ista definitione, nam medium circumstans magis iuvabit unum contrarium quam aliud. Et ad Commentatorem dico, quod Commentator loquitur de mixto imperfecto X *Metaphysicae*

24 Commentator…29 invenitur] Cf. Aver., *In De celo*, I, com. 7, 17, 27—32. | 36 Commentator…46,8 diminuitio] Cf. Aver., *In Metaph.*, X, com. 23, 71vb.

commento 24, ubi dicit contra Galienum sic, quod omnia componentia ex duobus debent habere ex altero istorum minus et ex reliquo istorum maius. Et hoc denotat ex suo sermone, quoniam impossibile est, quod composita ex contrariis sint contraria aequalia, sed alterum debet esse dominans. Et hoc demonstrat impossibile esse invenire commixtionem mediam aequaliter ex contrariis, ut existimavit Galienus, quoniam si ita esset nec accideret transmutatio nec diminuitio.

105 Hoc tantum de primo articulo.

De secundo articulo scilicet de potentia activa

106 Sequitur de secundo qualiter potentia activa habet terminari per suum ultimum. Et dico, quod debet per minimum in quod non potest.

107 Et quando dicitur "quod impotentia terminatur ad illum terminum, ergo potentia non terminatur ad eundem" – dico quod non sequitur, quia impotentia terminatur per illum tamquam per terminum intrinsecum, et potentia terminatur per eundem tamquam per terminum extrinsecum.

108 Et quando dicebatur ultra "quod ista dicta Philosophi videntur repugnare, quod potentia activa videtur terminari per maximum in quod potest et impotentia per minimum in quod non potest" – dico, quod Philosophus intelligit ista sic quod potentia activa terminatur per maximum in quod potest. Item dare est maximum in quod certa potentia data potest, ita quod in maius illo sensibiliter non potest aut notanter, et est dare minimum in quod illa potentia non potest, et in quodlibet notabiliter et sensibiliter minus potest et hoc cum certis circumstantiis. Et sic non repugnant ista dicta, et ista glossa satis convenit cum processu suo propter exempla quae potest demonstrare "per centum leucas et fere quinque librorum".

109 Et ad aliam formam: "accepto minore in quod non potest potentia data" – dico quod in quamlibet eius partem potest, si separetur a toto et per se poneretur.

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11 qualiter…13 potest] Cf. Buridanus, *Quaestiones in 'De coelo et mundo'*, 110, 6—8. | 23 dare…27 circumstantiis] Cf. <Pseudo-Duns Scotus>, *Ioannis Duns Scoti in octo libros Physicorum quaestiones et expositio*, qu. 15, 95a: potentiam terminari est ipsam finiri maximo in quo sic, vel minimo in quo non.

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10 De…activa] *marg.*

110 Et quando supponebatur ultra: “quod agens datum incipiat agere in illud minimum in quod non potest, quod sit B” – tunc dico, quod aget in ipsum B et continue tardius. Et si posset durare in infinitum numquam complete ageret per totum nec terminaret sicut A actio in istud passum, si tam agens quam passum fuerint aeterna. Quod tamen terminaret A actionem ad aliquod punctum B, hoc est per accidens quod non potest ulterius durare.

88ra **111** Et ad aliud quando dicitur ultra: “agat illud | agens in B tantum quantum potest, et reddit argumentum” – dico, quod non potest agere quantumcumque potest, quia si in aeternum duraret in aeternum posset agere in B. Ideo non potest agere tantum quantum potest agere, sed quantum potest agere in B tantum potest agere.

112 Et ad ultimam formam quando dicitur: “si B sit minimum in quod non potest A, tunc A est maximum quod non potest in B” – concedo. Et nego ultra consequentiam: “potentia resistiva B terminatur per maximum cui potest resistere et impedire totaliter eius actionem ne agat per totum B, ergo potentia activa terminatur per maximum simpliciter in quod potest.”

De tertio articulo

113 Sequitur de tertio articulo qualiter potentia passiva habuerit terminari. Dico quod intellectus Philosophi est, quod terminetur per minimum a quo potest pati cum certis circumstantiis.

114 Et quando arguebatur ultra, quod “non plus debet terminari per parvum quam per magnum, quia quantae virtutis est pati a minori in propinqua distantia cum aliis circumstantiis requisitis, tantae virtutis est pati a maiori et ulteriori distantia proportionabiliter” – concedo. Et ideo verum est, quod sicut potentia passiva terminatur per minimum cum certis circumstantiis, ita potest terminari per maius cum circumstantiis proportionabilibus. Et sic potentia activa terminari potest per minimum sicut per magnum cum certis circumstantiis proportionalibus. Verum tamen ex frequentiori usu loquendi terminamus potentiam activam tam per magnum quam per parvum, sicut quaerendo quam fortis est Socrates, terminamus magis eius potentiam dicendo quod potest tam magnum pondus levare quam quod potest movere minus pondus per magnam distantiam. Et ita de potentia passiva, ut si quaeratur quod bonum visum habet Socrates, terminamus eius potentiam passivam per ita

19 De…articulo] *marg.*

modicum quod vix posset videri, et sic intelligit Aristoteles. Concendendo tamen de virtute sermonis, dico quod est aliqua virtus eius passiva quae nec terminatur per minimum a quo potest pati similiter nec per maximum a quo non potest pati. Quia si esset passivum sic dispositum quod prima pars proportionalis esset alicuius resistentiae et illa subduplae resistentiae, et sic deinceps, tunc quodcumque agens posset incohare actionem ad illud extremum ubi terminantur omnes partes proportionales secundum primam progressionem.

115 Et ulterius potest concludi conclusio et satis rationabiliter, quia stramen cadens super lapidem durissimum aliquam partem potest dividere.

116 Et quando arguebatur quod "quaelibet gutta cadens super lapidem aliquam partem aufert' – concedo. Et ad Philosophum textu commenti 23 – dico quod intelligit sic, quod ultima gutta tangit lapidem sensibiliter et | praecedentes guttae disponunt ad 88rb
cavilationem sensibilem per ablationem partium insensibilium.

117 Et ad aliam formam quando arguebatur quod "si stramen cadens super lapidem durum aliquam partem dividat, tunc posset dividere lapidem per totum, tunc potentia resistiva sit minor et potentia activa sive divisiva aequalis" – dico tunc, quod potentia divisiva debilitatur in dividendo etiam debilitat resistentiam. Et si minoris extensive est passum per ipsam magis resistit intensive, sicut ligni modicum divisi latera inclinantur ut se iuvent, et determinent dividens ne ulterius possit dividere. Et ex hoc quod potentia divisibilitatis debilitatur in dividendo sequitur, quod non sit dare debilissimum dividens quod possit aliquam partem lapidis dati dividere, quia cum fuerit dividens ad medium partis dividendae, debilius erit quam in principio, et tamen aliquam partem dividet.

118 Et si dicatur, quod tale potest continuare actionem cum fuerit debilitatum, sed non potest actionem incohare in illud passum –

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31 incohare] *corr. ad sensum, ms.* continuare

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14 Philosophum…15 23] Arist., *Phys.*, VIII, 253b14—19. | 15 ultima…17 insensibilium] Julius Caesar Scaliger, *Exotericae exercitationes*, Parisii 1557, 35r: Hoc a praesceptore dicimus veritatis in libris Physicorum: sed manifestissime ubi loquitur de lapidis cavatione. Multae, inquit, cadentes guttae nihil auferunt de lapide, sed una tantum puta centissima, in aliarum virtute partem lapidis tollit , quae tamen tota simul aufertur, non autem particularim.

contra. Incohare actionem in illud passum non est in infinitum difficilius quam continuare, ergo si potest continuare actionem sub certa [actione] velocitate, potest incohare actionem sub aliqua minore velocitate. Et ideo concludo, quod quodlibet potens incohare actionem in aliquid passum potest continuare per totum, ubi potentia activa debilitetur in agendo, et potentia resistiva fortificetur ad resistendum. Ergo lignum quod est in dividendo, fortificatur ad resistentiam securis propter reclinationem laterum ligni detinentium securim.

119 Et ad ultimam formam quando augebatur: “si stramen descendens supra lapidem aliquam partem lapidis posset dividere et non totum, tunc vel esset dare maximam partem quam posset dividere vel minimam quam non posset dividere” – dico quod est dare maximam quam non potest dividere.

120 Et quando dicitur ultra quod “quamlibet partem illius partis potest dividere, ergo et istam partem” – patet responsio in secundo articulo.

121 Et ad aliam formam concludendo de re inanimata quod est dare ultimum instans sui esse – dico, quod Philosophus intelligit VIII *Physicorum*, quod non est dare ultimum <instans> rei permanentis in esse, cuius corruptio est subita, sicut accidit de corruptione hominis, ubi anima non <est> extensa nisi in materia, sed in corruptione rerum inanimatarum quae corrumpuntur ad corruptionem cuiuslibet partis. Et etiam in corruptione rerum animatarum in quibus anima
88va est extensa, sicut in anguilla est dare ultimum | instans esse sine permanentia, sicut si ignis approximetur parti anguillae in qua parte extenditur anima, tunc posito quod ista anguilla vocetur A, et immediate post hoc instans erit aliqua pars materiae corrupta et etiam aliqua pars formae a qua denominatur animal, ergo etc.

122 Ad ultimam formam – dico quod de virtute sermonis A non potest incipere dividere lapidem aut lignum, quodcumque agens forte fuerit. Quia si in quacumque divisione aliqua pars corrumpitur, tunc approximato dividente ad passum inanimatum immediate post hoc erit aliqua pars eius corrupta, et per consequens immediate post erit ipsum A corruptum, et nihil aget in ipsum. Verumtamen ad istum modum loquendi potest concedi, quod A agens potest incipere dividere passum inanimatum quod sit B; hoc modo potest incipere separare aliquas partes quae sunt partes ipsius B, post hoc modo tamen non essent partes ipsius B nec umquam post hoc erit ipsum B.

Sed de virtute sermonis nihil aget in B, si immediate post hoc erit corruptum.

De quarto articulo

123 Sequitur de quarto et ultimo articulo "numquid velocitas sequitur excessum vel proportionem." Et dico, quod sequitur proportionem.

<Ad primam conclusionem>

124 Et concedendo conclusionem primam ad quam deducitur quod alium duplum grave movetur plusquam in duplo velocius in eodem medio quam grave subduplum, ut posito quod grave supduplum excedat resistentiam medii in proportione minori dupla. Et alium grave duplum movetur minus quam in duplo velocius in eodem medio, ut posito quod grave minus excedat resistentiam medii in proportione maiori dupla. Et numquam movebitur duplum grave praecise in duplo velocius in eodem medio nisi quando grave subduplum excedat resistentiam medii in proportione praecise dupla.

<Ad secundam conclusionem>

125 Et sic dico ad secundam conclusionem, quod aliquando idem grave movebitur plus quam in duplo velocius in aere duplae subtilitatis quam in aqua duplae densitatis, ut si potentia motiva illius excedat potentiam aquae in proportione maiori quam dupla.

<Ad tertiam conclusionem>

126 Et per idem ad tertiam conclusionem dico, quod non oportet quod "si aliquis motor possit movere aliquod mobile per aliquod spatium in aliquo tempore, quod possit movere medietatem illius mobilis per duplum spatium in aequali tempore." Immo dico, quod Philosophus intelligit per medietatem mobilis talem partem mobilis quae habet se ad motum in proportione subdupla ad proportionem

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10 resistentiam] *corr. ad sensum, ms.* regulam

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13 numquam…16 dupla] Cf. Bradwardinus, *op. cit.*, Concl. 4, 112, 64—66. | 17 aliquando…20 dupla] Cf. Bradwardinus, *op. cit.*, Concl. 6, 112, 76—78. | 22 si…24 tempore] Cf. Arist., *Phys.*, VII, 250a25—28.

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3 De…articulo] *marg.* | 21 Et] nota totum quia utile *marg.*

totius mobilis ad eundem motorem. Et in aliis regulis intelligit per duplum agens habens proportionem <duplam> ad eandem 88vb resistentiam. Et sic intelligendo | verificentur omnes istae regulae, si cetera fuerint paria. Et sic loquitur Philosophus in locis praeallegatis, quod grave duplum movebitur praecise in duplo velocius in eodem medio, id est, grave habens praecise proportionem duplam ad resistentiam illius medii, et <haec est> regula proportionis inter grave subduplum et illud medium. Et per hanc glossam dissolvi poterunt conclusiones Archimedis et Iordani *De ponderibus* et Euclidis, quae sunt in contrarium allegatae.

<Ad quartam conclusionem>

127 Ad quartam conclusionem dico, quod duo motus duorum gravium aequalium quorum unus descendit in aqua pedalis quantitatis et aliud in aere bipedalis quantitatis duplae tamen subtilitatis, non sunt aeque veloces nec aequalis est proportio motoris ad suam resistentiam utrobique. Et quando dicitur quod "resistentia aeris et resistentia illius aquae sunt aequales" – dico quod non, quia non sunt aequales extensive, notum est, nec intensive quia iste aer propter eius subtilitatem non potest tantum resistere suo motori ad causandum motum retardari, sicut potest ista aqua propter eius densitatem propter hoc quod partes eius propinquius iacent et melius poterunt applicari ad causandum motum tardum.

128 Et ad aliam formam argumentandi, quando dicitur quod "medietas aeris dati est subduplae resistentiae ad aquam datam – concedendo <dico quod> intensive sed non extensive; et ista medietas uniformis est subduplae resistentiae ad suum totum extensive non tamen intensive. Et tunc non procedit argumentum: "tota enim resistentia aeris dati uniformis non est in duplo intensior quam resistentia suae medietatis", nec tota albedo uniformis est in duplo intensior sua medietate.

<Ad quintam conclusionem>

129 Ad quintam conclusionem, concedo quod aliquod mobile potest moveri velocius alio in aliquo medio denso, quod medium posset in tantum subtiliari quod idem mobile moveretur tardius quam illud

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5 grave…8 medium] Cf. Bradwardinus, *op. cit.*, 100, 323—328. | 8 Et…10 allegatae] Cf. Bradwardinus, *op. cit.*, 100—102, 338—343.

mobile quod prius movebatur velocius in medio densiori, si unum illorum mobilium sit simplex et aliud mixtum, sicut in quaestione sufficienter fuerat demonstratum. Si tamen utrumque mobile esset simplex, sic non contingeret.

<Ad sextam conclusionem>

130 Ad sextam conclusionem quod "nihil movetur aeque velociter in pleno, sicut posset moveri in vacuo"; et quando supponebatur casus de tribus mixtis A B D proportionalis compositionis motis in C medio – concludo quod B movebitur velocius A in C medio et D velocius quam B, sed in minori proportione velocius B quam B velocius A. Nec aliquod mixtum mundi proportionalis compositonis ex gravitate et levitate, hoc est habens eandem proportionem gravitatis ad levitatem sicut habet A, posset moveri in triplo velocius in C medio quam A, quia omnia talia mixta proportionalis compositionis aeque velociter moverentur in vacuo; sed A non potest moveri in duplo velocius in vacuo quam movetur in C medio cuius resistentiam excedit in triplo. Et similiter A excedit suam resistentiam intrinsecam in triplo et ideo excedit illas duas resistentias congregatas in sexquialtera proportione, sicut tria excedunt duo. Unde | si ponatur A in vacuo se habebit ad suam 89ra
resistentiam in proportione tripla sicut tria se habent ad unum, et proportio tripla est maior quam dupla sexquialtera, ut per exemplum in numeris patet, nam proportio sex ad quattuor est sexquialtera, et proportio novem ad quattuor est sexquialtera duplicata, quae est praecise dupla ad proportionem sexquialteram, et proportio duodecim ad quattuor est tripla, quae est maior quam dupla ad sexquialteram et minor quam tripla ad sexquialteram.

131 Et ad aliam formam per quam arguebatur quod "A posset moveri velocius in C medio, in quacumque proportione volueris, quia accipiatur illud grave simplex quod propter eius parvitatem moveatur aequali velocitate cum A in C medio, quod simplex sit B, tunc B potest moveri velocius in C medio, in quacumque proportione volueris, propter subtiliationem medii; ergo et A potest per consimilem subtiliationem" – dico, quod non sequitur eo quod A habet resistentiam intrinsecam et B non.

132 Et ad formam ultra: "illi motus sunt aequales in velocitate" – dico quod non sequitur. Immo ista communis autem conceptio "si ab aequalibus aequalia demas, etc." intelligitur de ablatione partis communis utrique aequalium. Sed medietas resistentiae medii non est pars motus, et ideo non procedit argumentum ex ista communi conceptione.

<Ad septimam conclusionem>

133 Ad aliam conclusionem et ultimam dico quod verum est: "infinita velocitate movebitur grave descendens" loquendo de infinito sincategorematice, tamen non movebitur infinita velocitate ponendo ly 'infinitum' a parte praedicati, quia tunc idem est ac si diceretur: "illud grave movebitur non finita velocitate", quia tunc non est verum.

134 Et ad modum arguendi Philosophi *Physicorum* textu commenti 60, ubi arguit quod, si esset aliquod corpus quod posset habere quantitatem duplam ad istam quam habet, et triplam et quadruplam et sic in infinitum, tunc posset esse infinitus actu – dico quod consequentia est bona accipiendo antecedens prout est de copulato praedicato. Accipiendo tamen antecedens prout est copula, consequens non valet de forma, at tamen tunc antecedens est falsum et impossibile, quia minimum corpus potest habere duplam quantitatem ad quantitatem totius coeli. Et sic ex isto antecedente tamquam impossibili sequitur conclusio, consequentia tamen possibilis. Et ita potest dici, quod Philosophus intelligit in I *De coelo* ubi arguit, quod terra movetur infinita velocitate si distaret per infinitam distantiam a suo centro et moveretur aliquando in duplo velocius et in triplo et sic in infinitum, secundum quod est propositio de copulato praedicato. Verumtamen possibile est in casu, quod aliquid movebitur in duplo velocius quam nunc et in triplo et sic in infinitum, secundum quod est copula. Numquam tamen erit haec propositio vera: 'hoc movetur in duplo velocius et in triplo et sic in infinitum quam prius movebatur', secundum quod est propositio de copulato extremo.

135 Et ad aliam formam quando dicitur "quod accepta tota velocitate qua terra pura movebitur per aerem uniformem antequam attingat suum locum, tunc ista tota velocitas est infinita in actu" – dico quod non. Et quando dicitur quod "tota ista velocitas terminata

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1 medietas…3 conceptione] Cf. Bradwardinus, *op. cit.*, 120, 223—241. | 10 Philosophi…13 actu] Cf. Arist., *Phys.*, III, 207b1—6 | 20 Philosophus…23 infinitum] Arist., *De coelo*, I, 277a | 30 quod…54,2 uniformem] Cf. § 80.

ad ultimum instans illius motus non habuerit aliquam certam proportionem ad aliquam | velocitatem uniformem” – dico quod 89rb quacumque velocitate uniformi accepta adhuc velocitas terminata ad ultimum motum praedicti excessit in infinitum istam velocitatem uniformem. Non tamen excessit illam in infinitum secundum quod ly ‘infinitum’ stat a parte praedicati, quoniam tunc stat categorematice, et ideo falsa est propositio.

<Ad instantiam>

136 De causa velocitationis motus versus deorsum in suo descensu – dico quod minoritas resistentiae est causa principalis, et pulsus medii aliquando est causa partialis et coadiuvans non tamen est causa necessario acquisita. Quoniam etsi post terram continue descendentem derelinqueretur vacuum, et medium quod deberet insequi foret continue corruptum in nihil, adhuc velocitaretur motus gravis propter minoritatem resistentiae et tamen propter pulsum medii.

137 Dico etiam quod continuatio motus aliquando est causa velocitationis motus, unde duo gravia aequalis virtutis descendentia in eodem medio quorum unum incipit a loco superiori et aliud a loco inferiori, adhuc illud quod incipit a loco <superiori>, cum fuerit in aequali distantia a terra cum illo quod incipit a loco inferiori, citius attinget terram quam illud quod posterius incipit moveri.

138 Quaedam argumenta in ista materia valent ad opposita, quae dissolvi potuerunt per iam dicta per illos articulos propositos. Plures sapientes opiniones contrarias et ambiguas habuerunt. Et ubi sapientes discordant, difficile est veritatem investigare, nam quaedam falsa sunt probabiliora, quaedam verum et falsum, quandoque propter apparentias veri plures iudicant esse verum. Ideo opiniones aliorum in hac materia diligenter studens dixi recitandas et ponderandas; hinc de rationibus dictarum opinionum velut ponderibus in bracchiis aequlilibrae, illae opiniones firmius teneantur quae evidentioribus rationibus poterunt confirmari.

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8 De] nota bene *marg.*

Guilelmus Heytesbury

De motu locali

Regulae solvendi sophismata

SIGLA:

A = Vatican, Biblioteca Apostolica Vaticana, Vat. Lat. 2136, ff. 24va-27va;
B = Brugge, Stedelijke Openbare Bibliothek 497, ff. 56ra-57rb;
C = Oxford, Bodleian Library, Cannonici miscellaneous 409, ff. 14rb-16ra;
D = Venezia, Biblioteca Nationale Marciana, lat. VIII 38(3383), ff. 66va-68vb;
E = Erfurt, Wissenschaftliche Allgemeinebibliothek, Amploniana Cms 2o 135, ff. 13rb-14rb;
G = Leipzig, Universitätsbibliothek 1370, ff. 35v-39v;
K = Krakow, Biblioteka Jagiellońska 621, ff. 40rb-43ra;
R = Erfurt, Wissenschaftliche Allgemeinebibliothek, Amploniana Cms 4o 270, ff. 27v-30v;
U = Padua, Biblioteca Universitaria di Padova 1123, ff. 60vb-62va.

Guilelmus Heytesbury
Regulae solvendi sophismata

<De tribus predicamentis: De motu locali>

1 Tria sunt praedicamenta vel genera in quorum quolibet contingit proprie motum esse: mutatur enim localiter, quantitative, et qualitative quodlibet quod movetur. Et cum universaliter motus quilibet successivus velox sit vel tardus, nec aliquid est idem univocum penes quod attendi poterit velocitas in his tribus, conveniens erit ostendere qualiter quaecumque huiusmodi mutatio quoad eius velocitatem seu tarditatem ab alia sui generis distinguatur.

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1 vel] etiam(?) R | quolibet] *om.* G | contingit] convenit C quolibet *add.* G 2 proprie] proprium U | proprie…esse] motum esse proprie DR | mutatur] movetur R | localiter] solum aut D | quantitative…qualitative] qualitative aut quantitative D | et] aut ABCGKRU 3 universaliter] uniformiter BD motus quilibet] *inv.* CH 4 quilibet successivus] *inv.* D | tardus] tardius ABCGR | nec…idem] nihil enim est G | aliquid] *om.* D aliquod K | aliquid est] quod sit C | idem] *om.* ABK 5 univocum] univoce B | quod] quid ABCDGU | poterit] potest CD | his] hiis R | tribus] generibus *add.* CD 6 conveniens] *om.* CD convenientiis K | erit] est DKRU esset G ostendere] assignare *sed. corr.* C *om.* R | qualiter quaecumque] qualitercumque R | huiusmodi mutatio] *inv.* ACDGKRU huiusmodi…quoad] praesens B 7 quoad…tarditatem] *om.* C | seu] eius *add.* B | ab] eius *add.* D | alia] altera B 8 distinguatur] quoad eius velocitatem vel tarditatem *add.* C

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1 Tria] Sextum capitulum *marg.* E

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1 Tria…movetur] *Cf.* Arist., *Phys.*, V.1, 225b6–9, f. 213vb–ra: Si igitur praedicamenta divisa sunt substantia, et qualitate, et ubi, et quando, et ad aliquid, et quantitate, et ipso agere, et pati, necesse est tres esse motus, eum qui quantitatis, et eum, qui qualitatis, et eum qui secundum locum est.

2 Et quia motus localis naturaliter praecedit alios tamquam primus circa ipsum in hac parte transcurrens saltem intentio terminis praemittatur.

3 Loci autem mutatio quamvis habeat diversas species et tam essentialibus quam accidentalibus differentiis pluribus varietur, ad propositum tamen sufficiet motum uniformem distinguere a difformi.

4 Motuum ergo localium dicitur uniformis quo aequali velocitate continue in aequali parte temporis spatium pertransiretur aequale.

5 Difformis quidem in infinitum variari potest et respectu magnitudinis et etiam quoad tempus.

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1 praecedit] omnes *add.* CD | alios] *lin.* C | primus] prius G 2 circa] tunc G | transcurrens] discurrens GKU | intentio] *om.* C | terminis] ceteris ADGKRU circa(?) B *om.* C 3 praemittatur] praemedicatur(*sic!*) K 4 Loci] localis G | habeat diversas] *inv.* ABCDGKRU | tam] in *add.* GU 5 essentialibus…accidentalibus] *inv.* D | quam] etiam *add.* ABKR etiam in *add.* G in U | accidentalibus] *om.* G | differentiis] et *add.* C 6 tamen] *om.* G sufficiet] sufficiat G sufficit K | uniformem] difformem R distinguere…difformi] a difformi distinguere D | a difformi] ab uniformi (motus motuum autem localium *sed del.*) R 8 Motuum] motus R | ergo] autem ARU | localium] ille *add.* A quidam *add.* K localis *add.* R iste *add.* U dicitur] *om.* BC uniformis et quidam difformis motus igitur localis dicitur *add.* K | uniformis] esse *add.* G | quo] quae B est *add.* D | aequali] aequaliter BE *marg.* C aequale in G aequaliter et aeque velociter K velocitate] velocius B 9 continue] quando C *lin.* R | parte temporis] tempore vel temporis parte A | temporis] ipsius *add.* G aequale *add.* *lin.* R pertransiretur] pertransitur AG | aequale] *om.* DR 10 quidem] autem CDK | in infinitum] infinities G | variari] etiam C variatur G | variari potest] poterit variari A poterit vel potest variari U | potest] poterit D 11 et] *om.* KR | tempus] in univocum *add.* *sed del.* R

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1 motus…primus] *Cf.* Arist., *Phys.*, VIII.7, 260a23–29, f. 394vb–395rb: Manifestum enim est quod, si necessarium quidem est semper motum esse, primus autem hic, et continuus est, quod primum movens movet secundum hunc motum, quem necessarium est unum, et eundem esse, et continuum, et primum. Tribus autem existentibus motibus, alio quidem secundum magnitudinem, et alio secundum passionem, et alio secundum locum, quem vocamus lationem, hunc necessarium est esse primum.

D 66vb E 13va **6** In uniformi | itaque penes lineam a puncto velocissime moto descriptam, si quis huiusmodi fuerit, | quanta sit totius magnitudinis motae velocitas universaliter metietur.

7 Et penes hoc quod punctus talis uniformiter seu difformiter mutat situm totus motus uniformis dicitur vel difformis.

8 Unde data magnitudine cuius punctus velocissimus uniformiter moveatur, quantumcumque difformiter residua omnia differantur, uniformiter moveri conceditur tota proposita magnitudo.

9 Posito nempe casu, quo motae magnitudinis nullus sit punctus velocissime motus, penes lineam quam describeret punctus quidam K 40va | qui indivisibiliter velocius moveretur quam aliquis in magnitudine illa data tota, totius velocitas attendatur, sicut posito quod continue incipiant corrumpi puncta extrema, aut quod nulla sint ultima puncta illius sicut accidit in linea girativa quae ponitur infinita.

10 Movetur enim omnis magnitudo mota localiter ita velociter sicut aliqua pars illius, aut sicut aliquis eius punctus.

U 61ra **11** Unde in casu illo indivisibiliter | velocius movetur talis magnitudo quam aliquis eius punctus.

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1 itaque penes] penes istam U | lineam] descriptam *add.* CK | velocissime] velocissimo GRU | moto] motu R 2 descriptam] *om.* CK | quis] qua K huiusmodi] hoc modo U | fuerit] fuit R | sit] sic K 3 motae] moti B velocitas] velocitatis B | universaliter] uniformiter BDG | metietur] mentietur B mensuratur CDU movetur R 4 Et] item G | punctus] pectus(*sic!*) U | punctus talis] *inv.* G 5 situm] totius *add.* A scitum B totius (magnitudinis *marg.*) *add.* C suum *add.* GK | totus] totius BKR totius/totus(*sic!*) U | dicitur…difformis] vel difformis dicitur R | difformis] dicitur *add.* B 6 punctus] pectus(*sic!*) U | velocissimus] velocissime motus B 7 moveatur] intenditur G | quantumcumque] quamcumque A quacumque B quandumcumque C | difformiter…omnia] omnia residua difformiter R | residua] reliqua A | omnia] *om.* B | differantur] maneantur K moveantur R 8 moveri conceditur] *inv.* G | tota] *marg.* C | proposita magnitudo] *inv.* ABCK 9 casu] in *add.* U | quo] quod K | motae] notae AB | motae magnitudinis] *inv.* K | magnitudinis] motus *add.* B nullus] illius *sed corr.* R | punctus] pectus(*sic!*) U 10 motus] tunc *add.* AD *om.* B | describeret] describit D | punctus] pectus(*sic!*) U 11 velocius] *marg.* C | aliquis] alius *add.* G 12 tota] *om.* DK | totius] magnitudinis *add.* C eius *add.* K | attendatur] attendetur ACR *om.* D attenditur K tenderetur U sicut] similiter B 13 puncta] *om.* C | extrema] extremalia R | ultima puncta] *inv.* A 14 puncta] pecta(*sic!*) U | accidit] accidit AKU 15 enim] autem B mota localiter] motu locali G 16 pars illius] *inv.* C eius pars G | aliquis eius] *inv.* U | eius punctus] punctus illius R | punctus] pectus(*sic!*) U

12 Et ideo | iuxta idem conceditur tamquam possibile quod A 24vb
G 36r continue | tardius et tardius movebitur | A mobile per horam, et R 28r
tamen continue per eandem horam erit ita, quod quilibet punctus illius qui movetur velocius movetur quam prius et intendit motum suum. Et similiter, quod B magnitudo per horam continue aequaliter movebitur et uniformiter saltem quoad tempus, et tamen quilibet punctus illius continue per eandem horam tardabit motum suum.

13 Primum per continuam corruptionem punctorum extremorum ipsius mobilis satis poterit verificari et secundum absque inconvenienti concedi poterit de linea girativa.

14 Potest etiam concedi tamquam imaginabile similiter quod A magnitudo per horam continue velocius et velocius movebitur, et continue erit ita per eandem horam quod quilibet punctus motus ipsius A remittit motum suum. Et hoc per adventum continuum
novarum partium secundum extremum velocius | motum. B 56rb

15 Et quamvis universaliter omnis magnitudo mota localiter ita velociter moveatur sicut aliqua eius pars, ex hoc tamen non sequitur quod etiam <ita> tarde moveatur sicut aliquid eiusdem. Tarditas enim est quasi privatio velocitatis, non obstante quod de virtute

1 Et] *om.* R | idem] illud ACDKRU 2 movebitur] movebant G movebatur K | A] *marg.* C *om.* K | per] A *add.* R 3 eandem] illam G punctus] pectus(*sic!*) U 4 illius] eius D eius mobilis G | qui movetur] *om.* R | velocius movetur] *inv.* R | movetur^{2}] movebitur D *marg.* E | et] sive U | intendit] intendet R 5 per…aequaliter] aeque velociter per horam G | aequaliter] velociter K | aequaliter] aeque velociter ABCDRU 6 et^{1}] etiam D | tamen] cum R 7 punctus] pectus(*sic!*) U | illius] eius CDG continue] *om.* CD | eandem] illam K | horam] continue *add.* D | suum] patet *add.* U 8 Primum] patet *add.* D puta R | continuam corruptionem] *inv.* C punctorum] pectorum(*sic!*) U | punctorum extremorum] *inv.* G 9 poterit] primum *add.* R potest U | verificari] videri D | et] etiam *add.* K 10 inconvenienti] inconvenientia K | concedi poterit] *inv.* G | de] B *add.* CD | girativa] conclusio *add.* U 11 etiam] enim R | etiam concedi] *inv.* B | similiter] simpliciter D *om.* K 12 per] istam *add.* G | et velocius] *marg.* C | et^{2}] tamen *add.* ABCDGKRU 13 continue…ita] erit ita continue U | ita] quod *add.* C | punctus] pectus(*sic!*) U | motus] *om.* BCG 14 A]continue *add.* A *om.* C | remittit] remitteret B remittet G 16 universaliter] *om.* B *marg.* C uniformaliter(*sic!*) G | ita velociter] *om.* C 17 moveatur] *om.* B | pars] sed *add.* A moveatur *add.* B | ex hoc] *om.* B ex…eiusdem] *marg. a.m.* C 18 etiam] *om.* BGU | ita] *om.* E | aliquid eiusdem] aliqua eius pars G aliquis eius punctus R | Tarditas] *om.* G sic(?) nec sicut(?) alia eius pars eiusdem tarditas *add.* U 19 enim…quasi] est enim tarditas G

sermonis omnis tarditas sit velocitas, et econverso, sicut quaelibet parvitas est magnitudo. Et universaliter ab habitu et perfectione sua denominanda est res quaecumque habens illam. Unde omne continuum quamvis quamcumque parvitatem habeat quam habet aliqua sui pars, non est tamen aliquid quod infinitae parvitatis dicitur. Consimiliter, quamvis omni tarditate qua movetur pars alicuius moveatur et totum, non tamen requiritur quod ita tarde moveatur totum sicut aliqua pars eius.

16 In motu autem difformi in quocumque instanti attenditur velocitas penes lineam quam describeret punctus velocissime motus, si per tempus moveretur uniformiter illo gradu velocitatis quo
C 14va movetur | in eodem instanti, quocumque dato.

17 Posito enim quod A punctus per horam continue intendat motum suum, non oportet quod in aliquo instanti totius horae attendatur velocitas illius penes lineam quam describit punctus ille in illa hora.

K 40vb **18** Ad hoc quod aliqua duo puncta seu aliqua alia mobilia aeque |
D 67ra velociter | moveantur non requiritur quod in aequali tempore

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1 sit] est BCG | econverso] econtra AKR | sicut] etiam quod U 2 parvitas] punctus *sed corr. a.m.* D | magnitudo] et econverso *add.* G | Et] *om.* G universaliter] enim *add.* GK | et] *om.* K | perfectione] dispositione G | sua] *marg.* C *om.* G 3 denominanda est] nominanda B | est] qualibet *add.* R | res] *om.* D | res quaecumque] quaelibet res C | quaecumque] est *add.* B | illam] eam ipsam C et etiam a sua perfectione *add.* G 4 habeat] *om.* D | habet] *om.* R 5 sui] eius C suas(*sic!*) K | sui pars] *inv.* G | pars] habeat *add.* D | est tamen] *inv.* G | parvitatis] parvitate B 6 dicitur] dicatur CG dicuntur D Consimiliter] dicitur *add.* K | omni] quacumque G | movetur] aliqua *add.* CG moveatur aliqua U 7 alicuius] *om.* D | moveatur] *om.* G movetur KR | et] ipsum A ipsum *add.* C | tamen requiritur] *inv.* D 8 moveatur] movetur K | pars eius] *inv.* ABCGRU | eius] ipsius DK 9 In] locali *add.* C | autem] locali *add.* AB 10 describeret] discribit U | punctus] pectus(*sic!*) U 12 eodem] *om.* D | quocumque] instanti *add.* C 13 punctus] *om.* D pectus(*sic!*) U | per…continue] continue per horam DGK | intendat] moveri *add.* B intendet KR 14 suum] isto gradu velocitatis *add.* C | aliquo] quocumque AG | instanti] totius lineae *add.* R | totius] illius *add.* C istius *add.* D | attendatur] attenderetur K attendetur RU 15 describit] describeret DGK discriberet U | punctus] pectus(*sic!*) U | punctus ille] *inv.* R | ille] motus *add.* C iste DGK *om.* U | illa] una G 16 hoc] enim *add.* ABCDGKRU | duo] moveantur *add.* B | puncta] pecta(*sic!*) U | aliqua[2]] duo *add.* G | alia] duo A duo *add.* BC | alia mobilia] *inv.* G 17 moveantur] *om.* B

aequales pertranseant magnitudines, sed stat quod inaequales pertranseant in quacumque | proportione volueris imaginari. Posito A 25ra enim quod A punctus continue uniformiter moveatur C gradu velocitatis per horam pertranseundo pedalem quantitatem, et B punctus a quiete incipiat uniformiter intendere motum suum in medietate illius horae acquirendo illum C gradum, et in secunda <medietate> uniformiter remittat ille ab eodem gradu ad quietem; tunc notum est quod in medio instanti totius horae movebitur B punctus C gradu velocitatis et aeque velociter omnino cum ipso A puncto. Et tamen in instanti medio illius horae non erit tanta linea in pertransiri per B punctum sicut per A, ceteris paribus.

19 Similiter, pertranseundo lineam solummodo finitam quantumcumque modicam volueris potest B punctus in infinitum velocitare motum suum, quia super primam partem proportionalem aliqua velocitate, super secundam dupla, et sic in infinitum.

20 Ex quo satis manifeste sequitur quod huiusmodi velocitas difformis seu instantanea non attendit penes lineam pertransitam,

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1 aequales pertranseant] *inv.* R | pertranseant magnitudines] *inv.* BD magnitudines…pertranseant] *om.(hom.)* D | stat] sufficit C | inaequales] aequales C inaequaliter G 2 pertranseant] *om.* U 3 punctus] pectus(*sic!*) U | uniformiter moveatur] *inv.* BU 4 pertranseundo] *om.* B pedalem quantitatem] pedalis quantitatis B 5 punctus] pectus(*sic!*) U incipiat uniformiter] *inv.* G | uniformiter] *om.* B | uniformiter intendere] *inv.* D | in] prima *add.* AGKRU(*marg.*) 6 illum] *om.* G isti U | gradum] velocitatis *add.* G | et] *om.* G 7 medietate] *exp.* E | ille] motum suum AC usque B *om.* DGKR motum suum universaliter U | ab…ad] usque ab eodem gradu R | gradu] usque *add.* BCDGKU 8 in] illo *add.* B | instanti] *om.* G | totius] istius D illius G | movebitur…punctus] B punctus movetur C 9 punctus] pectus(*sic!*) U | et] *om.* CD 10 puncto] pecto(*sic!*) U | tamen in] cum R | in[1]] *om.* U | instanti medio] *inv.* AB | illius horae] *om.* G | linea] hora K | in pertransiri] pertransita A | in[2]…punctum] pertransita ab puncto G 11 per[1]] *om.* K | punctum] pectum(*sic!*) U | per[2]] ab G 12 solummodo] *om.* D 13 modicam] modicum R | B] A B | punctus] pectum(*sic!*) U 14 super] supra D 15 velocitate] et *add.* AU | super] supra D et R | dupla] et quadrupla super tertiam *add.* AK duplam et super tertiam quadruplam BCGU duplam et super quartam tertia C duplam E velocitate et quadruplo super tertiam *add.* R 16 satis] *om.* BGKU sequitur] apparet D | huiusmodi] haec D 17 seu] sive K | seu instantanea] *om.* C | attendit] attenditur DGKRU

sed penes lineam quam describet punctus talis si per tantum tempus vel per tantum uniformiter moveretur illo gradu velocitatis quo movetur in illo instanti dato.

21 Est autem circa intensionem et remissionem motus localis G 36r advertendum quod motum | aliquem intendi aut remitti dupliciter contingit: uniformiter scilicet aut difformiter.

22 Uniformiter intenditur motus quiscumque cum in quacumque aequali parte temporis aequalem acquirit latitudinem velocitatis. Et uniformiter etiam remittitur motus talis, cum in quacumque aequali U 61rb parte temporis aequalem deperdit latitudinem velocitatis. |

23 Difformiter vero intenditur motus aliquis vel remittitur cum maiorem latitudinem acquirit vel deperdit in una parte temporis E 13vb quam in alia sibi aequali. |

24 Iuxta illud sufficienter apparet quod cum latitudo motus seu velocitatis sit infinita, quod non est possibile aliquod mobile illam uniformiter acquirere in aliquo tempore finito.

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1 sed] licet C | sed…lineam] *om.* G *marg.* U | penes lineam] *marg.* C describet] describeret A discriberet U | describet punctus] pertransit C punctus] pectus(*sic!*) U | talis] scilicet velocissime motus *add.* R | tantum] totum G | tantum tempus] *inv.* R 2 vel…tantum] *om.* D 3 in] *om.* G 4 autem] igitur C | intensionem…remissionem] intencionem et remicionem(*sic!*) U | et…localis] motus localis et remissionem C 5 advertendum] adducendum R | aut] vel DGKRU | dupliciter] *om.* C 6 contingit] *om.* D | uniformiter scilicet] *inv.* ACGU | scilicet] *om.* BR | aut] vel AG et C 7 Uniformiter] autem *add.* A enim *add.* CDR | intenditur] vel remittitur *add.* C | quiscumque] quicumque ABGKRU quocumque C *om.* D | quacumque] quaelibet D *om.* G 8 temporis] intensiori(*sic!*) R acquirit] vel perdit *add.* C | Et] *om.* BG | Et…velocitatis] *om.* C 9 etiam] *om.* BK | remittitur] remittet R | motus talis] *inv.* D | talis] quiscumque B quacumque] aliqua B | aequali] tali G inaequale K 10 temporis] qua(?) R deperdit] deperdet R 11 vero] autem BG | motus aliquis] *om.* AB *inv.* C aliquis] *om.* G 12 latitudinem] velocitatis *add.* ABCDGKR | acquirit] asquirat(*sic!*) R adquiret velocitatem U | deperdit] perdit C deperdat R | in] aut G 13 sibi aequali] *inv.* BC 14 sufficienter] satis R 15 velocitatis] velocitas K | quod] *om.* C | non] *om.* B | possibile] impossibile B quod *add.* R | aliquod…illam] ipsam aliquod mobile C | illam uniformiter] uniformiter ipsam G 16 acquirere] acquirat R

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7 Uniformiter] quid est uniformiter intendi motus *marg. a.m.* E

25 Et quia quilibet gradus velocitatis per latitudinem | tantum R 28v finitam distat a non gradu seu termino privativo totius | latitudinis, A 25rb qui est quies, ideo a quiete ad gradum datum quemcumque contingit aliquid mobile uniformiter intendere motum suum. Et consimiliter a gradu dato contingit motum uniformiter remittere ad quietem. Et universaliter a quocumque gradu ad quemcumque gradum alium contingit utramque mutationem fieri uniformem.

26 Circa quod est advertendum, quod sicut nullus est gradus velocitatis quo uniformiter continue movendo plus pertransiretur in aliquo tempore aequali quam in alio, sic nec est aliqua latitudo velocitatis | incipiens a non gradu aut ab aliquo gradu certo per K 41ra quam complete ipsam uniformiter acquirendo in aliquo tempore assignato plus pertransitur quam per eandem pertransiretur in aliquo tempore aequali ipsam uniformiter deperdendo. Omnis enim latitudo sive a non gradu incipiat, sive a gradu aliquo, dum tamen ad gradum aliquem terminetur finitum, et uniformiter acquiratur seu deperdatur, correspondebit aequaliter gradui medio sui ipsius, sic scilicet quod mobile illud ipsam uniformiter acquirens vel deperdens in aliquo tempore dato aequalem omnino | magnitudinem D 67rb pertransibit, sicut si ipsum per aequale tempus moveretur medio

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1 Et] *marg.* C sed G | quia] *om.* G | gradus] finitus *add.* R | velocitatis] finitus *add.* A | latitudinem] in *add.* K | tantum] tantummodo ABCDKRU 2 seu] a *add.* A | termino] *om.* R | latitudinis] motus *add.* G 3 qui] quae K gradum…quemcumque] quemcumque gradum finitum C quemcumque gradum datum KR | quemcumque] *om.* B quaelibet D 4 aliquid] aliquod U | intendere] actendere(*sic!*) R | intendere…uniformiter] *om.* D motum] *om.* A | Et] etiam BDR *om.* C | a…dato] *om.* B 5 motum] a gradu B | motum uniformiter] uniformiter motum suum R | remittere] motum *add.* B remitti C 6 universaliter] uniformiter U | gradu] *om.* R gradum] *om.* ABCGKR | gradum alium] *inv.* BD 7 contingit] *om.* B 8 quod est] quem R | quod^{2}] *om.* B | nullus] non B | est gradus] *inv.* C 9 continue] aliter *add.* C | movendo] modo *add.* C mota G permovendo R pertransiretur] pertransitur G 10 aequali] aequaliter K | sic] sicut A | nec] non B 11 gradu^{2}] *om.* C | gradu certo] *inv.* BDGU termino K 12 quam] quem CK | ipsam] propriam A | aliquo] quo G 13 assignato] signato D pertransitur] pertranseatur DK pertransiretur R 15 aliquo] *om.* C 16 gradum aliquem] *inv.* G | terminetur finitum] *inv.* G | et] etiam *add.* R uniformiter] universaliter G 17 correspondebit] correspondet C | ipsius] *om.* B 18 ipsam uniformiter] *inv.* K 20 sicut] ac G | si ipsum] seipsum B tempus] continue *add.* ABCDGKRU | moveretur] uniformiter *add.* G

gradu illius. Cuiuslibet etiam talis latitudinis incipientis a quiete et terminatae ad aliquem gradum est gradus suus medius subduplus ad gradum eandem latitudinem terminantem. Ex quo sequitur quod cuiuslibet latitudinis terminatae ad duos gradus inclusive vel exclusive est gradus medius maior quam subduplus ad gradum intensiorem eandem latitudinem terminantem.

27 Ex praecedenti sequitur quod cum mobile aliquod a quiete uniformiter intendat motum suum ad aliquem gradum datum, quod ipsum in duplo minus pertransibit in tempore illo quam si ipsum per
C 14vb idem tempus uniformiter moveretur gradu | illo ipsam latitudinem terminante, quia totus ille motus correspondebit gradui medio illius latitudinis qui est subduplus praecise ad gradum illum qui est terminus illius eiusdem.

28 Item sequitur, quod cum mobile aliquod ab aliquo gradu
B 56vb exclusive ad alium | gradum inclusive vel exclusive intendat motum suum uniformiter, quod ipsum plus pertransibit quam subduplum
A 25va ad illud quod ipsum uniformiter pertransiret | in aequali tempore secundum istum gradum ad quem stabit intensio sui motus, quia totus ille motus correspondebit <gradui> suo medio qui maior est

1 gradu] gradi B | illius] ipsius latitudinis totius G | talis] *om.* U | latitudinis] illius *add.* C talis *add.* G | incipientis] scilicet *add.* BCDKR 2 terminatae] terminantis R | aliquem] certum *add.* U | aliquem gradum] certum gradum aliquem C | est] erit BG 3 sequitur] arguitur D 4 ad] cum D | gradus] *om.* K | inclusive…exclusive] *inv.* G 5 gradus medius] *inv.* R | quam] gradus *add.* G 7 praecedenti] quo G etiam *add.* R | mobile aliquod] *inv.* DU 8 uniformiter] *om.* D | aliquem] certum *add.* A | quod] per K 9 duplo] duplum U | si ipsum] supra K | per idem] per totum C | per…uniformiter] uniformiter per idem tempus AGR uniformiter per totum tempus D uniformiter per illud tempus KU 10 moveretur] movetur U 11 terminante] *om.* R | correspondebit] correspondet AU | gradui] isto *add.* G | medio] totius *add.* ABC | illius] *om.* AC 12 qui[1]] quae U qui[1]…eiusdem] *iter. sed del.* G | subduplus praecise] *inv.* ABCDRU praecise] *om.* K | gradum illum] *inv.* R | qui[2]…eiusdem] (intensiorem *sed del.*) eandem latitudinem terminantem R | est terminus] *inv.* ABCDU 13 illius] *om.* ABCDGKU 14 Item] *om.* ABCDGKRU latitudinis *add.* G | Item sequitur] similiter etiam convenit B | sequitur] etiam consimiliter *add.* ACDGKRU | mobile aliquod] *inv.* C | aliquod] *om.* B 15 alium] aliquem AG | intendat] intendet ABCDKRU | intendat…uniformiter] uniformiter intendet motum suum G | motum…uniformiter] uniformiter motum suum ABCGKR 17 ipsum] *om.* C | pertransiret] pertransibit R 18 istum] cum B *om.* D | sui] illius C | quia] cum *add.* C ut *add.* R 19 totus] *om.* B | correspondebit] aequaliter *add.* C | gradui] *marg.* E

quam subduplus ad gradum terminantem illam latitudinem acquirendam. Quamvis autem huiusmodi motus difformis correspondebit aequaliter sui medio gradui, ita velox tamen erit ille motus totus categorematice sicut est aliquis motus uniformis sub aliquo gradu intrinseco illius latitudinis acquirendae, et consimiliter ita tardus.

29 Ad probandum autem quod cuiuslibet latitudinis | incipientis a G 37r
quiete et terminatae ad aliquem gradum finitum est medius gradus praecise subduplus ad gradum eandem latitudinem terminantem est sciendum quod si aliqui sint tres termini continue proportionales, qualis est proportio primi ad secundum, aut secundi ad tertium, talis est proportio differentiae primi et medii ad differentiam medii et tertii, ut 4 2 1, 9 3 1, 9 6 4. Qualis enim est proportio 4 ad 2 aut duorum | ad unum, talis est proportio differentiae 4 ad duo ad K 41rb

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1 terminantem…acquirendam] istam latitudinem acquirendam terminantem B | illam] istam *sed corr. marg. in*: eandem D 2 autem] aliquando K | huiusmodi motus] *inv.* ABCDKRU motus ille G 3 sui medio] medio suo ACR *inv.* BU maximo suo D motus suo K | sui…gradui] gradui suo medio G | gradui] gradu B | tamen] *om.* BG cum K | tamen erit] est tamen U | ille…totus] totus ille motus ABCGRU totus iste motus DK 4 uniformis] uniformiter R | sub] ab GU 7 autem] igitur E tamen R 8 et] *om.* R | terminatae] terminantis K | terminatae…finitum] in alii gradum finitum terminante B | ad] in GK | medius] *om.* B | medius gradus] *inv.* G 9 subduplus] primo *add.* B | eandem] illam B | est] tamen *add.* GK autem *add.* U | est sciendum] sciendum est etiam R 10 aliqui sint] *inv.* G | sint] sunt AK | tres] *marg. a.m.* D | proportionales] proportionabiles U 11 aut] et ACK 12 est proportio] *marg.* C | differentiae] *om.* C | primi] primae G et medii] ad secundum A | medii^{2}…tertii] secundi ad tertium A 13 4^{1}] quattuor G | 4^{1}…4] quattuor ad duo et duo ad unum U | 2^{1}] duo G | 1^{1}] et G | 9^{1}] et/8(?) *add. lin.* R novem U | 9^{1}…1] *om.* G | 9^{1}…4] *om.* C | 3…9] *om.* K | 1^{2}] *om.* B | 9^{2}] 8 *add.* B | 9^{2}…4] 8 4 2 D | enim] *om.* G | aut…unitas] talis est proportio differentiae 2 ad 1 est enim dupla proportio differentiae 4 ad 2 et differentiae 2 ad 1 C 14 duorum] duo BK 2 DG duorum…unum] *del. et add. lin.*: ad differentiam illorum ad unum R | unum] 1 AD | differentiae] *om.* K | 4] quattuor G | ad^{2}…4] *om.(hom.)* R | duo] 2 ABK duplam G

differentiam duorum ad 1. Est enim differentia 4 ad 2 binarius, differentia duorum ad unum unitas, et consimiliter est in aliis.

30 Signato igitur aliquo termino sub quo infiniti alii termini sunt continue proportionales proportione dupla, quilibet ad sibi proximum coniugatur, et quanta fuerit differentia primi termini dati U 61va ad secundum, tantum praecise erit aggregatum | ex omnibus differentiis terminorum sequentium. Quanta est enim prima pars proportionalis alicuius quanti finiti, tantum est aggregatum praecise ex omnibus partibus proportionalibus residuis eiusdem.

31 Cum ergo quaelibet latitudo sit quaedam quantitas, et universaliter sicut in omni quanto medium aequaliter distat ab extremis, ita cuiuslibet latitudinis finitae medius gradus aequaliter D 67va distat ab utroque extremorum, | sive illa duo extrema sint duo gradus aut unum illorum fuerit aliquis gradus et alterum omnino privatio, seu non gradus illius. Sed sicut iam ostensum est, dato gradu aliquo sub quo <essent> infiniti alii continue proportionales, quilibet ad sibi proximum signetur, aequalis erit differentia seu

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1 differentiam] ad duo ad duplam G | duorum] 2 ABGK | 1] 4 enim ad 2 est dupla proportio et 2 est dupla proportio ad 1 *add.* A unum BCR | 2] duo BCDRU | binarius] et etiam *add.* A et *add.* K ad R 2 differentia] vero *add.* BCDU | duorum] 2 ABD duo K | unum] 1 AD est *add.* BCRU 1 est G | unitas] in tria(?) quos est consimiliter proportio dupla *add.* A 3 igitur] *om.* A | infiniti…sunt] sunt infiniti termini K | termini] *om.* B termini sunt] *inv.* C 4 proportionales] cum *add.* R | dupla] et *add.* AD quilibet] quaelibet A quibus R | ad] aliquid *add.* B | sibi] sui A *om.* C suum R 5 coniugatur] tantum *add. marg.* B coniungatur G | et quanta] in quantum U | fuerit] fuit R | primi] istius U | termini] illius E | termini dati] terminati K 6 secundum] terminum *add.* G duplum terminum R | tantum] terminum K | praecise erit] *inv.* A 7 sequentium] subsequenter B | est enim] *inv.* ABCDKRU enim erit G | prima] illa U 8 est] erit ABCGKRU aggregatum praecise] *inv.* BC 9 omnibus] aliis *add.* G | partibus…residuis] residuis partibus proportionalibus B 10 latitudo] finita *add.* AB *om.* G quaedam quantitas] *inv.* G | et] ut R 11 quanto] quantitate K | aequaliter distat] *inv.* G 12 extremis] extremo et A | ita] *corr. in*: sic D | finitae] est *add.* G | aequaliter distat] *inv.* U 13 distat] distans G | utroque] graduum *add.* B | duo[1]] *om.* G et K | sint] fuerunt A | duo gradus] *inv.* D 14 aut] sive G | fuerit] sit R | aliquis] *om.* A | et] *om.* C 15 Sed] *om.* K | iam] *om.* G | est] *om.* K 16 gradu aliquo] *inv.* U | essent] et C *om.* E etiam KR essent…alii] finiti alii sunt gradus G | infiniti] infinito R | infiniti alii] *inv.* C alii] alia R | proportionales] et *add.* D proportionali R 17 quilibet] quibus B | sibi] sui R | aequalis] inaequalis G | erit] et K differentia…latitudo] latitudo sive differentia A

latitudo inter primum et secundum suum sibi subduplum sicut
latitudo composita ex omnibus differentiis seu latitudinibus inter
omnes gradus | residuos, sequentes videlicet duos primos; ergo E 14ra
aequaliter praecise et per aequalem latitudinem distabit ille gradus
secundus subduplus scilicet ad primum | duplum ab illo duplo sicut A 25vb
distabit idem secundus gradus a non gradu seu ab extremo opposito
illius latitudinis datae.

32 Et sicut universaliter probatur de omni latitudine incipiente a non
gradu et terminata ad aliquem gradum finitum continente etiam
gradum aliquem et subduplum, et subquadruplum, et sic in
infinitum, quod eius gradus medius est praecise subduplus ad
gradum ipsam terminantem; unde non solum est hoc verum de
latitudine velocitatis | motus incipientis a non gradu sed etiam de R 29r
latitudine caliditatis, frigiditatis, luminis et aliarum similium
qualitatum, et consimiliter argui poterit et probari.

33 Quantum autem ad magnitudinem pertranseundam uniformiter acquirendo talem latitudinem motus incipientem a non gradu et

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1 suum] *om.* ABC | suum sibi] scilicet sibi R | sibi] scilicet BDU sicut G 2 composita] proposita scilicet B 3 sequentes] sequentis BG | videlicet] *om.* G | primos] gradus *add.* C proximos(*sic!*) E | ergo aequaliter] *inv.* C 4 praecise] *om.* D | et] *om.* GR 5 secundus] duplus CR *marg.* E primus G scilicet] *om.* BDKU | primum…illo] primi et secundum ad isto C | duplum] gradum *add.* A | illo] isto primo G 6 idem] iste D *om.* G | secundus] duplus CR | secundus gradus] gradus iste G | gradus] *marg.* U | ab] *om.* K opposito] aliquo G composito K 7 latitudinis] magnitudinis C 8 sicut] sic ADGKRU | universaliter] uniformiter G | incipiente] *om.* D 9 et…ad] terminatae in G | terminata] terminante ABU | ad] in ABR | gradum] certum *add.* A | finitum] certum et *add.* C | continente] continentem ABCDRU | etiam] *om.* CG 10 et[1]] *om.* DK | subduplum] et subtriplum *add.* D | subquadruplum] subquatriplum U 11 quod] quia C eius] eiusdem A 12 ipsam] illam latitudinem C ipsum K | ipsam terminantem] terminantem illam latitudinem AG | hoc verum] huiusmodi vero R 13 velocitatis motus] *inv.* G | sed etiam] *inv.* A 14 caliditatis] vel *add.* A | caliditatis frigiditatis] frigiditatis et caliditatis D | frigiditatis] vel *add.* A | luminis] lumini C | similium qualitatum] *inv.* A 15 qualitatum] sensibilium *add.* C | et[1]] *om.* AC | argui poterit] potest argui R | poterit] potest G 16 autem] *om.* G enim U | uniformiter] universaliter D 17 incipientem] incipientis BCR

terminatam ad aliquem gradum finitum dictum est prius quod totus ille motus seu tota illa acquisitio correspondebit gradui suo medio.

34 Et consimiliter etiam, si ab aliquo gradu exclusive uniformiter acquiratur latitudo motus ad aliquem gradum terminatum finitum [ex quo] satis cognosci poterit in huiusmodi uniformi intensione vel remissione quanta erit magnitudo pertransita, ceteris paribus, in prima medietate temporis et quanta in secunda.

35 Cum enim a non gradu ad aliquem gradum uniformis fiat alicuius motus intensio, subtriplum praecise pertransibit in prima medietate temporis ad illud quod pertransibit in secunda. Et si alias ab eodem K 41va gradu aut ab aliquo alio quocumque ad | non gradum uniformis fiat C 15ra remissio, triplum praecise pertransiretur | in prima medietate temporis ad illud quod pertransiretur in secunda.

36 Totus enim motus factus in toto tempore correspondebit medio G 37v gradui suo, illi scilicet quem habebit | in instanti medio illius temporis; et secunda medietas illius motus correspondebit gradui medio secundae medietatis eiusdem motus qui est subquadruplus ad gradum illum terminantem illam latitudinem. Ideo, cum ipsa

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1 terminatam] terminante CU terminanta D terminantem E | ad] in ABCDGKRU | finitum] ut *add.* C sicut *add.* G | quod] quia G 2 ille motus] remotus K | seu] vel C sive D | correspondebit] correspondet U suo] sui B | suo medio] *inv.* U 3 consimiliter] quod similiter R | etiam] *om.* CU | uniformiter acquiratur] *inv.* A 4 motus] terminata A terminatum] *om.* A terminata BDU est tamen C terminantem R terminatum finitum] *inv.* G finitum terminantem K 5 ex quo] *om.* C | quo] illo R | cognosci] cognosco K | poterit] posset U | in huiusmodi] *om.* K | vel] et G 6 quanta erit] quantum sit B | erit] est C 7 et] etiam *add.* G in…uniformis] *om.* G | secunda] et quanta in tertia *add.* K 8 enim] est K aliquem] certum C | uniformis] uniformem E | alicuius] A R alicuius…intensio] intensio alicuius motus A 9 intensio] si *add.* G intensior K | subtriplum] subduplum CR | pertransibit] *marg.* E 10 temporis] quo G | illud] *om.* DK | pertransibit] pertransiretur A secunda] dupla C | alias] aliquis A 11 ab] *om.* K | aliquo] *om.* U quocumque] gradu *add.* A 12 pertransiretur] pertransitur B pertransibit G 13 pertransiretur] transibit B 14 Totus] *om.* K | enim motus] *inv.* KR | toto] *om.* K | medio gradui] *inv.* DR | medio…suo] gradui suo medio G 15 gradui suo] *inv.* AC | illi scilicet] sive illi A *om.* K | instanti medio] *inv.* A medio] motus K 16 et] ita *add.* A 17 secundae] et sic K | medietatis] *om.* D | subquadruplus] subduplus D subquatriplus U | ad] istum *add.* D 18 illum] *om.* ABCDGKRU | terminantem] totam *add.* G terminantem…latitudinem] illam latitudinem terminantem B | Ideo] inde G

secunda medietas durabit solum per subduplum tempus, in quadruplo minus pertransiretur per illam secundam medietatem quam per totum motum. Igitur, totius magnitudinis pertranseundae toto motu pertransirentur tres quartae per primam medietatem totius motus et ultima quarta pertransiretur per secundam medietatem eiusdem. Sequitur igitur quod in huiusmodi uniformi remissione vel intensione alicuius motus ab aliquo gradu usque ad non gradum, vel a non gradu in aliquem gradum, in triplo plus praecise pertransiretur per medietatem illius latitudinis | intensiorem A 26ra
quam remissiorem.

37 Motum autem aliquem uniformiter intendi vel remitti | ab aliquo D 67vb
gradu in aliquem gradum infinitis modis contingit, quia ab aliquo gradu ad suum subduplum vel ad suum subquadruplum aut subquintuplum, aut ad suum | subsexquialterum, aut B 56vb
subsexquitertium, et sic deinceps. Ideo nullus potest esse terminus universalis penes quem universaliter cognosci poterit quanto plus pertransiretur per primam medietatem illius intensionis vel remissionis quam per secundam, quia respectu diversorum graduum

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1 secunda medietas] *inv.* GK | solum] solummodo CDGKRU 2 minus] praecise *add.* ACDGRU | pertransiretur] pertransibit A praecise *add.* B 3 pertranseundae] pertranseundo B pertransitae C a *add.* G 4 toto] totius C per totum U | motu] motae C motum U | pertransirentur] pertransiretur C pertranseantur U | quartae] initiae(?) B | medietatem] et *add.* A istius *add.* C 5 totius] totus D | et] *om.* A ista *add.* S | secundam] secunda U 6 quod…huiusmodi] *iter.* A | uniformi remissione] *inv.* B 7 remissione…intensione] *inv.* K | vel] et R 8 a…gradum] econverso D gradu] usque *add.* ARU | in[1]] ad GRU 9 praecise pertransiretur] pertranseatur praecise U | illius] secundus D | intensiorem] intensioris D 10 quam] per *add.* A | remissiorem] remissioris D remissionem U 12 in] ad BCDGKRU | aliquem] alium BCDKRU | modis] modum(*sic!*) R | quia] *om.* K quod R 13 gradu] *om.* D | suum[1]] gradum *add.* A *om.* B | vel] et B suum[2]] subtriplum *add.* D | subquadruplum] quatriplum(*sic!*) U 14 subquintuplum] sub suum subtriplum G ad suum subquatriplum U | ad] *om.* ACDK | subsexquialterum] sexquialterum ABCDG subsexduplum(*sic!*) K subsexquialterum *sed corr. in:* sexquialterum R | aut[2]] ad suum *add.* B *om.* D ad *add.* G | aut[2]…deinceps] et sic de aliis et C 15 subsexquitertium] *om.* D sesquitertium vel sesquiquartum G subsextriplum K subsexquialterum R | Ideo] deinde R | nullus] non AC potest] poterit A | terminus] talis *add.* D 16 universalis] naturalis A | quem] *om.* R | universaliter] alia lectio naturaliter *marg.* A | cognosci poterit] cognoscitur C *inv.* DK potest cognosci G 17 pertransiretur] pertranseatur K | medietatem] medietatis G 18 secundam] duplam R

extremorum diversa erit proportio magnitudinis pertransitae in prima medietate temporis ad magnitudinem pertransitam in secunda.

U 61vb **38** Cognitis tamen gradibus extremis, | ita scilicet quod cognoscatur quantum pertransiretur uniformiter in tanto tempore vel in tanto per gradum extremum intensiorem terminantem et consimiliter respectu gradus extremi remissioris, cognosci poterit per calculationem quantum pertransiretur in prima medietate et etiam quantum in secunda, quia cognitis isto modo extremis gradibus haberi potest etiam gradus medius inter istos et gradus medius inter illum gradum medium et gradum intensiorem illam latitudinem terminantem. Sed huiusmodi calculatio maiorem sollicitudinem ageret quam profectum.

39 Et ideo sufficit quod in omni huiusmodi casu respondeatur generaliter quod plus pertransiretur per medietatem talis latitudinis intensiorem quam remissiorem, tantum scilicet quantum per gradum medium istius medietatis intensioris uniformiter pertransiretur in aequali tempore sicut si ipsa medietas uniformiter acquiritur vel deperditur.

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1 erit] est K 4 extremis] *om.* B 5 uniformiter] in tanto tempore vel in tanto per gradum extremum intensiorem terminantem et consimiliter *add.* D | tempore…tanto] *om.* U 6 intensiorem] alia lectio intensius *marg.* A intencius(*sic!*) U | consimiliter respectu] *inv.* U 8 quantum[1]] quanta U etiam] *om.* BDR | quantum[2]] quanta U 9 cognitis] cognitae G cognito K extremis gradibus] *inv.* KR | potest etiam] etiam poterit A poterit et G 10 etiam] *om.* B qui est iste D | gradus medius[1]] *inv.* G | inter[1]…medius] *om.(hom.)* A | istos…intensiorem] *om.* U | et] etiam *add.* BCDGKR | gradus medius[2]] gradum medium B *inv.* G | medius[2]] medium R | gradum] *om.* K gradum medium] *inv.* R 11 et] eodem modo *add.* B | gradum] medium *add.* B | illam…terminantem] terminantem illam latitudinem R 12 huiusmodi] ista C illa G | calculatio] in *add.* K | maiorem] minorem *sed corr.* C | sollicitudinem] solutionem B soliorem(?) R | ageret] faceret C arguet D 13 profectum] proficuum A perfectum B profectionem(*sic!*) C profactam(?) R 14 quod] *om.* R | omni…casu] omnibus casibus A respondeatur] alia lectio ostendatur *marg.* A respondeam C 15 pertransiretur] pertransitur A | per…latitudinis] talis latitudinis per medietatem B 16 quam] per *add.* A | tantum] tantam KR | quantum] quanta scilicet R 17 intensioris] plus *add.* B | pertransiretur] pertransitur A alia lectio pertransiri potest *marg.* A 18 sicut] *om.* U | si] *om.* ABGKR acquiritur] adquiratur U 19 deperditur] deperdatur U

40 De difformi autem intensione vel remissione, sive ab aliquo gradu usque ad non gradum, vel econverso, seu ab aliquo gradu usque ad aliquem alium, nulla potest esse regula quantum in tanto tempore vel in tanto pertransiretur, aut cui gradui | K 41vb intrinseco illius latitudinis correspondebit talis latitudo motus difformiter deperdita seu acquisita. Quia sicut infinitis modis contingit talem remissionem seu intensionem difformem variari, ita etiam infinitis gradibus intrinsecis eiusdem latitudinis, immo cuilibet gradui intrinseco illius latitudinis sic acquisitae vel deperditae poterit totus ille motus correspondere.

41 Unde universaliter gradus terminans talem latitudinem secundum extremum intensius est remississimus citra aliud extremum eiusdem latitudinis cui non potest totus huiusmodi motus difformiter difformis correspondere. | E 14rb Unde nec est possibile quod totus huiusmodi motus ita remisso | G 38r gradui correspondeat | A 26rb sicut idem motus poterit correspondere, nec ita intenso.

42 Comparando autem intensionem unius motus ad remissionem alterius, aut intensionem cum intensione, aut remissionem cum remissione multa contingit fieri sophismata.

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1 vel] et G 2 ad…gradu] *marg.* U | vel] et K | econverso] econtra ABDGKR 3 usque] *om.* ABCK | aliquem alium] non gradu *marg.* D | alium] gradum *add.* A gradum GU | regula] resistentia(*sic!*) R 4 vel…tanto] *iter.* R | in] *om.* A 5 correspondebit] correspondeat A corresponderet C | deperdita] perdita(*sic!*) E 6 remissionem…intensionem] *inv.* AC 7 variari] variaretur B | ita etiam] *om.* B | etiam] *om.* A | infinitis] *om.* K 8 immo] etiam *add.* C | cuilibet] cuiuslibet R 9 latitudinis] et *add.* E | sic acquisitae] *inv.* G | acquisitae] acquisitus C | deperditae] deperditus C | poterit…motus] totus ille motus poterit G poterit…correspondere] totus ille motus correspondere potest A 11 talem] illam A istam *sed corr.* D totam illam R 12 est] vel CR remississimus] remissius GKR | citra] erit C tunc K | aliud] aliquod B aliquid K | aliud extremum] *inv.* G | eiusdem] illius C 13 huiusmodi] ille ACD(*marg.*)R iste U | difformiter difformis] *inv.* B 14 nec] non A 15 huiusmodi] iste K | idem] ille GK 16 poterit] potest B | correspondere] corresponderi R | ita] tam K | intenso] intensio BDU intense CR 17 autem] *om.* G | motus] *om.* D | ad] intensionem aut B 18 alterius] motus *add.* A | aut intensionem] *om.* B | intensione] remissione K 19 remissione] remissionem U | multa] infinita *add. marg.* C finita *add. marg.* D *om.* U contingit] contingant A convenit B *om.* C contingunt K | contingit fieri] *inv.* GRU

43 Posito enim quod ab eodem gradu, scilicet C, incipiat motus A uniformiter intendi, et sic uniformiter intendatur usque ad gradum duplum ad C, et B motus | R 29v in eadem hora uniformiter remittatur ab eodem C gradu ad quietem; arguitur quod A motus non aequaliter intendetur | D 68ra sicut remittetur motus B. Quia ab ipso C gradu remittetur B motus ad gradum subduplum et subquadruplum, et suboctuplum, et sic in infinitum; igitur si ab eodem C gradu aeque velociter intendetur motus | C 15rb A in aequali tempore, sequitur quod A motus intendetur ad gradum duplum ad C gradum, et ad quadruplum, et sic in infinitum. Consequens falsum et contra casum quia A non intendetur nisi ad gradum duplum ad C, scilicet ad gradum G.

44 Et per idem argumentum arguitur quod non sit possibile aliquem motum uniformiter remitti ad quietem ab aliquo gradu signato, quia tunc esset possibile quod aeque velociter intendatur alius motus ab eodem gradu ad alium gradum intensiorem. Consequens arguitur esse falsum per argumentum factum de A et B motibus in casu posito.

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1 ab...C] a C gradu G | scilicet C] *om.* K 2 intendatur] intenditur K 3 ad C] a se K | c] C] qui sit G *add.* ABCDGKU qui sit D *add.* R | B] G G | in] *om.* C eadem…remittatur] uniformiter remittatur in eadem hora AC 4 ad…gradu] *iter.* G | quietem] tunc *add.* DG | aequaliter] aeque velociter ABCDGRU aeque K 5 intendetur] intenderetur BR | remittetur] remittitur G remitteretur R | remittetur...B] motus B remitteretur B | motus B] *inv.* D | B] G G | Quia…motus] *om.(hom.)* D | ipso] eodem R 6 B] *om.* C G G | B motus] *inv.* A | et[1]] *om.* R | et[2]] *om.* R | et suboctuplum] *om.* DU 7 suboctuplum] subquintuplum A subsextuplum G | infinitum] ad C gradum *add. sed exp.* C | aeque] ita C 8 motus A] *inv.* G | aequali] eodem D aequali tempore] *inv.* C aequalitate K 9 ad[2]…gradum] *om.* B | ad[2]…ad] *om.* C | gradum[2]] *om.* A 11 A] *om.* A | scilicet...G] *om.* G 12 G] octuplum K B R 13 idem] illud G | quod] *lin.* R 14 ad…signato] ab aliquo gradu signato ad quietem B | quia] *del.* G 15 tunc] *om.* G | esset] erit D | motus] gradus G modus K 16 ad] aliquem *add.* B 17 argumentum] prius *add.* A modo *add.* BCDGKRU | de] per G | et] de *add.* R | motibus] *om.* G

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1 Posito] primum *marg.* A primum sophisma *marg.* CE

45 Item, posito quod A et B motus uniformiter intendantur ab illo C gradu aut ab alio aliquo gradu, et A in duplo velocius ipso B continue, arguitur quod A motus aliquando erit duplus ad B, quia quocumque gradu intensiori isto C signato in duplo citius acquiret A gradum illum quam faciet B; igitur gradum quadruplum ad C in duplo citius acquiret A quam faciet B. Igitur cum A fuerit sub gradu quadruplo ad C, erit B sub gradu praecise duplo ad C. Igitur tunc erit A motus duplus ad B.

46 Et per illud argumentum potest argui quod A motus per huiusmodi | intensionem poterit esse quadruplus ad B et octuplus, K 42ra et sic deinceps quacumque proportione signata, quod est manifeste repugnans sicut probatur statim. Quia si A motus in principio fuisset duplus ad B et incepisset intendi uniformiter in duplo velocius quam B, et hoc a gradu duplo ad illum gradum a quo incepisset B uniformiter intendi in duplo tardius quam A ab illo gradu duplo, <tunc> continue stante illa uniformi intensione respectu utriusque continue maneret motus A in duplo intensior motu B praecise. Igitur cum in casu prius supposito in quolibet | instanti erit B aeque U 62ra

1 posito] ponatur U | motus] B *add.* K | motus uniformiter] *inv.* B uniformiter intendantur] *inv.* A | intendantur] intendatur G 2 alio aliquo] *inv.* ABCDGKRU | gradu[2]] ita quod *add.* D | et] *om.* ABCGKRU ita quod D | A] continue *add.* D aut G | ipso] quam D | B] gradu *add.* A A in duplo velocius ipso B *add.* G 3 continue] *om.* D | aliquando erit] *inv.* R | B] motum *add.* BC 4 signato] assignato C gradu dato G | A...illum] istum B 5 faciet] faceret A | igitur...B[1]] *om.(hom.)* D 6 faciet] *om.* B 7 quadruplo] quarto G quatriplo(*sic!*) U | erit[1]] et G | sub…praecise] praecise sub gradu GKR 8 motus] praecise *add.* AGU | duplus...B] *om.* K 9 Et…argumentum] *marg. a.m.* E | illud] idem ABCDGRU | argui] probari A | A] *om.* R | per[2]] *om.* U 10 intensionem] intransitionem(*sic!*) A poterit] potest U 11 deinceps] in infinitum ABR | quacumque] quacumque K | signata] data vel assignata U 12 sicut] quod B sed K probatur statim] *inv.* ABCDKU statim probabitur GR | in] a K in…fuisset] fuisset in principio D 13 duplus] duplum U | et] vel ACU incepisset] B *add.* E | intendi uniformiter] *inv.* C | uniformiter…intendi] *om.(hom.?)* G 14 gradum] *om.* ABCDKRU | a[2]] in AU cum D *om.* R incepisset] cepisset(*sic!*) U 15 duplo[2]] vel primo *add.* C et *add.* D *om.* U 16 tunc] *om.* E | uniformi intensione] *inv.* R | utriusque] cuiuscumque CG 17 continue] *om.* C | motu B] *inv.* R | motu…praecise] praecise motu B C | B praecise] *inv.* D 18 cum] *om.* R | in[1]…intensus] *om.* D | supposito] posito G | B] motus *add.* G

1 Item] secundum sophisma *marg.* C

intensus sicut esset in casu secundo pro eodem instanti, et A in quolibet instanti remissior quam esset in eodem instanti iuxta secundum casum, sequitur quod A in primo casu numquam erit in A 26va duplo velocior seu intensior ipso B, quod erat probandum. |

47 Antecedens enim satis probari poterit, quia posito quod A et B motus incipiant uniformiter intendi uterque a quiete, A continue in duplo velocius B praecise et sequitur – notum est – quod A erit B 57ra continue in duplo | velocior B praecise, quia cum cuiuslibet latitudinis incipientis a non gradu et terminatae ad aliquem gradum finitum sit medius gradus praecise subduplus ad gradum illam G 38v latitudinem | terminantem et ad quemcumquem gradum intrinsecum illius latitudinis in duplo citius deveniet praecise A quam B, sequitur quod A continue erit motus in duplo praecise velocior ipso B, quod prius fuit assumptum et probandum.

48 Aliud est etiam sophisma consimile quod tamen ex iam visis satis faciliter potest solvi, scilicet posito quod A motus uniformiter incipiat intendi et intendatur a gradu duplo ad C, et continue aeque velociter praecise intendatur sicut B motus qui uniformiter continue

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1 intensus] intensum C intensius K | esset] erit D *om.* G | A] in casu *add.* A in[2]] *om.* R 2 instanti[1]] erit A est C esset *add.* R | remissior] remissius K 3 secundum casum] *inv.* R | A] *om.* G | A...casu] in primo casu A A 4 seu intensior] *om.* U | erat] est C 5 enim] *om.* R autem U | satis] statim C bene *add.* R 6 incipiant] incipiunt D | incipiant uniformiter] *inv.* A | uterque] utrique C *om.* D numquam G | quiete] et *add.* G | in…praecise] velocius B praecise in duplo C 7 velocius] velocior R | praecise] *om.* K | et] tunc A *om.* BCDU | sequitur] et *add.* A ut nunc *add.* C sicut *add.* G sequitur…praecise] *om.(hom.)* D | A] *om.* K | erit continue] *inv.* ABCGKRU 8 quia] *om.* B | cum] *om.* D 9 terminatae] terminantem B terminante E | ad] in ABCDKR 10 sit…gradus] medius gradus erit K | gradus] motus *add.* G 11 et…latitudinis] *om.* C | quemcumquem] quemlibet R 12 illius] huius G citius] velocius C | deveniet] proveniet D devenit R | deveniet praecise] *inv.* ABCGKU | praecise] *om.* R 13 quod...B] *om.(hom.?)* B | continue] *om.* D motus RU | motus] continue RU | motus…praecise] praecise motus in duplo C | in duplo] *om.* R | praecise] *om.* RU | praecise velocior] *inv.* AG velocior] in primo *add.* U 14 quod] quam R | prius…probandum] est ad probandum assumptum C | et] *om.* AD ad G 15 est] *om.* B | est etiam] *inv.* DKR | etiam] enim A | quod] *om.* B | tamen] tantum C *om.* DGK | visis] dictis G | satis] *om.* GU 16 potest solvi] *inv.* ABCDGK solvi poterit R dissolvi poterit U | scilicet] quod *add.* E | posito] *om.* K | A] B E | motus] *om.* B | uniformiter incipiat] *inv.* B 18 praecise intendatur] *inv.* B intendatur] A motus *add.* R | motus] *om.* U | uniformiter continue] *inv.* B continue] *om.* ADR aeque velociter *add.* C

| intendetur ab ipso C gradu; arguitur tunc quod A motus continue D 68rb erit duplus ad B, quia incipit a gradu duplo et aeque continue intendetur sicut B a gradu subduplo, igitur continue aequaliter distabunt A et B ab invicem sicut in principio. Et in principio fuit A motus duplus ad B, ergo continue manebit etiam duplus ad eundem – quod tamen consimiliter improbatur omnino sicut prius.

49 Consimiliter etiam arguitur comparando remissionem ad remissionem, ut posito quod A incipiat uniformiter remitti in hora ab aliquo gradu ad gradum subduplum, et B ab eodem gradu quo A in duplo velocius continue uniformiter remittatur, arguitur quod B non remittetur ad quietem sed solummodo ad gradum subquadruplum ad illum a quo incipit | ipse A motus remitti. Quia K 42rb ex quo B praecise in duplo velocius remittetur quam A, in duplo minori tempore praecise deveniet ad quemcumque gradum remissiorem illo a quo incipit remitti quam faciet A; ergo cum A motus fuerit remissus ad gradum subduplum erit B remissus praecise ad gradum subquadruplum.

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1 intendetur…gradu] *om.* D | ipso…gradu] illo gradu C G | arguitur] probatur D arguatur G *lin.* R | tunc] sic *add.* G *exp. et corr. in*: sic K | quod] quia tunc K | continue erit] *inv.* R 2 quia] qui K | aeque] velociter *add.* ABCDGKRU 3 intendetur] A *add.* R | a] *om.* R | subduplo] duplo *sed corr. marg.* C 4 ab] ad ACDK | Et…principio] *om.(hom.?)* D 5 etiam] *om.* ABCGKRU 6 consimiliter] simpliciter B | improbatur] probatur K improbatur omnino] *inv.* D | omnino] *om.* ABCGRU 7 etiam] *om.* U arguitur] quod *add.* K 8 remissionem] remissione D | A] *om.* R | incipiat uniformiter] *inv.* R 9 subduplum] ad B *add.* R | ab[2]] sub D | gradu[2]] *om.* B quo A] ad quietem sed quod C 10 continue] *marg.* D | uniformiter] *om.* D arguitur] igitur *add.* R 11 gradum subquadruplum] quadruplum gradum G 12 subquadruplum] subduplum ABR | ad illum] ab illo B | a[1]] gradum G incipit…motus] ipse A motus incipit G 13 praecise…velocius] in duplo velocius praecise ABCGKU in duplo velocius R | praecise…remittetur] in duplo velocius remittetur praecise D | remittetur] remittitur AGU | quam A] *om.* B 14 praecise] quam A *add.* B 15 a[1]] *om.* C *iter.* R | quo] A *add.* K incipit] A *add.* R | faciet] faciat AG faceret C | A[1]] B K | ergo] *om.* K et cetera *add.* R | ergo cum] cum igitur A | A motus] *inv.* R 16 remissus[1]] remissius BCR | erit B] *inv.* R | remissus[2]] remissius BGKR | remissus praecise] *inv.* CU | praecise] *om.* A | praecise…gradum] ad gradum praecise D 17 gradum] suum G | subquadruplum] subduplum R subquatriplum U

E 14va **50** Ad haec omnia faciliter apparebit responsio ex praedictis, | R 30r supposito | scilicet cum hoc quod velocitas intensionis motus et C 15va remissionis attendantur penes acquisitionem vel deperditionem | latitudinis motus et non penes proportionem graduum eiusdem latitudinis deperditae vel acquisitae. Sicut enim prius dictum est signatis tribus gradibus continue proportionabilibus proportione dupla erit latitudo inter maximum et medium praecise dupla ad A 26vb latitudinem inter medium et tertium.|

51 Quibus visis dicitur in primo argumento quod ita velociter praecise per totam illam horam intendetur A motus sicut remittetur B et econtra, quia aequalem latitudinem praecise acquiret A sicut deperdet B. Et quando arguitur quod B motus remittetur ad subduplum et ad subquadruplum, et sic in infinitum; igitur si A uniformiter et aeque velociter continue per eandem horam intendetur, sequetur quod A intendetur ad duplum, et ad quadruplum, et sic in infinitum, dicitur negando consequentiam, quia formaliter sequitur oppositum.

1 faciliter] *om.* C | faciliter…responsio] apparebit responsio faciliter BD respondeo faciliter patet G responsio apparebit U | praedictis] iam dictis faciliter C 2 hoc] *om.* U | motus…remissionis] et remissionis motus D vel remissionis motus G 3 attendantur] attendetur R | acquisitionem] aliquam *add.* R 4 proportionem] acquisitionem C 5 Sicut…est] *om.* C | enim] *om.* A iam G et U | prius…est] dictum est prius AG 6 signatis] enim *add.* CU | continue] *om.* A | proportione] *om.* B 7 erit] et C 8 medium] mediam *sed corr.* R | tertium] nec contrarium(?) A secundum *sed corr.* R gradum *add.* U 9 dicitur] tunc dicatur G | primo argumento] *inv.* DK | ita] quod *add.* K 10 remittetur] *marg.* R 11 B] motus *add.* R 12 ad] suum *add.* U 13 ad] *om.* A | subquadruplum] subduplum R subtriplum U igitur…infinitum] *om.(hom.)* D | igitur…consequentiam] *iter.* R 14 uniformiter et] *om.* R | aeque…continue] continue aeque velociter R per…intendetur] intendetur per eandem horam R | horam] praecise *add.* G 15 sequetur] sequitur AGKR | et ad] *om.* G | ad^2] *om.* CKR 16 quadruplum] quatriplum(*sic!*) U 17 quia] quoniam ABCDGKRU formaliter] forte K | sequitur] sequetur BCGRU

9 Quibus] ad primum *marg.* E

52 Ad secundum, cum arguitur quod in casu illo erit A motus aliquando in duplo illo B velocior dicitur negando, quod illud est repugnans, sicut sufficienter est ibidem probatum.

53 Et ad argumentum, cum arguitur quod quocumque gradu velocitatis dato sub quo erit tam A motus quam B quod in duplo citius erit A sub illo gradu quam B, dicitur concedendo illam propositionem et etiam quod in duplo citius erit A sub gradu quadruplo ad C gradum quam B.

54 Sed cum ulterius infertur ex illo quod cum A primo erit sub gradu quadruplo ad C, erit B tunc primo sub gradu duplo ad C gradum, dicitur negando consequentiam. Et causa est quia gradus duplus ad C non est gradus medius inter C gradum et gradum quadruplum ad C sed multo minor quam medius inter C et suum quadruplum. Quia, sicut prius dictum est, gradus medius inter quoscumque duos gradus per aequalem latitudinem praecise distat ab uno gradu extremorum sicut ab altero et econtra. Sed latitudo inter gradum quadruplum ad G 39r C et inter gradum duplum | ad ipsum C est praecise dupla | ad U 62rb latitudinem inter C gradum et gradum duplum ad ipsum. Ex quo satis manifeste apparet quod gradus duplus ad C non est medius inter C et suum quadruplum, et ita non sequitur conclusio intenta

1 secundum] tertium C | cum] casum A | A...aliquando] *om.* D 2 aliquando] velocior om. A | illo…velocior] velocior ipso B BDGKRU intensior ipso B C | velocior] *om.* A | negando] *om.* ABCDGKRU | illud] *om.* U 3 est…probatum] iam probatum est B | ibidem] alibi D ibi U 5 erit] A motus *add.* R | quod] quia K 6 A] motus *add.* A | dicitur concedendo] sic conceditur D | illam propositionem] istam consequentiam K 7 etiam] illam *add.* G | quod] quando B | sub] isto *add.* C 8 quadruplo] *om.* R | ad C] *iter.* R | B] erit *add.* B 9 Sed...B] *om.(hom.?)* B | cum[2]] quando ACKRU primo erit] *inv.* U 10 quadruplo] quatriplo(*sic!*) U | C] gradum *add.* U erit…tunc] tunc B erit C | erit…primo] et B tunc praecise erit D | primo] praecise AG 12 gradus] *om.* ABCGKRU | inter] quartum gradum et inter *add.* D | C[2]] quartum D | gradum[2]] *om.* U | gradum[2]...C] inter D 13 suum] gradum *add.* C 14 inter] etc G 16 ab altero] aliter K | altero] alio A gradum] *om.* R | quadruplum] quatriplum(*sic!*) U | ad] ipsum *add.* A 17 et] *om.* R | gradum duplum] *inv.* G | C[1]] totum C | ad[2]] totam *add.* C ipsam *add.* G 18 inter C] intelligit G | et gradum] *om.* C | gradum[2]] *om.* DF ipsum] C *add.* A 20 quadruplum] quatriplum(*sic!*) U | conclusio] consequentia K

1 Ad secundum] ad secundum *marg.* E

scilicet quod cum A erit primo quadruplus ad C gradum, erit B tunc primo duplus ad eundem.

55 Unde conceditur tamquam satis imaginabile seu possibile, quod A motus per infinitam latitudinem D 68va | velocitatis excedet B motum et erit K 42va | ipso velocior seu intensior, et tamen A motus nunquam erit in duplo velocior ipso B. Quamvis enim in infinitum maioretur distantia seu latitudo secundum quam fiat A motus velocior B, non tamen propter hoc sequitur quod in infinitum maiorabitur proportio A respectu B, nec etiam sequitur quod ad duplum attinget. Sicut positis duabus quantitatibus continuis aequalibus quarum una continue uniformiter acquirat duplum ad illud quod altera, non sequitur quod aliquando erit aliqua illarum dupla ad reliquam, quamvis per imaginabile per infinitam quantitatem excedit una illarum alteram, A 27ra | quia cum semper totum de novo acquisitum ab una sit duplum ad totum de novo acquisitum ab altera et antiquum est aequale antiquo, nunquam erit totum aggregatum ex novo et antiquo in una duplum ad totum aggregatum in altera. Oportet enim

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1 quod] *om.* U | A] motus *add.* G | erit primo] *inv.* ABG praecise erit CDR primo est K ideo erit U | quadruplus] quadruplum U | erit[2]] *om.* D et G | B] *om.* D | tunc primo] praecise C praecise tunc DGRU *inv.* K 3 imaginabile…possibile] *inv.* R 4 excedet] excedit AD 5 ipso] ipse RU ipso…erit] *om.(hom.?)* D | seu intensior] *om.* R 6 enim] infinita distantia sit seu *add.* A *om.* G | maioretur] illa *add.* G 7 non tamen] omnino tum G 8 tamen] *om.* C 9 A] *om.* K | duplum] duplam proportionem G 10 quantitatibus] vel quantitatibus *add.* C | continuis] *om.* G continue R 11 continue] praecise D *lin.* R | acquirat] acquiret G | quod] acquirat *add.* ACDKU *om.* B acquiret *add.* G requirit *add.* R 12 aliqua] altera GK 13 quamvis] quantum C enim *add.* KU cum forte R | imaginabile] imaginationem CKU sit quod *add.* D 14 illarum] *om.* R | alteram] reliquarum B reliquam R | quia] tamen *add.* C 15 de novo] *om.* BCD acquisitum] *om.* CD | et] quia tamen D 16 nunquam…antiquo] *om.(hom.)* R | totum] *om.* D | novo] acquisito *add.* D 17 duplum] *om.* C aggregatum] consimiliter *add.* DRU | altera] consimiliter *add.* A consimiliter enim add. B quia tunc consimiliter *add.* G | Oportet] oporteret D | Oportet enim] *inv.* B | enim] *om.* G *lin.* R

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2 eundem] exemplum potest haberi in numeros faciliter ponendo quod A et B moveantur a latitudine ut 2 usque ad latitudinem ut 8 et A in duplo velocius B ut dicit, tunc constat quod duplum ad 2 quod est 4 non est praecise subduplum ad latitudinem quae est inter 8 et 2 quia non aequaliter distat ab extremis, quod tamen respondetur quia 4 distat a 2 per 2 et ab 8 per 4 et istae sunt extrema igitur erit continuatio 5 sed cum 5 sit duplum ad 2 sicut intentum *marg.* D

quod antiquum similiter esset proportionale ad antiquum sicut novum ad novum.

56 Sed contra illud forte arguitur posito quod A et B sic intendantur saltem difformiter A continue <in duplo> velocius B quousque uterque illorum habuerit omnem gradum velocitatis imaginabilem, videlicet usque ad gradum velocitatis infinitum secundum imaginationem, et ab illo gradu postea per omnia consimiliter remittantur usque ad C gradum A continue velocius B in duplo. Et sequitur quod in via remissionis erit A motus continue subduplus ad B, igitur in via intensionis erit A aliquando duplus ad B.

57 Ad illud dicitur negando consequentiam, quia in remissione incipiunt A et B remitti quasi a non gradu et subito deperdet uterque illorum latitudinem infinitam, quod non accidit in via intensionis. Ideo argumentum non procedit.

58 Ad aliud consimiliter respondetur cum ponitur quod A motus uniformiter incipiat intendi et aeque velociter a gradu duplo sicut B a B 57rb subduplo gradu, negando quod A motus erit aliquando | duplus | ad R 30v B iuxta casum illum. Et negatur consequentia illa: A et B continue aequaliter distabunt secundum aequalem latitudinem velocitatis sicut in principio, et in principio fuit A motus duplus ad B; igitur continue

1 similiter] *om.* G | similiter esset] esset consimiliter A similiter…proportionale] proportionabile esset consimiliter U | antiquum[2]] antiquo B 3 Sed] forte *add.* GKR | forte] *om.* ABGKRU | B] motus *add.* ABCGKRU | sic] *om.* A | intendantur] intendatur G 4 difformiter] *om.* G | in duplo] *om.* BEK | B] sicut prius *add.* B | quousque] usque K 5 habuerit] habeat G | imaginabilem] imaginabile R 6 videlicet] *om.* CGU scilicet D 7 postea] prime(*sic!*) U | per…consimiliter] similiter D consimiliter] consimilia G 8 remittantur] remittatur D | A continue] et continue A G | velocius…duplo] in duplo velocius B ABCDGKRU 9 erit…intensionis] *om.(hom.)* K 10 igitur] *om.* G | A] *om.* G | aliquando] motus GR | duplus] duplum K 12 incipiunt] incipient R | quasi] *om.* KR a] ad BK | gradu] gradum B remissionis *add.* G | deperdet] perdet E uterque] utrique CD 13 illorum] iste K | accidit] accipit K | intensionis] et *add.* ACDGKRU 14 Ideo] ergo B 15 Ad aliud] *om.* G | ponitur] ponatur R 16 uniformiter incipiat] *inv.* AG | et] remitti et *add.* D et…velociter] *om.* G | velociter] uniformiter *add.* B 17 subduplo gradu] *inv.* ABCDGKU gradu subduplo dicitur R | erit aliquando] *inv.* AR 18 Et] etiam D | consequentia illa] ista consequentia C | illa] scilicet *add.* A ista DK *om.* G 19 aequaliter distabunt] *inv.* A 20 in[1]] a D | et…principio] *om.* K igitur] econverso *add.* G

3 Sed contra] contra responsionem *marg.* E | 15 Ad aliud] ad aliud *marg.* E

postea <aut> aliquando erit A duplus ad B quia aequalis excessus non continue servat aequalem proportionem, sicut satis apparet tam in quantitatibus continuis quam discretis, sed excessus proportionales. Unde continue maiorabitur proportio illius A respectu B quamvis latitudo secundum quam A excedit B continue manebit aequalis, aequalis enim est excessus quo duo excedunt unum et quo tria excedunt duo, et tamen proportio duorum ad unum est dupla, et trium ad duo est proportio sexquialtera, et plus adhuc continue variatur in ascendendo, ideo etc.

59 Ad ultimum, cum arguitur quod si A motus | E 14vb uniformiter remittatur in hora ab aliquo gradu ad suum subduplum, et B in eadem | K 42vb hora uniformiter ab eodem gradu quo et A in duplo velocius remittatur, quod B non | G 39v remittetur ad quietem seu ad non gradum, sed solummodo ad gradum subquadruplum ad gradum illum a quo incipit remittere, | A 27rb dicitur quod hoc est falsum et repugnans antecedenti. Quia tunc B deperdet duplam latitudinem ad illam quam deperdet A, et A deperdet medietatem istius latitudinis quae incipit a non gradu et terminatur ad illum gradum a quo incipit

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1 aut] vel saltem D *om.* E | aut aliquando] ante K *om.* R | aliquando] postea *add.* ABCDGU | A] motus *add.* BG | B] motum *add.* G | aequalis] aequales B 2 servat] servant B generat C | apparet] ex iam dictis *add.* B 3 sed] *corr. marg. in*: secundum D | sed…proportionales] *om.* K 4 proportionales] proportionalis G | Unde] ubi G | maiorabitur] minorabitur CDU maioratur G 5 respectu] et G | continue] *om.* G 6 aequalis^{1}] excessus *add.* G | est excessus] *inv.* B | duo] 2 B 7 tria] 3 BD tres CR | excedunt] *om.* D | duo] 2 BD | tamen proportio] *inv.* A | duorum] 2 ABD 8 unum] 1 ABD | est^{1}] proportio *add.* A | et^{1}] proportio *add.* A sed G | trium] 3 ABD | duo] 2 ABD | est^{2}] *om.* R | proportio] solummodo ABRU *om.* CDGK | proportio sexquialtera] sexgaltera(*sic!*) solummodo D | plus] proportio C 9 adhuc] ad C C *om.* DU | continue variatur] *inv.* A | variatur] variat BCGR variabitur D | in] *om.* AG | ideo] igitur ABDGKRU | ideo etc] *om.* C 12 uniformiter] *om.* D | ab] in E | quo] quadruplo D | et] etiam C | A] incipit remitti *add.* A | in] *om.* K | in…velocius] uniformiter C in…gradum^{2}] *lin.* E 13 velocius] alia lectio modo velociori *marg.* A remittatur] ita *add.* D et *add. lin.* R | quod] quam AK | B] alia lectio quod B *marg.* A | non^{1}] remittatur et B non *add.* R 14 subquadruplum] et *add.* K subquatriplum(*sic!*) U 15 illum] primum R | a] *om.* U | incipit] incepit R remittere] alia lectio remitti *marg.* A remitti K 16 Quia] et R | deperdet] perderet E | duplam…deperdet^{2}] *om.* U 18 quae] qui U | a^{2}] *om.* U

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10 Ad ultimum] ad ultimum *marg.* E

A | remittere, quia illius latitudinis est medius gradus praecise D 68vb
subduplus ad gradum terminantem illam latitudinem. Et ab illo gradu extremo usque ad suum subduplum remittetur A motus, igitur cum in aequali tempore continue uniformiter remittetur B ab eodem gradu et in duplo velocius A, sequitur quod B in illa hora remittetur ad quietem.

60 Et ad argumentum cum arguitur quod non, quia ad quemcumque gradum remittetur A et B in illo casu in duplo citius remittetur praecise B ad eundem gradum quam A, ergo cum B | primo fuerit U 62va
remissus ad gradum subquadruplum erit A tunc remissus ad gradum subduplum, dicitur negando consequentiam, quoniam sequitur oppositum propter inaequalitatem latitudinum inter gradus aeque proportionales, sicut prius dictum est.

61 Infinita possunt fieri sophismata de velocitate motus localis et de comparatione unius motus ad alium, et de comparatione intensionis ad remissionem, et coniungendo latitudinem intensionis et remissionis motus cum latitudine velocitatis.

62 Sicut enim contingit aliquid moveri velocius et aliquid tardius, et aliquid intendere motum suum, et aliquid remittere, ita etiam

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1 remittere] remitti A intendere K | quia] et K | est] erit C 3 extremo] exclusive A(alia lectio extremo *marg.*) 4 aequali] tali G | continue uniformiter] *om.* C | uniformiter...B] remittetur B uniformiter G | B] motus *add.* B uniformiter *add.* C | ab…gradu] ad eundem gradum *sed del. et corr. marg.* R 5 et] *om.* AK | illa hora] hora ista K 7 cum] quando BCDGU arguitur] dicitur D | quia] *marg.* A 8 remittetur[1]] remittuntur K | A...B] *inv.* G | illo…in] *om.* R | illo…citius] duplo citius in illo casu G | remittetur praecise] *inv.* ABCDGKU 9 praecise] *om.* R | B] *om.* AR | B] *om.* D | B primo] praecise B G | primo] praecise C | primo fuerit] prius fuit K primo…remissus] fuerit remissius primo R 10 remissus[1]] remissum G remissius K | ad gradum[1]] *om.* R | subquadruplum] ad A *add.* R subquatriplum(*sic!*) U | tunc] primo *add.* A *om.* R | remissus[2]] primo *add.* BRU praecise *add.* C remissum praecise G remissius primo K gradum[2]] tunc *add.* G 12 inaequalitatem] *om.* G aequalitatem U inaequalitatem latitudinum] latitudinem inaequalitatum C | latitudinum] latitudinem G | inter] tunc R | gradus] gradum G | aeque] inaequaliter C aeque proportionales] om. G 14 possunt] possent E | de comparatione] de perditione E 15 motus…comparatione] *om.* U | et] *om.* BDGK 16 remissionem] motus *add.* B | coniungendo] comparando D comparando intensionis G | et[2]] vel GR | et remissionis] *om.* D 19 aliquid[2]] aliud K etiam] *om.* BCDGKRU

contingit <et> aliquid velocius intendere et aliquid tardius et consimiliter remittere, sic quod sicut imaginata est latitudo velocitatis in motu incipiens a quiete in infinitum ascendens, ita etiam est imaginabilis una latitudo intensionis et remissionis secundum quam in infinitum velociter vel tarde contingit motorem aliquem intendere motum suum vel remittere, et illa latitudo consimiliter se habet respectu latitudinis motus sicut se habet latitudo motus respectu magnitudinis et quantitatis continue vere pertransibilis succesive. Et consimiliter respondendum est in una comparatione et in alia comparando etiam nomen accidentale motus respectu nominis essentialis signantis motum. Contingit etiam frequenter fieri sophismata, ut est possibile quod A punctus continue moveatur aeque velociter sicut prius et uniformiter, et tamen quod A aliquando in infinitum tardius ascendet vel descendet quam in tali instanti vel in tali descendebat, vel quod nec ascendat nec descendat. Et quod illa divisio est aeque velox motus sicut ipsa prius fuit, et tamen est multum tardior divisio, unde divisione K 43ra uniformi | continue dividet A tardius et tardius; et gradu velocitatis

1 contingit] accidit G | et[1]] *om.* E etiam GKR | velocius intendere] *inv.* AB tardius] intendere *add.* G 2 consimiliter…imaginata] ita sicut imaginabile G | remittere sic] *inv.* R | sic quod] similiter B | imaginata] imaginabilis K 3 incipiens] incipiente G | in[2]] *om.* U | ascendens] ascendendo G 4 imaginabilis] imaginata G imaginabile R | una] ista B *om.* G | et] vel GK 5 secundum] per G | in] etiam K | in…tarde] infinita velocitate vel tarditate G | tarde] tarditer D | motorem] mobilem R 6 vel] et K | illa] ita C 8 motus] *om.* D | et] in D | continue] *om.* R | vere] *om.* ABK ubi E 9 pertransibilis] pertransitae A pertransibit E | respondendum] *om.* G | respondendum est] *inv.* CDGKR | est] *om.* B 10 et] ac B *om.* C | in] *om.* ADK | etiam] *om.* G | nomen] nullum D 11 essentialis] motus *add.* A essentialiter E | signantis] significantis ABGKRU significatis D | motum] motus D | Contingit] contingenter C | etiam] om. D et U | etiam frequenter] *inv.* B 12 frequenter] simile C | frequenter fieri] *inv.* G sophismata] sophisma C | possibile] ponatur *add.* C | punctus] pectus(*sic!*) U 13 et[1]] B *add.* K 14 A aliquando] *inv.* BC 15 tali[1]] *om.* A vel[1]…descendebat] ascendat vel descendat *sed del.* A | descendebat] descendat C | quod] idem *add.* A | nec] *lin.* R | ascendat] ascendebat K 16 descendat] descendebat K | illa] aliqua C | est] *om.* R | est…motus] motus est aeque velox K | ipsa] *om.* AC ipse E 17 et tamen] *om.* G | est] *om.* A | est multum] *inv.* R | multum] multo BGU | tardior] tarda K | unde] in *add.* CDGU 18 A] *om.* BK et *add.* E | et[2]] cum C

in duplo tardiori dividit A multo velocius quam gradu multo velociori dividit B, et sic de talibus infinitis in quibus omnibus multa possunt fieri sophismata quae non expedit recitare.| A 27va

63 Ideo viso generaliter penes quid tamquam quoad | effectum C 16ra
attendatur velocitas universaliter in motu locali, quia secundum proportionem potentiae motoris ad potentiam resistivam generaliter attenditur velocitas in quocumque motu tamquam quoad eius causam.

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1 tardiori] B *add.* C tardior E | dividit] dividet AR | A] *om.* C et G | multo[1]] multum G | multo velocius] alia lectio duple velocius *marg.* A | gradu] *om.* R | multo velociori] *inv.* G 2 velociori] fortiori D | infinitis] quibuscumque et *add.* G | infinitis…recitare] *om.* D | in] de *sed corr.* C 3 possunt fieri] *inv.* R 4 Ideo] isto D | viso] iam *add.* ABCDKRU | quid] quod K | quoad] penes D 5 universaliter] *om.* ABCDGKRU 6 motoris] in motis E motivae K moventis R | potentiam] rei motae quia ad *add.* D resistivam] resistentiam D | generaliter] *om.* K 7 attenditur] attendetur C quocumque motu] *inv.* A

Anonimus

Utrum in motu locali sit certa servanda velocitas

Tractatus de sex inconvenientibus

SIGLA:

O = Oxford, Bodleian Library, Canonici Miscellaneous 177, ff. 203rb—212va;
R = Paris, Bibliothèque Nationale de France, fonds lat. 6527, ff. 156va—169vb;
P = Paris, Bibliothèque Nationale de France, fonds lat. 6559, ff. 28rb—42va;
V = Venezia, Biblioteca Nazionale Marciana, lat. VIII. 19(3267), ff. 117v-145ve

Anonimus

Utrum in motu locali sit certa servanda velocitas?

1 Et arguo primo quod non, quia ex isto tunc sequitur quod talis velocitas attenderetur penes excessus potentiarum moventium
P 28va ad potentias resistentes sicut ponit| una positio <i. e. prima>,
V 118r aut penes proportionem |excessuum potentiarum moventium ad potentias resistentes sicut ponit secunda positio, aut penes proportionem proportionum potentiarum moventium ad potentias resistentes sicut ponit tertia positio.

2 Primae duae positiones demonstrative a pluribus improbantur, praecise a duobus famosis: a magistro Thoma de Bradvardyn in tractatu suo *De proportionibus* et a magistro Adam Pippewelle qui subtiliter hoc demonstravit. Nec tertia est ponenda, quoniam ex illa sequuntur plura inconvenientia.

<Inconvenientia ad tertiam opinionem>

3 Primo quod A et B sunt duo gravia cuius proportio gravitatis A ad suam levitatem est tanta praecise sicut proportio gravitatis B ad suam levitatem. Et A et B ponuntur in eodem medio aequaliter resistente utrique et A sufficit moveri in isto medio, et B non sufficit in illo moveri.

4 Secundo quod C et D sunt duo gravia quorum ad suas resistentias est eadem proportio et aequaliter sunt extra loca sua naturalia in consimilibus mediis praecise aequaliter resistentibus, et tamen deductis omnibus iuvamentis et impedimentis extrinsecis A continue movetur velocius quam B.

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2 servanda] signanda K 3 et...primo] arguitur V | tunc] casu K 4 excessus] excessum KR excesum V 5 potentias resistentes] suas resitentias K | positio] opinio V 7 potentias] *om.* R | potentias resistentes] suas resitentias K 8 moventium] motivarum R 9 potentias resistentes] ad suas resistentias R | sicut…positio] *om.* R 11 Bradvardyn] Berdvardin K Bardvadin R 12 et] *om.* K | Pippewelle] Papavilie K Palpavic R Pyppewelle V 13 subtiliter] sufficienter K | hoc] hic R | quoniam] quia K 21 resistente] in superficie R 22 in…moveri] moveri in illo medio K moveri in illo V 24 eadem] tanta R 27 extrinsecis] intrinsecis V

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11 a[2]…12 proportionibus] Cf. Thomas Bradwardinus, *Tractatus proportionum*, 86-94 | 13 nec...ponenda] Cf. Thomas Bradwardinus, *op.cit.*, 112.

5 Tertio quod quaecumque fuerit proportio G gravis ad suam resistentiam ipsum G grave movebitur infinita tarditate.

6 Quarto quod E, F sunt duae potentiae aequales intensive et extensive, et se habent ad suas resistentias in proportione aequalitatis, et utraque intendetur una alia velocius usque ad finem alicuius horae, et tamen in fine horae erunt aequaliter | intensae. R 156vb

7 Quinto quod H, I sunt duae potentiae motivae aequales intensive et extensive, et H sufficit moveri aeque velociter praecise cum C resistentia sicut I, ita quod eadem est proportio H ad suam resistentiam sicut I ad suam resistentiam vel eandem, et tamen si aliqua certa resistentia fuerit addita ad C, H sufficit moveri cum illa alia certa velocitate, et si eadem fuerit addita ad I, nullo modo sufficit moveri cum illa, vel quaecumque resistentia mundi fuerit addita ad C, H velocius sufficit moveri cum sua resistentia quam I cum sua, si eadem fuerit addita ad resistentiam I mobilis.

8 Sexto quod L, M sunt duo mobilia et utrumque illorum sufficit moveri in C medio aliqua velocitate, et eadem est proportio L mobilis ad suum medium sicut M ad suum medium quantum ad medium, et si resistentia C medii fuerit duplicata, tunc L sufficit moveri et M nulla velocitate mundi, et si primo fuerit eadem resistentia subdupla, tunc M sufficit moveri velocius in illo quam L deducta rarefactione, condensatione; et omnia cetera erunt paria, quin illa sint inconvenientia et contra positionem ingenium nullum dabit.

<Primum inconveniens>

3 E[1]] et *add.* V | duae] *om.* V | potentiae] magnitudines praecise K 4 resistentias] potentias R 5 utraque] utrabique R 8 H I] A et B V | I] A R motivae] *om.* R 9 H] *correxi ex* A P 10 I] *correxi ex* B P *lin.* R | ita…I] *om.* R | est proportio] *inv.* K | proportio H] C H sufficit moveri cum illa *add. sed exp.* P | H] A KPV 11 I] B KP | eandem] eadem V 12 H] quod K B V 13 alia] *om.* KR 15 ad] ipsi K a et *add.* R | H] et K 16 ad…I] resistentia ad A K 17 I] A R 18 L] et *add.* R 19 aliqua] a K | est] *om.* V 20 quantum…medium] *om.* RV 22 M] *om.* tamen *add.* R 23 subdupla…L] *om.* V | velocius] *om.* K 24 rarefactione] et *add.* RV 25 quin…dabit] *om.* R | illa] alia K

P 28vb **9** Ad probationem primi inconvenientis | arguitur sic. Sit A unum V 118v mixtum | uniformiter difforme compositum aequaliter ex gravi et levi et sic situetur A quod pars magis gravis sit sub centro mundi. Sit B aliud mixtum uniforme cuius gravitas ad suam levitatem sit sicut gravitas A ad suam levitatem, et aequaliter compositum ex gravi et levi. Sed ponatur B totaliter extra centrum mundi. Et sit medium circa centrum aequaliter resistens A et B. Et sequitur tunc conclusio quod A et B sunt duo mixta cuius proportio gravitatis A ad suam levitatem est tanta praecise sicut proportio gravitatis B ad suam levitatem; et A, B ponuntur in eodem medio aequaliter resistente, sicut patet ex casu; et A sufficit moveri in isto medio. Quod si negatur, contra: A sic K 203va positum appetit moveri et non impeditur, igitur movetur. | Assumptum probatur. Nam tota levitas in A ultra centrum appetit ascendere et tota gravitas in A citra centrum appetit contiguari cum centro mundi, igitur omnia promoventia A quantum ad motum erunt sua gravitas citra centrum et levitas ultra centrum, et nihil est impediens nisi solum levitas citra centrum, quia ponitur citius(*sic!*) vacuum citra centrum, vel quod se habeat ad medium extrinsecum in valde magna proportione maioris inaequalitatis, igitur nihil est quod impediat ipsum A quantum ad motum nisi solum levitas citra centrum; sed maior est proportio gravitatis in A citra centrum cum levitate in A ultra centrum ad movendum quam est levitas citra centrum ad resistendum, igitur ab ista proportione sufficit moveri; et ultra, igitur A sufficit moveri in isto medio et B non sufficit, nam B est mixtum uniforme per totum ita quod cuiuslibet partis B gravitas illius R 157ra partis ad suam levitatem est sicut | totius gravitatis B ad totam

1 arguitur sic] *om.* R | A] *om.* R 2 difforme] difformiter K 3 sit] situetur K 4 mundi] et similiter *add.* R 7 circa] citra | aequaliter] *iter.* R | resistens] resistent R 9 est…levitatem] *om.* R 10 levitatem] gravitatem K | A] et *add.* R 11 medio] et *add.* R 12 isto] uno R | contra] probatur nam *add.* R 13 positum] compositum R | impeditur] ipse R 14 nam] et iam V levitas] gravitas R | levitas…et] *om* (*hom*) K 15 ascendere…appetit] *om.* R citra] ultra K | contiguari] *lac.* V 16 omnia promoventia] iam impedientia R | promoventia] impedientia K 18 solum] sola V 19 citius] totius K 21 ipsum] o*m.* R | A] in *add.* K 23 A^{1}…levitate] *iter* K | in A^{2}] *lin.* R 24 movendum] motum KR | quam] quo V 26 et] cum K | est] unum *add.* K 27 illius] ipsius V 28 suam] aliam K | B…B] *om.* K | totam] suam R

1 primi inconvenientis] Cf. § 3.

levitatem in B, sed totius gravitatis B ad suam levitatem est proportio aequalitatis a qua proportione non est motus possibilis nec B habet aliunde iuvamentum ad motum, igitur B non sufficit moveri in isto medio, igitur propositum.

<Secundum inconveniens>

10 Ad probationem secundi inconvenientis sumitur sicut prius quod C sit unum corpus uniformiter difformiter mixtum et aequaliter compositum ex terra et aqua, et D unum aliud corpus uniformiter difforme aequaliter compositum ex aqua et terra, et quod O sit una superficies rotunda ad quam omnis aqua naturaliter inclinatur. Et quod C D sic applicantur ad O superficiem quod utrumque illorum habeat aequalem partem sub O et etiam C aequalem resistentiam de suo medio sicut D ex suo, et quod pars C habens plus de aqua sit supra O, et pars C habens plus de terra sit sub O, et | pars D habens plus de terra sit | supra P 29ra|V
O, et pars D habens plus de aqua sit sub O, et quod tam C quam D 119r
sufficiant secundum applicationem superpositam descendere per aliam certam partem sui medii. Istis positis et deductis iuvamentis et impedimentis extrinsecis sequitur quod C per aliquod tempus descendet velocius quam D per idem tempus. Quod probatur sic: C per aliquod tempus habebit maius iuvamentum et minus impedimentum ad descendendum quam D, igitur C per aliquod tempus velocius movebitur descendendo quam D. Consequentia palam patet. Antecedens probatur sic: C in praesenti instanti habet maius iuvamentum et minus impedimentum ad descendendum quam D, igitur C per aliquod tempus habebit, etc. Consequentia est manifesta, cum nullus excessus iuvamenti

1 in B] *om.* R | gravitatis] in *add.* K 3 nec] et R | B[1]] non *add.* R | aliunde] aliud KR 7 sumitur] assumitur V | sicut] ut R 8 difformiter] difforme K *om.* V | et] *om.* R 9 compositum] *om.* R | corpus] *om.* R 11 O] *lin.* K *om.* V 12 inclinatur] inclinat P | O] *om.* V 14 O] A V | et] quod *add.* R | ex] de KR | suo[2]] medio *add.* K 15 O] *om.* V | C[2]…plus] plus habens K 16 sub] supra R | O] A V | et…O] *om.* K | pars…O] *om.* R 17 O[1]] C V C…D] D quam C K | C…secundum] D quam C secundum sufficiant V 19 medii] *om.* R | iuvamentis…impedimentis] *inv.* KR 24 velocius movebitur] movebitur velocius in K | consequentia] omnia K 25 palam patet] patet et R patet manifeste V | in] ex K | instanti] *om.* K 27 etc] *om.*R

7 secundi inconvenientis] Cf. § 4.

descensus C aut impedimenti descensus D possit subito deperdi. Antecedens probatur, quia C in primo instanti habet aequale iuvamentum de tota terra in C ad descendendum sicut habet D de tota terra ad descendendum. Et C in praesenti instanti habet maius iuvamentum de aqua supra O ad descendendum quam habet D de aqua in D supra O ad descendendum, cum tota aqua in C sit multum intensior et maioris virtutis quam tota aqua in D supra O.

11 Item, omnes partes aquae in C supra O et aquae in D supra O nituntur esse, et aqua in D super C nititur esse immediate ipsi O, et si sic, igitur totum iuvamentum in C ad descendendum est maius quam totum iuvamentum D ad descendendum. Et eodem modo contingit probare quod in praesenti instanti totum impedimentum D ad descendendum est maius quam totum impedimentum C ad descendendum, cum C et D aequaliter habeant de impedimento extrinseco ad descendendum ex parte medii. Et D plus habeat de impedimento intrinseco ad descendendum ex parte aquae in D sub O inclinantis econtra ad C R 157rb quam habet C ex parte aquae in C | sub O inclinantis econverso, K 203vb sicut patet ex suppositis. Et si sic, cum C et D nitantur | naturaliter locari sub O, et utrumque habet aequale iuvamentum ad movendum ulterius per aliquod tempus, et C maius per idem tempus, sequitur conclusio principalis quod C, D mobilia sunt extra sua loca naturalia in consimilibus locis et consimilibus mediis praecise aequaliter resistentibus, et tamen deductis omnibus iuvamentis et impedimentis extrinsecis C continue movebitur velocius D.

<Tertium inconveniens>

1 descensus^{1}] descensui K | aut] sic V 3 in] per KR 5 O] D V 6 D^{1}…aqua] aqua A V | aqua^{1}] de aqua *add.* K | O] C V 7 C] se K 9 supra^{1}…nituntur] supra D nitentur V | et…O] *om.* (*hom.*) KR 10 esse^{1}…esse] *om.* (*hom.*) V | et aqua] aquae K | C] O K 12 totum] *lin.* P 15 cum…descendendum] *om.* (*hom.*) KR 18 in…O] a D sub B K | D] in C *add.* V | O] B V | inclinantis] *om.* R 19 C^{1}] *om.* K | inclinantis] *lac.* R 22 ulterius] *om.* K 23 C] et *add.* V 24 sua loca] *inv.* V | consimilibus^{2}] *om.* R 25 praecise aequaliter] *om.* K | tamen] *om.* V 26 omnibus…extrinsecis] iuvamentis extrinsecis et impedimentis V 27 D] B K

12 Ad probationem tertii inconvenientis sumatur casus iste quod G sit una terra pura sphaerica, quantaecumque magnitudinis volueris, quae sit extra locum naturalem citra centrum mundi et D medium, quantaecumque modicae resistentiae volueris, ita quod G ad suam resistentiam sit sufficiens proportio ad movendum et quantacumque volueris. Sit tamen gratia argumenti sua resistentia medii | uniformis per totum, ut pono, signata per 2. Cum isto V 119v pono quod deductis iuvamentis extrinsecis G moveat | ex seipso P 29rb D medium quousque idem G grave fuerit in suo loco naturali ut medium eius sit medium mundi et sit potentia motiva G gratia argumenti signata per 3. Tunc arguo sic: G movebit D ex se quousque devenerit ad locum suum naturalem ut medium eius sit medium mundi et antequam idem G deveniet ad locum naturalem ut medium eius sit medium mundi, habebit idem G aliquam resistentiam quae erit maior quam sua potentia motiva ad suum locum naturalem, igitur G per tempus movebitur infinita tarditate. Consequentia est manifesta et maior patet ex casu et minorem probo. Nam tota potentia motiva G in aliquo instanti antequam idem G deveniet ad centrum mundi ut centrum eius sit centrum mundi, excedat suam resistentiam intrinsecam per minus quam per 2 et plus quam per unitatem, ut ponitur, et si sic, cum tota resistentia medii fuerit uniformis et signata per 2, igitur tota resistentia intrinseca et extrinseca G antequam centrum eius sit centrum mundi erit maior quam tota potentia motiva eiusdem; nam tota resistentia G intrinseca et extrinseca erit plus quam 3, et non tanta fuit umquam eius potentia motiva, igitur antequam G deveniet ad centrum mundi per modum dictum, movebitur G cum maiori resistentia quam sit eius potentia motiva, igitur

1 sumatur] supponitur K 2 magnitudinis] frueris vel *add.* K 3 quae…volueris] *om.* R 4 modicae] *om.* K 5 G ad] inter G et V | ad[1]] *correxi ex* et P in R | ad[1]…resistentiam] in summum resistentia K 7 uniformis] uniforme K | pono] per totum *add.* R | per[2]] ut R 8 ex…10 sit[1]] *om.* per *add.* R 9 quousque] *om.* R 10 motiva] *om.* V 11 argumenti] exempli R 12 ut] et K 13 et…mundi] *om.* (*hom.*) KV | locum] suum *add.* R 14 mundi] et *add.* R | habebit…18 instanti] *om.* R 15 quam] *om.* R 17 est] satis *add.* V 18 antequam] numquam R 19 centrum[1]] medium K sit] medium sui *add.* K 20 intrinsecam] *om.* V | per] *om.* R 22 et] *om.* K 23 eius…centrum] *om.* R 24 erit] et R | motiva] *om.* K 25 tota] *om.* V | G] eius *add.* eidem *add. sed exp.* R 26 motiva] *om.* K 27 modum] medium K

1 tertii inconvenientis] Cf. § 5.

infinita tarditate. Et confirmatur argumentum sic: quam cito aliqua pars G fuerit ultra centrum mundi, habebit G resistentiam intrinsecam ex qualibet tali parte continue acquirendo maiorem resistentiam intrinsecam et maiorem. Tunc sic: tota resistentia G crescet continue usque ascendet ultra 3 et in principio fuit potentia G signata per 3 a qua proportione diminuetur eius potentia, et continue quam cito aliqua eius pars fuerit ultra centrum mundi, igitur G in aliquo instanti antequam centrum eius
R 157va sit centrum mundi movebitur a | proportione minoris inaequalitatis, igitur infinita tarditate.

<Quartum inconveniens>

13 Ad probationem quarti inconvenientis: pono quod E, F sint duae potentiae motivae aequales quae se habeant ad suas resistentias in proportione aequalitatis, et quod O sit gradus duplus ad illum gradum quem iam habet ipsum F et E, et etiam pono quod E potentia intendatur et hoc per uniformem acquisitionem potentiae quousque habuerit O gradum. Et sequitur conclusio quod E, F iam sunt aequales intensive et extensive per casum et se habent ad suas resistentias in
V 120r proportione aequalitatis, et una continue intenditur velocius | alia. Si concedatur, et tamen in fine erunt aequaliter intensae, quia in fine solum habebunt O gradum, igitur, etc. Ideo, si negatur, una illarum alia velocius intendetur, tunc nulla illarum velocius
K 204ra intendetur E | et istae duae intendentur, igitur E, F aeque
P 29va velociter intendentur et ultra, igitur E | ita velociter continue intendetur sicut F, et F uniformiter intendetur, igitur E

1 tarditate] movebitur *add.* R | sic] *om.* R 2 aliqua] aliqualiter R | G[1]] C R centrum] *iter.* V 3 ex…intrinsecam] *marg.* R | tali] simili R sui K 4 maiorem] et arguitur *add.* R | tunc sic] igitur si V 5 usque] quousque R et] igitur R tunc V 7 continue quam] cum KR 8 G] *om.* R 9 minoris] maioris V 13 pono] sumatur iste casus R | pono…F] *om.* V | pono…sint] sumatur K | E F] *om.* R 14 aequales] *om.* R 15 O] A V 16 habet] F *add. sed del.* R 17 E] *om.* K | potentia] *om.* V 18 habuerit…gradum] fuerit o signum et e uniformiter difformiter K 19 E[1]] et *add.* V | iam] *om.* V sunt] possibile (*sic!*) *add.* R 21 aequalitatis] *om.* R 23 igitur etc] ideo ei (*sic!*) R 24 una] *lac.* V | illarum[2]…intendetur] intenditur velocius R 25 duae] *om.* V | intendentur] sic *add.* R | igitur…27 intendetur[1]] *om.* K | igitur…27 et] *om.* R 26 E] 3 V

13 quarti inconvenientis] Cf. § 6.

uniformiter intendetur, igitur E acquirit medietatem potentiae acquirendae in medietate horae. Sed potentia acquirenda in tota hora erit sicut 2, igitur in medietate horae acquiret medietatem moti, supple unitatem. Et si sic, igitur potentia E assignabitur per 3 et sua resistentia per 2, igitur in medio instanti totius horae erit proportio totius potentiae E ad totam resistentiam sexquialtera<m>, et in fine erit dupla ad illam, quam habebit in medio instanti, igitur proportio dupla est dupla ad proportionem sexquialteram. Sed istud est impossibile et istud sequitur quod E potentia velocius intendetur quam potentia F, igitur etc. Et quod potentia E erit dupla in fine temporis ad illam quae erit in medio instanti, arguitur sic: E movetur certa latitudine motus uniformiter difformis incipiente a non gradu, igitur medius gradus est proportionaliter subduplus ad gradum ad quem terminatur in extremo intesiori, quia est accipere tria continue proportionabilia ex remississimo qui non est in isto medio gradu et intensissimo qui non est in isto, igitur remississimus qui non est in illo est duplus ad quemlibet illorum, igitur est duplus ad medium, igitur potentia acquirenda in fine horae erit dupla ad illam quae acquiretur in medio instanti et a proportione dupla, quod fuit probandum.

<Quintum inconveniens>

14 Ad probationem quinti supponitur casus iste quod H, I sint duae potentiae motivae, ut puta duae terrae purae, et assignetur potentia H per 6 et similiter potentia I per alia 6, et sit C una resistentia mixta ex terra et igne, ita quod terra sit sicut 3 et ignis sicut 3 similiter, et sit D resistentia simplex cuius resistentia sit signata per 2, et applicentur H ad C et I ad D optima applicatione, et ponantur omnia cetera paria, et sequitur conclusio manifeste.

3 2] E K 4 moti] motus K | supple] supra R | E] *om.* KRV 5 medio] *om.* V | instanti] *om.* R 6 E] *om.* R | resistentiam] potentiam R 9 istud[1]] idem K 12 instanti] substanti R 13 medius] medietas R 15 quia] quod K accipere] acquirendum V | tria] illa R 16 ex] extremo R | remississimo] remisso R | isto…18 in] *om.* R 17 isto] iste V | igitur] gradu K 18 quemlibet] aliquem V 19 igitur] a V | erit dupla] et duplum V 20 quae] continue V 24 I] et *add.* R 25 duae[2]] *om.* V 26 et[1]] *om.* R similiter] *om.* K 29 et[2]] *om.* R 30 omnia] *om.* R

24 quinti] Cf. § 7.

Nam H, I sunt duae potentiae motivae aequales extensive et intensive, ut patet ex casu, nam tanta est una, quanta est alia et intensive et extensive, quia utraque assignatur per 6. Et | [R 157vb] H movetur aequaliter cum C sicut I cum D, nam aequalis est proportio H ad suam resistentiam quae est solum ignis in C cum iuvamento terrae in C sicut I ad D, quia utraque proportio est proportio tripla. Et motus sequitur proportionem talem iuxta positionem, igitur H sufficit aeque velociter moveri cum C resistentia sicut I cum D resistentia. Iam addatur resistentia signata | [V 120v] per 3 ad C, ita quod tota resistentia H signetur per 6, et sequitur adhuc quod H cum iuvamento terrae in C sufficit moveri cum tanta resistentia, quia adhuc se habebit ad illam in proportione sexquialtera. Et si eadem resistentia fuerit addita ad D, nullo modo sufficeret I moveri cum illa, cum a proportione aequalitatis non sit motus. Et tunc inter illas potentias foret proportio | [P 29vb] aequalitatis, igitur, etc.

<Sextum inconveniens>

15 Ad probationem sexti supponitur casus iste quod C sit medium aereum cuius resistentia assignetur per 2 et sit L unum mixtum ex terra et igne, ita quod gravitas assignetur per 8, levitas per 2 et ponatur in C. Et sit M terra simplex cuius potentia assignetur per 4 et ponatur in eodem medio | [K 204rb] C ubi L ponitur, et sint omnia cetera paria ex parte mobilium et ex parte medii. Et sequitur conclusio manifeste, quoniam L movetur aeque velociter praecise in C sicut M et econtra secundum illam positionem, quia solum movetur ab aequali proportione praecise, quia utriusque illorum potentia motiva ad totam resistentiam est praecise dupla et motus sequitur proportionem, igitur etc. Tamen si resistentia

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1 H] et *add.* R 2 et] *om.* K 3 assignatur] signatur R 4 aequaliter] per *add.* K | I] *om.* K 5 C] est R *om.* V 7 et motus] *om.* R | iuxta positionem] *om.* K 9 iam] nam V 10 signata per] D *add. sed exp.* P 12 adhuc se] ad C H habet *add.* R 14 cum^2] et K 16 etc] *om.* R 19 sit] unum *add.* K 20 assignetur] signetur V | L] B V 21 assignetur] sit assignata K signetur R | 8] et *add.* KR 7 V 22 2] B V | ponatur] ponitur R | M] ibi K 23 assignetur] signetur R | medio] *corr. ex* extremo P extremo *add.* K | L] B V 24 omnia] *om.* KR | mobilium] l mobili K | et] *iter. sed del.* R 25 L] B V 26 C] fuit *add. sed exp.* R | M] in E K | et] *lin.* R 27 ab…proportione] a proportione aequali K 29 etc] et *add.* KV

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19 sexti] Cf. § 8.

medii fuerit dupla adhuc sufficit L moveri cum illa, quia adhuc potentia motiva L ad totam resistentiam extrinsecam et intrinsecam se haberet in proportione sexquialtera sicut 8 ad 6, igitur adhuc potentia motiva L excederet suam resistentiam et hoc divisibiliter, et cum quilibet excessus divisibilis sufficit ad movendum, sicut patet per Philosophum et Commentatorem, igitur L sufficit moveri cum C et duplicata sic resistentia medii M non sufficeret moveri in isto, sicut patet, igitur sequitur conclusio manifeste.

<Inconvenientia ad secundam opinionem>

16 Secundo ad quaestionem arguo sic: si quaestio foret vera, sequuntur inconvenientia ampliora contra positionem iam dictam quae sequuntur ex conclusione.

17 Primo quod A et B mobilia dividerent sua media inter se omnino aequalia et continue ab eadem proportione, et tamen A continue in duplo velocius quam B ceteris paribus.

18 Secundo quod C descendet in isto medio aliqua velocitate proveniente a certa proportione potentiae et hoc per certam horam, et in aliqua parte illius horae potentia C augmentabitur, et numquam diminuetur, et tamen sua potentia augmentata ipsum descendet et tardius in isto medio quam prius deducta condensatione medii.

19 Tertio quod D descendet in isto medio certa proportione velocitatis per aliquam horam et per aliquam partem illius horae | R 158ra
diminuetur potentia illius D, et numquam maiorabitur sua

1 L] *om.* V 2 L] *om.* KR B V | totam resistentiam] potentiam resistentem K | totam…et] potentiam resistentem R 3 8] 7 V 4 igitur] *om.* K adhuc] *marg.* tunc *add.* K | L] H R 6 movendum] motum V | sicut] sic V Commentatorem] *lac.* P et in physicis *add.* R 7 L] *om.* R | C] *lin.* K | et] *om.* R etiam K cum *add.* V | sic] sicut KP similiter R 13 arguo] arguitur V quaestio] *om.* V 14 ampliora] applicata K *om.* R 15 quae…conclusione] *om.* V | conclusione] et *add.* R 17 omnino] non K *om.* R 18 continue] *om.* K 20 potentiae] *om.* R 21 potentia] potentiae K 22 tamen] cum omnis V | augmentata] aucta K 23 et] *om.* RV 25 D] *om.* R | in] *om.* K 26 aliquam[2]] aliam V 27 illius…condensato] *om.* KR

5 quilibet…6 Commentatorem] Cf. Ricardus Kilvington *Utrum in omni motu potentia motoris excedit potentiam rei motae*, § 26.

potentia medio non condensato ceteris paribus, et tamen D descendet velocius quam prius.

20 Quarto quod aliqua terra pura ut E movetur naturaliter et
V 121r solum ex se aliquo gradu motus, quae | eadem terra pura E non appetit moveri.

21 Quinto quod F est fortissimum, quod non sufficit agere in B et idem F est similiter fortissimum, quod non sufficit agere in C, et tamen C est duplum ad B.

22 Sexto quod G est una potentia quae iam sufficit agere in B et continue resistentia illius B intendetur usque ad aliquod instans, forte ad duplum, et tamen post talem intensionem sufficit agere
P 30ra velocius quam prius | vel saltem aeque velociter.

<Primum inconveniens>

23 Ad probationem primi inconvenientis arguitur sic et ponatur casus iste quod A, B sint duo gravia simplicia et quod sint aequalia omnino, et quod sint duo media aequalia et aeque intensa praecise, et applicetur utrumque ad extremum unius medii, et quod utrumque tam A quam B se habeat ad suum medium in proportione dupla quantum ad movendum, et quod medium illius A continue ascendat a tanta proportione sicut se habet B ad suum medium. Hiis positis sequitur conclusio quod A et B ab eadem proportione divident sua media, quia utrumque a proportione quam habet ad suum medium, sed utrumque ad suum medium se habet in proportione dupla, igitur, etc. Et tamen quod A in duplo velocius dividet, etc. quam B dividet suum medium, probatur, quia si medium illius A continue quiesceret, ceteris paribus, A et B aeque velociter moverentur, sed medium illius A movetur ascendendo tanta velocitate quanta movetur ipsum B, igitur A movetur in duplo velocius B.

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1 D] *om.* K B R 4 se] *om.* V | pura E] *om.* R 7 similiter] idem R simpliciter V 10 intendetur] augetur V 11 tamen] tunc R *om.* V 16 A] et *add.* RV sint^2] sit R 17 sint] sit R | sint…media] media sint V | et^2] *om.* R | aeque] *add.* praecisa R 24 habet…medium] habent ad sua media R sed…medium] *om.* (*hom.*) K | ad^2…habet] se habet ad suum medium V 26 etc] *om.* KR | dividet^2] *om.* R 28 B] continue *add.* R

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15 primi inconvenientis] Cf. § 17.

24 Item, si A grave continue quiesceret et medium sic ascendat tanta velocitate sicut iam A dividit illud medium, tunc A moveretur ita velociter sicut B, sed iam descendet tanta velocitate in isto medio sicut B propter | ascensum medii, igitur in duplo K 204va velocius A dividet suum medium quam B.

25 Item, A dividet suum medium a proportione gravitatis A medii et ascensus illius medii et B solum a proportione gravitatis suae ad medium, sed proportio gravitatis A ad suum medium cum ascensu eiusdem medii ad proportionem gravitatis B ad suum medium est proportio dupla, nam quaelibet illarum proportionum est tanta sicut proportio B ad suum medium, igitur istae duae proportiones sunt duplae ad proportionem B ad suum medium, et A dividet suum medium secundum illas proportiones, igitur A in duplo velocius dividet suum medium quam B, et tamen ab eadem proportione, igitur sequitur conclusio.

<Secundum inconveniens>

26 Ad probationem secundi supponitur quod C sit unum mobile per omnia aequale ipsi A et supponantur omnia de C quae sunt supposita de A, et retineatur | casus prior, tunc ponatur C R 158rb augmentata se habet ad suum medium in maiori proportione quam potentia non augmentata, et hoc, positis paribus, medio non condensato, igitur maiori velocitate | movebitur in isto V 121v medio. Tunc ponatur quod C sit unum grave simplex et B sit unum medium in quo sufficiat descendere aliqua certa velocitate uniformi et quod medium ascendat aliqua velocitate, et quod per secundam medietatem illius horae augmentatur potentia C, et ascendat B medium per illam secundam medietatem horae velocius, et maiori proportione quam augmentetur potentia C.

1 continue] iterum R 2 A^2] *om.* V 4 propter] post *sed add. marg.* ascendo propter K | medii] medium R 6 A^1] *lin.* R | A^3…gravitatis] *om.* (*hom.*) K 11 igitur] *om.* R 13 et] cum K 14 B] D K 15 igitur…conclusio] G etc. V 18 supponitur] casus iste *add.* V 19 aequale] aequalia R | omnia^2] a R 21 augmentata] aucta K | habet] *om.* PV 22 augmentata] aucta K | hoc] ceteris R 23 condensato] condensaretur V | igitur] *om.* V 26 uniformi] uniformiter R | uniformi…velocitate] *om.* K | aliqua] certa *add.* V 27 secundam] priorem K illam R | medietatem illius] partem ipsius V illius] *om.* KR 28 B] *om.* V | horae] *om.* V

18 secundi] Cf. § 18.

P 30rb **27** Isto posito sequitur conclusio quod | A descendet aliqua velocitate in isto medio et in secunda medietate illius horae eius potentia augmentabitur, et tamen tunc movebitur tardius quam prius movebatur. Quod probo sic: nam prius movebatur tardius quam si medium quiesceret, quod arguo sic: ascensus medii aliqualiter impedit descensum C et magis quam si quiesceret, igitur non illa velocitate descendet C in illo medio sicut tunc descenderet. Consequentia est manifesta, antecedens probatur sic: quia si non, sequitur quod medium quantumcunque fuerit densum non impediret grave quantum ad motum descensus, quod est falsum et contra Aristotelem IV *Physicorum*, ubi ponit quod per subtiliationem medii convenit motu<m> velocitari in infinitum, igitur per densitatem medii potest motus tardari in infinitum. Sed magis impedit quam si foret densius quam iam est, et quiesceret, igitur talis ascensus medii impedit motum descensus C. Ex quo arguitur tunc ulterius sic: C in prima medietate movebatur tardius quam si medium quiesceret, sed in secunda medietate horae medium a maiori proportione ascendit quam prius ascendebat, igitur C nunc tardius descendit quam prius descendebat. Cosequentia nota est et antecedens sequitur ex casu, igitur consequens, igitur conclusio.

<Tertium inconveniens>

28 Ad probationem tertii ponatur quod D sit unum grave simplex et quod B sit unum medium uniforme per totum, et quod D moveatur motu descensus in isto medio, et quod medium continue ascendat certo gradu uniformi, et quod per secundam

1 isto] medio *add. sed exp.* P illo R 2 isto] illo R | illius] ipsius V 4 prius[1]…tardius] *iter.* R 7 illa] aliqua R | illa velocitate] ita velociter V velocitate descendet] velocior descendit K | illo] *lin.* P 11 Aristotelem] in *add.* V 12 per] propter R | subtiliationem] subtilitatem KR | convenit] contingit V | motum velocitari] moveri aeque velociter K 13 densitatem] descensum R | tardari…infinitum] retardari infinite K 14 quam[1]…foret] si fuerit V | iam] nunc K 15 talis] *om.* V 16 C[1]] *om.* KR | arguitur] arguo R 17 si] B K B *add.* R | medium] mere V | sed] sic V 25 D] B K

11 quod…13 infinitum] Cf. Arist., *Phys.,* IV, 8, 215b-216a; Aver., *In Phys.*, IV, com.72, f. 163ra. Cf. Ricardus Kilvington, *op. cit.*, § 58.

24 tertii] Cf. § 19.

medietatem illius horae diminuatur potentia istius quantum ad descensum, et quod illud medium ascendat tardius et a minori proportione quam sua potentia augmentetur. Et sequitur conclusio, nam prima pars est vera, scilicet quod D descendet in isto medio aliqua velocitate certa, et etiam illud est verum quod per aliquam partem illius sua potentia diminuitur et numquam
augmentetur. Et tamen quod | tunc movebitur velocius, probo: R 158va
quia si medium aequaliter ascendat, hoc est ab aequali proportione sicut potentia diminuitur, tunc continue aequali velocitate moveretur iuxta positionem, sed nunc tardius ascendit medium quam tunc descenderet D, igitur nunc movetur velocius D quam prius, igitur sequitur conclusio.

<Quartum inconveniens>

29 Ad probationem | quarti inconvenientis supponitur quod E sit V 122r
una terra pura naturaliter mota et solum ex se versus locum suum naturalem, et hoc aliquo certo gradu motus qui sit D. Tunc quaeritur, an E appetit moveri D gradu, an aliquo alio; si D gradu,
contra: E quantum est ex sui natura appetit sic moveri, ut | K 204vb
immediate post praesens instans esset in loco suo naturali, igitur E appetit moveri infinite velociter, igitur non D gradu.

30 Item, si E appetat moveri D gradu, cum E non appetat moveri
alio gradu quam D, igitur E tantum D gradu | appetit moveri, P 30va
quod tamen arguo esse falsum, quia sequitur E appetit moveri tantum D gradu, igitur E movetur ita velociter sicut ipsum appetit moveri, ex quo sequitur quod nullum medium impediet appetitum seu inclinationem quibus E appetit moveri. Et si sic, igitur E non habet resistentiam ex medio ad movendum, et per

1 illius horae] *om.* V 2 minori] maiori R 3 sua] si A V | augmentetur] augetur KV diminuatur R 5 etiam] tunc R 6 illius] ipsius horae V 7 augmentetur] augetur KV 9 tunc] et R 10 moveretur] intenderetur R 11 tunc] nunc R | nunc] tunc R | movetur…D] D movetur velocius R 15 inconvenientis] *om.* R | supponitur] proponitur K ponitur R casus iste V 16 mota] moveatur K motiva V 17 gradu] *lin.* P | sit] dicitur R 18 aliquo] aliqua R 19 ex] de R | moveri] sic *add.*V 20 immediate] prius et *add.* V 22 item] sic *add.* R | E[1]] D PV | D] E PV | E[2]] D PV 23 alio…quam] *om.* V | igitur E] *om.* V 24 quia] quod R | sequitur] E *add. sed del.* K *om.* R | E] D PV 25 tantum] *om.* R | D] E PV | E] D PV 26 impediet] appeteret R 28 E] D KPRV

15 quarti inconvenientis] Cf. § 20.

consequens, cum E non habeat resistentiam aliquam ex aliquo alio quam ex medio, sequitur quod E movetur infinita tarditate et non D gradu.

31 Item, si sic, tunc appeteret simul quiescere et moveri. Si aliquo alio gradu appeteret moveri quam D et non est maior ratio de quocumque alio gradu quam de D, igitur sequitur quod vel nullo gradu appeteret moveri, et tamen moveretur naturaliter, vel quod omni gradu appeteret, et sic sequeretur quod simul velocius et tardius, uniformiter et difformiter, finite et infinite appeteret moveri.

<Quintum inconveniens>

32 Ad probationem quintam supponitur casus iste quod F sit unus ignis simplex et B una aqua, et C unus aer, et sint aequalis potentiae, et sit quod F non possit agere in B, sed quodlibet eo fortius sufficiat; et sequitur tunc quod F sit fortissimum quod non sufficit agere in B et nec etiam in C. Tunc educatur caliditas de C et inducatur frigiditas tanta sicut est humiditas praecise. Tunc adhuc F est fortissimum quod non sufficit agere in B nec in C, quia adhuc non sufficit agere in C, et per quantumcumque foret maior, sufficeret, igitur, etc. Antecedens arguitur sic: quia frigiditas in ipso C est tanta sicut caliditas praefuit et humiditas sicut siccitas, et sunt aequales in quantitatibus, igitur sicut B C sunt aequalis potentiae, igitur si F sit fortissimum quod non sufficit agere in B, igitur F est fortissimum quod non sufficit agere in C, et tamen quod C est duplum ad B; probatur, quia C est duplae resistentiae quo ad ipsum F ad illud, quod fuit in principio, sed in principio fuit tantae resistentiae sicut B, igitur iam est C duplae resistentiae ad B, igitur, <etc.>.

33 Item, C aliqualiter resistebat in principio, sed tamen est sibi
V 122v additum | de resistentia sicut prius resistebat, igitur, etc.

4 simul] *om.* R | aliquo] *om.*R 8 appeteret] moveri *add.* K 9 infinite] hoc *add.* K 13 quintam] quinti KRV 14 et[1]] A R | aequalis] aequales R 15 sit] *om.* R 17 tunc] *om.* R 19 quod] et *add.* R | in[1]…nec] *om.* R 20 quia…C] *om.* R | per] *om.* R 21 etc] *om.* R 23 igitur] sui *add.* R 24 aequalis] aequales RV 25 B…in] *om.* (*hom.*) K 26 C[1]] B R 28 C] *om.* KR 29 igitur…31 etc] *om.* R 30 item...igitur] *om.* (*hom.*) K

13 quintam] Cf. § 21.

<Sextum inconveniens>

34 Ad probationem sexti supponitur | quod G sit unum calidum R 158vb
in summo scilicet ignis et capio unum aerem qui sit B, ita quod G ad B sit proportio dupla. Pono tunc quod aliquod agens inducat frigiditatem in B minorem tamen quam subduplam ad caliditatem in aere, tunc in fine se habebit ad frigiditatem ingeneratam in proportione maiori quam dupla et continue crescet resistentia, et numquam minorabitur proportio agentis ad passum, igitur sequitur conclusio.

<Inconvenientia ad primam opinionem>

35 Tertio sic: si quaestio, sequuntur adhuc inconvenientia multo plura praedictis et contra positionem iam dictam.

36 Primo quod A mobile continue intenderet motum suum per tempus et solum a proportione potentiae | motivae A ad suam P 30vb
resistentiam, et tamen per totum idem tempus inter potentiam motivam A et eius resistentiam est proportio aequalitatis.

37 Secundo quod nullum grave mundi potest intendere motum suum versus finem motus et hoc ubi grave movetur versus locum suum naturalem naturaliter vel, si intendat motum suum, velocius movebitur a proportione minori quam a proportione maiori. Et conitnue intendit motum suum ubi continue minoratur proportio potentiae motivae ad potentiam resistentem.

38 Tertio quod in infinitum egit A in B, et tamen post hoc aget C in B velocius quam A egit in B.

39 Quarto quod in infinitum velociter A incipit agere in B et continue A aget in B velocius et velocius quam ipsum incipit agere.

3 capio] pro KR 4 dupla] et *add.* V 5 caliditatem…aere] quantitatem aliquam K 6 in[1]] *om.* R | in aere] maiorem V | aere] aerem R frigiditatem] ibi *add.* R | ingeneratam] ibi generatam K generatam R 7 crescet] crescit R 8 numquam] et non R 13 tertio] ad quaestionem *add.* R | quaestio] aliquid etc. K aliquid sequitur *add.* R foret vera *add.* V multo] *om.* V 14 et] *om.* V 16 A[2]] *om.* K 18 est] erit K 21 naturaliter] *om.* K 22 minori] inaequalitatis *add. sed exp.* V | maiori] maioris V 23 proportio] proportione P 24 potentiae] activae *add.* K | potentiam resistentem] suam resistentiam R 25 quod] *om.* KR 26 in B[1]] *om.* K 27 A] *om.* R 28 A[1]] *om.* K | et velocius] *om.* V 29 agere] in B *add.* KRV

2 sexti] Cf. § 22.

40 Quinto quod A, B sunt duo puncta quae continue per certum K 205ra tempus movebuntur motu recto et super spatia |quiescentia, et A continue movebitur velocius B, et tamen non plus pertransiet in aequali tempore.

41 Sexto quod A et B sunt duo mobilia aequaliter distantia a terminis suis fixis et aeque cito devenient ad terminos suos fixos per motum rectum ad illos terminos, et A per totum tempus movebitur velocius B, et tamen B per idem tempus nec umquam movetur tardius A.

<Primum inconveniens>

42 Ad probationem primi inconvenientis supponitur casus iste quod A sit una potentia motiva et B sua potentia resistiva inter quas sit proportio aequalitatis, deinde augmentetur potentia et sicut crescit potentia, ita crescat eius resistentia proportionaliter, ut inter illam et eius resistentiam continue sit proportio aequalitatis. Et sequitur conclusio, nam potentia A continue intendetur per tempus, cum per aliquod tempus erit eius potentia continue maior quam est in praesenti instanti, quia crescit continue per casum. Et tunc arguo sic: A velocitabit motum suum per tempus et solum a proportione potentiae motivae ad suam V 123r resistentiam iuxta positionem, sed inter illas est | proportio aequalitatis, igitur conclusio.

<Secundum inconveniens>

43 Ad probationem secundi inconvenientis arguitur sic: quia si aliquod grave mundi existens extra locum suum naturalem possit

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1 quinto] *om.* V | A] et *add.* K | B sunt] et sint V | certum] totum R 2 et A] *lin.* R 3 velocius] quam *add.* R | pertransiet] pertransit R 4 aequali] eodem R | tempore] vel aequali *add.* R 5 mobilia] moventia V 7 illos] illo R 8 nec…movetur] numquam movebitur K 13 potentia[1]] pars V resistiva] resistive R 14 augmentetur] *correxi ex* augmenter P eius *add.* K potentia] A *add.* KV B *add.* R 15 crescit potentia] eius potentia crescit K potentia] A *add.* V | resistentia] potentia resistiva R 16 inter…continue] continue in ipsum et eius resistentiam V | sit…20 casum] maneat aequalis proportio K 17 aequalitatis] aequalis R | et...continue] *om.* R 18 erit…continue] continue erit eius potentia V 19 quam] sit *add.* V 20 et] *om.* RV | arguo] *om.* R 22 illas] *correxi ex* illa 23 conclusio] *om.* R etc. K 26 inconvenientis] *om.* KR 27 mundi] *om.* R

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12 primi inconvenientis] Cf. § 36. | 26 secundi inconvenientis] Cf. § 37.

intendere motum suum versus finem motus, sit illud A et sit A grave simplex extra locum suum naturalem, | R 159ra et sit medium circa centrum mundi, quod est eius locus naturalis uniformis resistentiae per totum, quod sit B. Et ponatur A in B, ita quod A secundum se et secundum quamlibet sui partem sit supra B, et sit C certum tempus quo sic movebitur et ita quod in prima medietate contingat centrum mundi, et in secunda moveatur ulterius quousque medium eius sit medium mundi, ita quod in fine temporis primo sit medium eius medium mundi. Tunc A grave non intendet motum suum versus finem, quod arguo sic: A per totam secundam medietatem | P 31ra C temporis movebitur cum maiori resistentia et maiori continue, igitur per totam secundam etc. minorabitur continue proportio potentiae motivae A ad suam resistentiam et motus sequitur proportionem, igitur A per totam secundam etc. tardabit motum suum, igitur per multum ante finem motus non intendet motum suum, quod est contra Philosophum et Commentatorem. Assumptum primum probatur: A per totam secundam medietatem C temporis movebitur cum aequali resistentia extrinseca et cum resistentia intrinseca maiori continue et maiori - ut demonstratum est superius - quam umquam prius movebitur, igitur, etc.

44 Item, si A intendat motum suum et hoc versus finem per aliquam partem secundae medietatis C temporis, et quanto appropinquat ad finem temporis, tanto movetur cum maiori resistentia intrinseca et maiori et aequali resistentia extrinseca, igitur A velocius movebitur cum resistentia maiori quam cum resistentia minori.

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1 motum] suum *add.* KRV 2 sit] M *add.* KR 5 secundum[2]] *om.* R 6 certum tempus] centrum K | et] *om.* KRV 7 in] *om.* K 8 medium[2]] centrum R | ita…mundi] *om.* (*hom.*) K 10 arguo sic] arguitur R 11 secundam] *om.* R | cum] et K 12 et…continue] *om.* R | secundam etc] *om.* K tempus R 13 minorabitur] manebitur K movebitur R | proportio] a proportione R | A] *om.* R 14 totam secundam] tantam illam R 15 igitur…suum] *om.* (*hom.*) KR 16 non] *om.* V 18 temporis] cum *add.* P *om.* R 19 movebitur] *iter.* K | cum[2]] quod R 20 intrinseca] *om.* R | et] *lin.* K | est superius] *om.* R 21 prius] post R 24 appropinquat] propinquat K ad finem] partes V | temporis] *om.* K motus R | cum] et R 25 et[2]] vel R 26 movebitur] continue *add.* R | cum resistentia[1]] quare quare si a R

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16 quod…17 Commentatorem] Arist., *De coelo*, I.8, 277a; Aver., *De celo*, com 88, 160-161.

45 Item, per totam secundam medietatem C continue remittetur potentia A et in fine C erit eius potentia motiva remissa ad non gradum potentiae. Igitur si A intendat, etc., igitur A intendet motum suum continue, ubi continue minoratur proportio potentiae ad potentiam resistivam. Et sic sequitur conclusio et antecedens patet, quoniam post medium instans C temporis A secundum sui medietatem inferiorem descendet sub centro mundi quousque medietas inferior sit totaliter sub centro et medietas superior supra centrum et centrum eius centrum mundi, sed continue usque centrum eius sit centrum mundi crescet resistentia ex parte partium ultra centrum, quousque sua resistentia sit aequalis potentiae motivae et centrum eius cum
K 205rb centro mundi, igitur, etc.|

<Tertium inconveniens>

46 Ad probationem tertii supponitur casus iste quod A sit unum calidum uniforme remissum, quod assimilavit sibi B deductis quibuscumque iuvamentis et impedimentis extrinsecis et quod A
V 123v egit continue secundum ultimum | sui, et quod C sit unum calidum in summo approximatum ad B, quod continue agit in B secundum ultimum sui quousque B fuerit assimilatum ipsi C, et quod C se habeat in maiori proportione ad B quantum ad
R 159rb assimilandum sibi B quam numquam habuit A ad B. | Et sequitur inconveniens, quoniam C continue velocius et velocius aget in B quam A egit in B, quod arguo sic: a maiori et maiori proportione C aget in B continue post hoc instans quam A egit in B, ut ponit casus, et velocitas motus sequitur proportionem iuxta positionem illam dictam, igitur C continue velocius et velocius aget in B quam

2 erit] est R 3 A[1]] non K *om.* R 5 potentiae] motivae *add.* R ad…resistivam] activae ad resistentiam K | potentiam resistivam] suam resistentiam R | et] *om.* K 6 instans] scilicet K 7 inferiorem] infinities V 8 sit totaliter] naturaliter sit V 9 et] cum K | eius] sit *add.* K 10 usque] cum K 12 resistentia] potentia K | et] cum K 13 mundi] *om.* K mundi…etc] *om.* R | etc] sequitur conclusio V 16 supponitur] sumitur KV 19 sui] suae potentiae R | sit…calidum] *om.* R 20 approximatum] appropinquatur R | in B] *om.* R 21 C] *om.* R 22 quantum] quo R 23 assimilandum] similandum R | sibi] ipsi K | A] *lin.* R | et] quo K ex quo V 25 egit] agit R 26 continue] *om.* R | instans] videlicet K | ut…22,1 B[1]] *om.* (*hom.*) V | ut…22,1 B[2]] *om.* (*hom*) R

16 tertii] Cf. § 38.

A egit in B. Et tamen quod in infinitum velociter A egit in B, arguitur sic: aliquando | maxima resistentia A fuit aliqualiter P 31rb magna et aliquando in duplo minor, in triplo minor, et sic in infinitum, et etiam ipsamet sua potentia non debilitata continue egit secundum ultimum sui, igitur in infinitum velociter A egit in B. Consequentia patet et minor ponenda est in casu, et maior probatur sic: quia A per partem ante partem assimilavit sibi B, igitur sequitur quod prius assimilavit sibi medietatem propinquiorem ipsius B quam medietatem remotiorem ipsius B et eodem modo assimilavit sibi prius primam partem proportionalem quam secundam, et secundam quam tertiam, et sic deinceps, et cum medietas propinquior fuerit assimilata ipsi A, tunc solum resistebat sibi medietas assimilanda, et cum secunda pars proportionalis istius A fuerit assimilata ipsi A, tunc solum resistebat ipsi A totum sequens illam partem proportionalem, et sic deinceps. Et per consequens sequitur quod aliquando resistebat sibi alia pars et alia in duplo minor, et alia in triplo minor, et sic in infinitum, et si sic, igitur infinite permutata fuit una resistentia, et per consequens in infinitum velociter egit A in B, quod fuit probandum.

<Quartum inconveniens>

47 Ad probationem quarti supponitur casus iste quod B sit unum calidum uniformiter difforme terminatum in extremo intensiori ad gradum summum exclusive et quod A sit unum calidum in summo approximatum ad extremum intensius B, et quod A se habeat in magna proportione ad agendum in B, et quod A aget continue in B a proportione maiori et maiori. Tunc sequitur quod A continue velocius et velocius aget in B, cum continue ipsum

2 arguitur] arguo V | maxima resistentia] maximam resitentiam K | A] *om.* V 3 minor^{1}] et aliquando *add.* K 5 egit^{1}] agit R | velociter] continue *add.* R 6 maior] ma *lac.* V 8 sequitur quod] *om.* V | medietatem] partem R 11 secundam^{2}] et 3^{am} *add sed exp.* P 13 solum] *om.* K | sibi] solum *add.* K 14 pars] medietas R | istius A] *om.* V | A^{1}] B K 15 A] *om.* K 16 aliquando] sibi *add.* R 17 alia^{1}] aliqua R 18 permutata] *correxi ex* permutatis P 19 in infinitum] infinite KV 23 quarti] quinti K 24 calidum] *om.* KR 27 A] *om.* KR | aget] *om.* V 29 A^{1}] *om.* V | cum continue] et tamen R | cum…aget] tamen ipsum continue aget K

23 quarti] Cf. § 39.

aget in B a proportione maiori, et velocitas motus sequitur proportionem iuxta positionem, et tamen in infinitum velociter A incipit agere in B. Quod arguo sic: quoniam B secundum extremum sui intensius secundum nullum gradum resistentiae resistit ipsi A, quia ad idem extremum terminatur alia frigiditas aliqualiter resistens et alia in duplo minus resistens, et alia in triplo minus resistens, et sic in infinitum, et cum ibi nulla sit resistentia nisi frigiditas, igitur secundum nullum gradum resistentiae B secundum extremum sui intensius resistit. Et tunc arguo sic: A approximatum ad extremum intensius B aget in B V 124r secundum illud extremum | et nullo gradu resistentiae resistit secundum illud extremum, igitur A infinite velociter aget secundum illud extremum calidum.

48 Item, in infinitum velociter A incipit agere in B, quia quodlibet summum approximatum extremo intensiori B sufficit sibi R 159va assimilare | B, et cum contingit assignare | aliquod calidum |K 205va finitum maius alicuius potentiae et aliquod in duplo maioris, et aliquod in triplo maioris, et sic in infinitum, igitur secundum nullum gradum potentiae B secundum extremum sui intensius B resistit ipsi A. Consequentia patet et antecedens arguitur: quia quodlibet calidum summum approximatum extremo intensiori B P 31va quibuscumque extrinsecis deductis | ipsum assimilabit sibi B, quod arguo sic: tota caliditas in isto extremo cum caliditate in B sufficienter dominatur supra frigiditatem, et per consequens quodlibet calidum summum assimilabit sibi B, et hoc immediate post hoc, igitur in infinitum velociter A assimilabit sibi B et cum A sit unum calidum in summo, sequitur propositum.

<Quintum inconveniens>

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1 B] continue *add.* R | maiori] et maiori *add.* V 2 A] *lin.* P 4 extremum sui] sui secundum extremum R 5 quia] quod V | alia] aliqua RV 6 aliqualiter] aequaliter R | et[1]] in R 7 resistens] *om.* KR 10 intensius] ipsius *add.* V 11 extremum] resistit *add.* K | et] B *add.* K | nullo] non V | resistit] duplum R 13 calidum] *om.* KV *marg.* P intensius R 14 quodlibet] calidum *add.* KRV 15 sufficit] *om.* R 16 cum] *om.* KR 17 finitum maius] summum KRV 19 B[2]] *om.* KRV 22 extrinsecis] impedimentis *add.* V ipsum] *om.* V 24 frigiditatem] superficiem R | consequens] sequitur quod *add.* K 25 et] cum A *add.* R 26 A[1]] *om.* V | cum] quod R 27 calidum…summo] summe calidum R

49 Ad probationem quinti supponitur quod E, F sint duo corpora luminosa, et recito argumentum illud ratione sexti et proximi argumenti, aequalia intensive et extensive, et C, D sint duo obstacula aequalia, et aequaliter distet C ab E sicut D ab F, ita quod C, D causent umbras aequales, et corrumpantur C, D obstacula continue aequaliter quousque illa fuerint totaliter corrupta. Sed pono quod quamdiu aliquod utriusque manebit, quod illud aliquid causet umbram, tum continue minoretur et minoretur usque ad non gradum quantitatis. Tunc illud suppono quod E luminosum continue maioretur nulla remissione facta in E nec intensione nec aliqua transmutatione facta in F luminoso, et ponatur A in cono umbrae C et B in cono umbrae D et continue moveatur A in cono umbrae C et B in cono umbrae D, et A continue moveatur mensurando conum C, ita quod A semper tangat conum illum et B similiter tangat conum D, et sequitur conclusio quod A et B sunt duo mobilia aequaliter distantia a terminis suis fixis, ut sequitur ex casu, et aeque cito devenient ad terminos suos fixos. Nam tam cito erunt A, B mobilia ad terminos suos quam cito erunt C, D umbrae corruptae et non prius aliquod illorum quam alterum, sed C, D umbrae erunt simul et aeque primo corruptae. Igitur A, B mobilia simul erunt ad terminos suos, ita quod neutrum citius altero, et A per totum tempus movebitur velocius B, nam A continue movebitur ita velociter sicut conus umbrae C, et B ita velociter sicut conus umbrae D, sed conus umbrae C continue movebitur velocius cono umbrae D,

1 supponitur] casus iste *add.* V | F] et B R 2 recito] recita K recitat R retineo V | illud…argumenti] ad rationem sibi argumenti proximi R sexti et] sibi K 4 aequalia] interaequalia K | distet C] distent K | D] *om.* V ita] it R 5 causent] tenent K | C[2]…aequaliter] *om.* K 6 illa] ipsa R totaliter] aequaliter disposita K 7 aliquod] aliquid K 8 causet] *lac.* V 9 quantitatis] et *add.* K quantum R | tunc illud] ut R 10 maioretur] minoretur K | remissione] resistentia K | facta] *om.* R 11 intensione] nec extensione *add.* R 12 B] B F R | et[2]…D] *om.* (*hom.*) K | continue…A] a continue moveatur R 13 in[1]…moveatur] *om.* (*hom.*) R 14 conum] umbrae *add.* V 15 illum] illud R C *add.* K | et[1]…conum] *om.* V | B…et] *om.* (*hom.*) R | similiter] semper K | conum[2]] illius *add.* K 16 aequaliter] aequalia R 17 ex] A R 18 tam] ita R | A] et *add.* KRV | mobilia] *om.* V 19 suos] fixos *add.* V | non] tamen R | aliquod] unum R 20 alterum] aliud V | umbrae] *om.* K 21 A] et *add.* RV 22 citius] in *add.* K | et] cum K

1 quinti] Cf. § 40.

igitur, etc. Antecedens arguitur: quia si E luminosum non maioretur aliis ceteris paribus, tunc aeque velociter moverentur | [V 124v] illi duo coni cum umbra, tunc praecise aeque velociter corrumperentur versus illa obstacula, sed iam conus umbrae C velocius movebitur quam tunc moveretur cum umbra C, propter maiorationem E velocius continue corrumpetur quam corrumperetur, si non foret huius maioratio, et tamen non plus erit pertransitum ab A in aequali tempore quam a B cum spatia quiescentia et per quae solum distant a terminis suis sint aequalia, et illa solum erunt pretransita ab A et B punctis motu recto, igitur, et sic sequitur conclusio quinta.

<Sextum inconveniens>

50 Ex quo similiter | [R 159vb] sequitur conclusio sexta quod A et B sunt duo mobilia aequaliter distantia a terminis suis fixis et aeque cito devenient | [P 31vb] ad terminos suos fixos motu recto, et A per totum movebitur velocius B, sicut totum est determinatum, et tamen B per idem tempus non movetur nec movebitur tardius A. Quod arguo sic: A per totum tempus a primo instanti movetur velocius B et utrumqe movebitur motu recto versus suum locum et terminum, et in principio distabant ab illis terminis aequaliter, igitur A per totum tempus minus distabit a termino suo quam B a termino suo. Signo tamen aliquod instans intrinsecum istius temporis, quod sit C, in quo inaequaliter distant a terminis suis, et arguo sic: A et B iam inaequaliter distant a terminis suis et B plus distat a termino suo quam A et aeque cito deveniet motu recto ad terminum suum sicut A ad terminum suum, igitur B per totum tempus ab hoc instanti usque in finem motus | [K 205vb] movebitur

1 quia] quod R 2 maioretur] minoretur R 3 cum umbra] et umbrae R umbra] umbris K 8 pertransitum] pertranseundi K | cum] *om.* R 9 per quae] ipsa R 10 illa…motu] *om.* R 11 conclusio] F *add. sed del.* K 14 similiter] *om.* K | sexta] *om.* R 15 mobilia] aequalia et *add.* V aequaliter] aequalia R | et] *om.* R 16 A] *om.* R continue *add.* K 17 sicut] *om.* KV | totum] *om.* R 18 movetur nec] *om.* K 19 sic] a *add.* KV 20 locum et] *om.* V | locum…terminum] *om.* P | et^{2}] vel K 21 illis] suis V 22 termino] loco R | quam…suo] *om.* (*hom.*) K 23 termino] *om.* R tamen] *om.* K | instans] *om.* R 24 in] a V | inaequaliter] aequaliter R distant] distet R 25 iam] *om.* R | inaequaliter] aequaliter RV 26 deveniet] B *add.* R | motu] non R 28 in] ad *add.* R

14 sequitur…sexta] Cf. § 41.

velocius A. Consequentia illa patet, nam B in aequali tempore pertransiet maius spatium lineale, igitur velocius movebitur. Ex quo arguo ultra sic: iam A et B inaequaliter distant a terminis suis, B plus quam A et utrumque movetur motu recto versus terminum suum, et aeque cito praecise devenient ad terminos suos, igitur B movetur vel movebitur velocius A, tunc illa consequentia est bona et formalis. Et in quolibet instanti a primo instanti erit antecedens verum, igitur in quolibet instanti a primo instanti erit consequens verum. Et ultra, igitur per totum tempus quo sic movebuntur A et B erit hoc verum quod B movetur et movebitur velocius A, ex quo sequitur ultra quod B per idem tempus non movetur nec movebitur tardius A, ex quo sequitur conclusio cuius oppositum sequitur directe ex conclusione proxima, sicut patet.

<Ad oppositum quaestionis>

51 Ad oppositum est Aristoteles et Commentator IV *Physicorum* capitulo de vacuo, textu commenti 71 et 74 et etiam illis commentis, et primo *De caelo*, commento 33 et 51. Item, Jordanis *De pensis ponderibus* propositione prima, ubi dicitur quod inter quaelibet gravia, etc.

52 Nunc antequam respondeatur ad illa restat iuxta processum praehabitum tangere quosdam certos articulos de materia iam incepta.

<Articulus primus: Utrum velocitatio motus gravis sit ab aliqua causa certa>

3 A[1]] *om.* K | distant] in *add.* R | a[2]] l*in.* R 4 B plus] *inv.* R | et] *om.* K utrumque] utraque V 6 movetur vel] *om.* V 7 a…instanti] *om.* R instanti[2]] *om.* K 11 non] nec K 12 sequitur] igitur *add.* R 16 etiam…commentis] in illis etc. K in aliis commentis R 18 *pensis*] *om.* R propositione] proportione R 19 gravia] extrema V 21 praehabitum] praehabitam R

15 Aristoteles…17 commentis] Aver., *In Phys.*, IV, com. 71, ff. 158va-162rb, cf. Arist., *Phys.*, IV. 8, 215a-215b; Aver., *In Phys.*, IV, com. 74, ff. 164va-165ra, cf. Arist., *Phys.*, IV. 8, 216a. | 17 primo…51] Aver., *De celo*, I, com. 33, 62-65; com. 51, 102. | Jordanis…19 etc] Jordanus de Nemore, *Elementa Jordani super demontrationem ponderum*, P.01, 154; cf. Ricardus Kilvington, *op. cit.,* § 62.

V 125r **53** Primus articulus erit iste| Utrum velocitatio motus gravis sit ab aliqua causa certa. Et arguitur primo quod non, nam ex illo sequuntur plura inconvenientia impossibilia.

54 Primo quod aliquis, puta Socrates, non debilitata potentia potest saltare ad concavum orbis Lunae.

55 Secundo quod aliquis motus continue intenderetur in quo tamen motu continue minoratur proportio.

R 160ra **56** Tertio quod aliquod mobile continue intenderet | motum suum per tempus, per quod tamen tempus infinite tarde moveretur.

57 Quarto quod nullum grave simplex naturaliter intenderet motum suum versus locum suum naturalem.

P 32ra **58** Quinto quod pondus in aequilibra | foret simul et semel se ipso gravius et levius secundum situm.

59 Sexto quod aliquod grave moveretur naturaliter aliquo certo gradu quo nullo modo appeteret moveri.

<Primum inconveniens>

60 Ad probationem primi inconvenientis arguitur sic: si velocitatio motus gravis sit ab aliqua causa certa, igitur minoratio resistentiae foret causa velocitationis motus gravis, sicut ponit una positio, quod arguo esse falsum. Nam ex isto sequitur inconveniens primum ductum, quod probo sic: sit aliquis, puta Socrates, qui stans supra terram saltet superius versus concavum orbis Lunae, et signo spatium quantumcumque modicum, quod Socrates potest sic pertransire versus superius absque hoc quod in aliquo debilitetur eius potentia et hoc quantum ad motum,

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1 primus...iste] primus erit et ille K et primus articulus est iste R et erit primus iste V 2 causa] extrinseca *add.* K | causa certa] extrinseca certa causa R *inv.* V 3 inconvenientia] et *add.* R 7 minoratur] maioratur V 9 per^{2}] et R | tempus^{2}] potest V 12 versus…naturalem] *om.* R | suum^{2}] *om.* V 13 simul…semel] *om.* K 19 probationem…inconvenientis] primum inconveniens V 20 velocitatio] velocitas V | causa certa] certa extrinseca causa R | certa] extrinseca K 21 velocitationis] *om.* K 23 primum] primo K | ductum] dictum V 25 modicum] motum K magnum R 26 Socrates] Si V | superius] superficiem K | hoc…et] *om.* R quod…hoc] *om.* (*hom.*) KR 27 in] ab V

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19 ad...22 falsum] Cf. Ricardus Kilvington, *op. cit.*, § 81.

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19 primi inconvenientis] Cf. § 54.

sicut est satis possibile, et terminetur illud spatium per A et B puncta et sit A terminus a quo, B vero terminus ad quem. Deinde arguo sic: Socrates, cum pervenerit ad B punctum, erit tantae potentiae ad movendum, quantae umquam fuit ab initio et resistentia a B puncto versus concavum orbis Lunae minor per multum quam praefuit, et minoratio resistentiae est causa velocitationis gravis, igitur Socrates per eandem potentiam sufficit ulterius moveri et velocius. Et per consequens, si applicet se ad motum et ad saltandum ulterius, in aequali tempore pertransiet maius de spatio. Sit igitur A, B pars aliquota totius spatii intercepti inter terram et concavum orbis Lunae, puta centesima gratia argumenti vel prima medietas, et sequitur quod in centesima parte temporis vel in secunda medietate temporis erit Socrates non debilitata eius potentia ad concavum orbis Lunae.

61 Item, si minoratio resistentiae sit causa velocitationis gravis, ut ponitur, igitur grave existens in concavo sphaerae ignis velocius ibi moveretur quam in sphaera aeris, et in sphaera aeris quam in sphaera aquae, et in sphaera aquae quam in sphaera terrae, quia maiorem resistentiam habet grave in sphaera aeris quam in sphaera ignis, quia aer est medium densius | quam sphaera ignis K 206ra et in aqua maiorem resistentiam quam in aere a consimili ratione. Igitur tale grave non impeditum ab aliquo alio extrinseco quam a mediis elementaribus continue tardaret | motum suum et V 125v

1 sicut] et hoc V | et[2]] *om.* K 2 quo] et *add.* V | vero] *om.* V 3 erit] est KR 4 movendum] ascendendum R 5 Lunae] est *add.* V | minor] minoretur K minoratur R 7 velocitationis] motus *add.* V | igitur] sequitur quod *add.* R 8 et] *om.* K 9 motum] modum R 10 maius] plus K | B] spatium *add.* P | aliquota] acquisita R 11 puta] *om.* R 12 argumenti] exempli KR 13 medietate temporis] parte V 14 Socrates] potentia Socratis K | eius] *om.* R 16 item] secundo K | velocitationis] motus *add.* RV | ut] *om.* R | ut ponitur] vel motus ponatur K 17 igitur] tamen R concavo…ignis] sphaera ignis K 18 ibi] *om.* R | aeris[2]] velocius *add.* R 19 et] velocius *add.* R 20 in sphaera[2]] *om.* RV 21 sphaera[2]] *om.* V 22 aqua] habet *add.* V | a consimili] pari K 23 impeditum] impedito K alio] *om.* K 24 elementaribus] elementariis K *om.* V

numquam velocitaret. Consequens contra sensum et contra Commentatorem *Caeli et mundi* commento.

62 Item, minoratio resistentiae est causa velocitationis gravis, sed contraria non sunt causae eiusdem effectus, igitur cum maius et minus quodammodo sint contraria, igitur minoratio resistentiae non est causa, quare velocitetur motus gravis. Et patet R 160rb assumptum, nam fortius et velocius curreret homo | super terram quam super aquam; et iterum, fortius sagittaret aliquis ad distantiam aliquam magis remotam quam ad distantiam magis P 32rb propinquam, et multa talia experimenta possent adduci ad | hoc quod in multis casibus aliquid moveretur velocius in medio magis resistente quam in medio minus resistente.

Item, ad illam partem et contra illam positionem possent adduci argumenta ducta superius ad quaestionem.

<Secundum inconveniens>

63 Ad probationem secundi inconvenientis arguitur sic: si velocitatio motus gravis sit ab aliqua causa certa, igitur continuatio motus esset causa velocitationis motus gravis, sicut ponit alia positio, quod arguo esse falsum, quoniam ex isto sequitur secundum inconveniens. Quod probo sic: sit aliquod grave simplex in sphaera ignis, quod descendat versus terram movendo continue, tunc motus istius gravis erit continuus, et continuatio est causa velocitationis gravis, igitur motus istius gravis continue intendetur, et tamen in isto motu continue

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1 consequens] est *add.* R 2 Commentatorem] tertio *add.* KV libro *add.* R Caeli…mundi] De caelo et mundo R | et mundi] *om.* V | commento] *lac.* P 73 K *om.* RV 3 item] *om.* K ita *add.* R | minoratio] maioratio PRV 4 non] *om.* V 6 velocitetur] velocitatur V 7 curreret homo] moveretur hoc K 8 et] *om.* V 9 aliquam magis] *om.* R | magis remotam] et remotam magis K 10 talia] alia R 13 et] *om.* R 14 ducta] adducta KRV 18 velocitatio] velocitas V 19 continuatio motus] motu continue R | velocitationis] velocitatis V | gravis] *om.* V 20 positio] opinio V 21 quod] illud K | sic sit] *om.* R 22 simplex] *om.* K | quod] et KRV 23 tunc] *om.* V | tunc…25 continue[2]] *om.* (*hom.*) R | istius] ipsius V 24 velocitationis] motus *add.* K gravis] motus V | istius] ipsius V 25 gravis] *om.* V

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1 consequens...2 commento] Aver., *De celo*, vol I, com. 89, pp. 161-163; Cf. Arist., *De coelo*, I.8, 277a-b. | 17 ad...19 gravis] Cf. Ricardus Kilvington, *op. cit.*, § 81.

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17 secundi inconvenientis] Cf. § 55.

minoratur proportio, quoniam in isto motu continue crescit resistentia versus terram, ut patuit superius, et si sic, igitur continue minoratur proportio, igitur, etc.

64 Item, si continuatio sit causa velocitationis gravis, cum terra ab initio sui et sol fuit et erit in continuo motu propter calorem solis, igitur terra ab initio velocitabit motum suum, igitur nunc velocissime et sensibiliter movetur terra, et per consequens continue erit motus terrae sensibilis, et sic reverteret aedificia magna, domus et castra.

65 Item, si sic, cum motus caeli et orbium planetarum sit continuus, igitur caelum cum ceteris orbibus velocitaret motum suum continue. Consequens falsum, igitur, etc.

66 Item, si sic, cum motus horologii sit continuus, igitur motus talis esset intensior et intensior, et per consequens motus talis per tempus esset sensibilis motus valde.

67 Item, sit aliquod grave quod eodem gradu velocitatis continuaret motum suum, tunc, si continuatio talis motus esset causa velocitationis motus eiusdem gravis, sequitur quod aliquod grave continue velocitabit motum suum, et tamen numquam acquiret gradum intensiorem motus quam prius.

68 Item, sit aliquod grave, quod continue tardet motum suum per tempus, tunc, si continuatio talis motus velocitet ipsum motum, igitur aliquod continue velocitat motum suum, quod continue tardat ipsum.

69 Item, si sic, tunc hoc foret verum in casu <in> "quo duo gravia aequalis virtutis descendunt in eodem medio, et unum incipit a loco superiori et aliud a loco inferiori, adhuc cum fuerint inaequalia | distantia a terra, non aeque cito attingunt ipsam V 126r

2 et…etc] *om.* KR 4 continuatio] velocitatio K continue R velocitationis] motus *add.* KR 5 sui…ab] *om.* (*hom.*) R | et sol] cum sole K 6 initio] sui *add.* V 8 reverteret] ruerent V 10 et] aliorum V | orbium] et *add.* R | planetarum] *om.* V 12 consequens] est *add.* R 14 esset] *om.* R intensior[1]…intensior] velocior et velocior R | et[2]…valde] consequens falsum, igitur etc. V 15 motus] valde sensibilis *add.* K | valde] grave R 16 velocitatis continuaret] veloci continuet K 17 tunc] igitur V 18 velocitationis] talis *add.* K 22 tunc] sequitur quod R | continuatio] continue R 23 igitur] quam R 24 ipsum] motum suum R eum K 25 tunc] *om.* R | hoc…verum] foret hoc V | casu] in *add.* R

13 item...15 valde] Cf. ibidem, *op. cit.*, § 83. | 16 item...20 prius] Cf. ibidem, § 84. | 21 item...24 ipsum] Cf. ibidem, § 82.

terram", quo duo gravia aequalis virtutis descendunt in eodem medio, et unum incipit a loco superiori et aliud a loco inferiori, V 126r adhuc cum fuerint inaequalia | distantia a terra, non aeque cito attingunt ipsam terram, sed illud quod magis distat citius contingit terram quod non foret verum, nisi maior continuatio motus illius, quod sic plus distat argueret maiorem velocitatem. Sed contra. Si illud foret verum, igitur aliquod grave velocius moveretur cum maiori resistentia quam grave aequalis virtutis cum minori resistentia, quod non videtur rationabile.

<Tertium inconveniens>

K 206rb P 32va R 160va **70** Ad probationem tertii inconvenientis arguitur sic: | si velocitatio motus gravis | sit ab aliqua causa certa, igitur propinquitas gravis ad locum suum | naturalem esset causa velocitationis eiusdem, sicut ponunt tertii. Sed hoc est falsum, nam ex isto sequitur inconveniens tertium, quod probo sic: sit aliquod grave descendens a convexitate aeris ad centrum mundi, et sit A B tempus descensus cuius temporis A et B instantia sunt termini. Sit A praesens instans et B instans terminans totum tempus in quo instanti erit primo hoc grave in loco suo naturali et in loco quietis. Tunc sic: ab A instanti, quod est praesens, erit hoc grave continue propinquius et propinquius loco suo usque ad B instans continue et talis propinquitas velocitat motum gravis, igitur usque ad B instans movebitur hoc grave velocius et velocius continue intendendo motum suum; et ultra, igitur immediate ante B instans intendet motum suum et immediate ante B instans infinite tarde movebitur hoc grave. Quod arguo sic: nam in B instanti erit hoc grave sub non gradu motus et sub quiete in

1 duo] *om.* R 3 inaequalia] in aequali V 4 attingunt] attingat R | ipsam] *om.* R 5 contingit] attingit ipsam V 7 si...verum] *om.* R 8 resistentia] distantia K | grave…resistentia] cum minori resistentia grave aequalis virtutis K 15 eiusdem] *om.* V | tertii] tertia opinio V 17 aeris] usque *add.* V 18 A[1]] et *add.* R | cuius] *iter.* R | temporis] et *add.* V 19 termini] terminantia R | instans[1]] *om.* R | instans[2]] *om.* R | instans terminans] *om.* K 21 tunc] arguitur *add.* V 22 hoc…continue] continue hoc grave R | et propinquius] *om.* R 23 continue] *om.* R 25 et ultra] *om.* V 26 instans[2]] *om.* R 28 erit hoc] est hic R | hoc] *om.* K | motus] medio V

1 quo…4 terram] Ibidem, § 82. | 12 ad...15 terti] Cf. ibidem.

12 tertii inconvenientis] Cf. § 56.

termino motus, igitur immediate ante infinite tarde movetur. Et confirmatur illud sic: quoniam si in B instanti erit hoc grave sub non gradu motus, igitur movebatur prius aliquo certo gradu et gradu subduplo ad istum, et gradu subtriplo, subquadruplo, et sic in infinitum. Et si sic, igitur prius infinite tarde movebatur, igitur, etc.

71 Item, si ex appropinquatione istius gravis ad locum suum sequitur ipsum velocius moveri continue, contra: hoc grave sic descendens tendit ad quietem et quo proximius est loco suo, tanto propinquius est quieti. Igitur per totum tempus vel aliquam eius partem versus finem remittet motum suum, igitur non intendit continue motum suum.

72 Item, hoc grave antequam erit sub non gradu motus prius remittet motum suum successive, igitur ante finem motus tardabit motum suum et continue ante finem motus erit propinquius loco suo et propinquius; igitur ad talem propinquitatem non sequitur intensio motus vel si sic, sequitur quod idem grave in eodem tempore quo movetur velocius, eo tardius movetur.

73 Item, signetur aliquod instans quo aliqua pars illius gravis erit ultra centrum mundi, tunc ab hoc instanti, quod sit C, quousque medium illius gravis sit medium mundi movetur tardius et tardius, sicut demonstratum est in tertio principali, et nihilominus per totum illud tempus erit propinquius et propinquius loco suo | naturali, quod est medium mundi, igitur, etc. V 126v

74 Item, si sic, sequeretur quod inaequalitas appensorum faceret motum in aequilibra, contra tertiam positionem Jordanis *De*

2 illud] hoc R | si] *om.* R 3 prius] sub *add.* R 4 gradu^2] *om.* R | subtriplo] et *add.* KV 5 et...sic] *om.* R 6 etc] *om.* R 7 istius] ipsius V 8 continue] *om.* R | hoc] o*m.* K 9 descendens] crescens R | quo] quanto RV proximius] proximior V 10 propinquius est] est magis propinquius R 13 antequam erit] prius quam sit K | non] *lin.* K | motus] prius *add.* K 14 suum] non *add.* KV | igitur] *om.* K 15 et] *om.* K 18 eo] *om.* R 22 illius…medium] eius fuerit centrum R 23 est] *om.* K 24 suo] *om.* K 26 quod] in *add.* V | inaequalitas] aequalitas K | appensorum] appendictorum R appetitus V 27 motum] *correxi ex* mitum P | contra] per *add.* V | tertiam] quartam R | positionem] propositionem KV | Jordanis] Jordani KR

27 contra…33,3 inaequalitas] Jordanus de Nemore, *Elementa Jordani*, P. 03, 156.

R160vb *ponderibus* quae |est quod cum fuerint appensorum pondera aequalia, non faciet motum in aequilibra appensorum inaequalitas; cuius oppositum ita sequi probatur: sit aequilibra cuius appensibilia sint inaequalia, sed tamen longitudine, non gravitate, deinde appendantur pondera aequalia per omnia, et sit P 32vb A pondus appensum in appensibile longiori | et propinquiori centro mundi, et sit B aliud pondus, et dimittantur pondera versus centrum mundi ponderibus ipsis fixis continue in extremis appensorum. Tunc sic: A grave per totum tempus descensus erit propinquius centro mundi quam B grave et appropinquatio erit causa velocitationis motus, et ad ipsam sequitur velocitatio motus gravis, igitur A continue movebitur descendendo velocius B. Et si sic, igitur ex illa parte inclinabit cantherium, quod etiam arguo sic: A in suo descensu aut pertransiet spatium lineale aequale et solum tamen in aequali tempore cum B, et sic non movetur velocius B aut in aequali tempore plus pertransiet de spatio lineali, et si sic, patet quod A deprimet cantherium elevando B. Et per consequens agit motum et solum ex inaequalitate appensorum, nam ex inaequalitate appensorum accidit inaequalitas in K 206 va approximando et per consequens in velocitate, igitur, etc.|

75 Item, si sic, sequitur quod aequis ponderibus in aequilibra appensis, si alterum deprimatur ab aequidistantia orizontis versus centrum, velocius altero moveretur. Consequentia patet ex hoc quod pondus depressum esset propinquius loco suo quam pondus econtrario positum. Et patet falsitas consequentis, nam

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1 quod] *om.* R 2 aequalia] *add.* aequilibra *et iter. sed exp.* R | motum] *correxi ex* mitum PR 3 inaequalitas] in- *add. lin.* K | ita] videtur R | sequi] sequitur V | probatur] *om.* V | aequilibra] aequalia R 4 inaequalia] aequalia RV | tamen] in tantum R tantum V | non] illius *add.* R 5 deinde] vero *add.* R 6 appensibile] appendiculo K appensibila V | propinquiori] propinquior R 7 pondus] *add.* appensum in breviori R 8 extremis] ceteris R 9 appensorum] appendiculorum KR | sic] si R 11 motus[1]] in motu V et] *om.* KR | et…motus] *om.* (*hom.*) V | velocitatio] velocitas R | velocitatio motus] velocius et velocius K 13 cantherium] ad concavum K | etiam] *om.* VK 15 tamen] tantum K 17 patet] sequitur V | deprimet] comprimet K | cantherium] concavum K 18 motum] *correxi ex* mitum P suum *add.* R *om.* V 21 aequis] aliquibus R | aequis…aequilibra] in aequlibra aliquibus ponderibus K 22 appensis] et *add.* KR | alterum] altera V 23 centrum] unum *add.* V | altero] movebitur vel *add.* K 24 depressum esset] repressum et K

pondus depressum non plus capiet de directo nec etiam de obliquo in aequali tempore quam faciet pondus elevatum, et si sic, igitur ipsum non movebitur velocius quam alterum. Adhuc illa pondera sunt aeque gravia secundum situm et aeque gravia simpliciter, igitur unum non movebitur velocius altero. Adhuc sequitur quod illis ponderibus ab aequidistantia separatis numquam revertuntur ad aequidistantiam, et multa alia sequerentur quae propter expeditionem maiorem pertranseo consequenter.

<Quartum inconveniens>

76 Quarto. Si velocitatio motus gravis esset ab aliqua causa certa, igitur propulsus medii esset causa velocitationis huius, sicut ponunt quarti. Sed contra. Ex isto sequitur primo quartum inconveniens adductum, quod arguo sic: si pulsus medii velocitaret motum gravis in descendendo, igitur velocitatio talis gravis esset a principio extrinseco et non a principio intrinseco quae est forma gravis. Et si sic, igitur talis velocitatio non esset naturalis et idem arguo de quolibet gravi, ex quo sequitur quod nullum grave naturaliter | intenderet | motum suum versus V 127r|R 161ra
locum suum naturalem.

77 Item, sit grave prope sphaeram ignis et descendat, tunc hoc grave in suo descensu pellitur ab aere medio, igitur aer insequitur, et sequitur ultra: aer recedet a loco intermedio insequens motum gravis nec ignis subsequitur, igitur ibi relinquitur vacuum et sic

1 non plus] *iter.* K | de directo] *om.* K | directo] diamtero R 2 obliquo] aliquo K 4 sunt] *om.* R | aeque[2]…simpliciter] simpliciter aeque gravia V 5 altero] alio V 6 aequidistantia] aequali distantia R 8 sequerentur…consequenter] quae propter expeditionem maiorem transeo sequerentur K | expeditionem maiorem] experientiam manifestam dicto et R 9 consequenter] *om.* V 13 velocitationis] motus gravis *add.* R huius] *om.* R 14 quarti] opinio *add.* V | contra] si sic *add.* K | primo] unum R 15 adductum] *om.* RV | pulsus] propulsus V 16 gravis] igitur non K velocitatio] motus *add.* R | talis] motus *add.*R 22 item] si *add.* R si sic *add.* V 23 grave] *lac.* V | pellitur] *om.* V 24 motum] nec *add.* K 25 et] si *add.* R

12 si...13 huius] Cf. Ricardus Kilvington, *op. cit.*, § 86.

12 quarto] Cf. § 57.

sequitur quod ad motum cuiuscumque gravis relinquitur vacuum in alia parte aeris, quod negant plerique philosophi.

78 Item, corrupto medio supra lapidem in vacuum non P 33ra moveretur tardius descendendo, sed velocius, quia aer | et medium superius pellens lapidem deorsum condensat medium in inferius, et per consequens facit quod medium inferius plus resistat. Et si sic, igitur talis pulsus medii plus impedit motum gravis quam promovet, igitur, relicto vacuo supra velocius moveretur grave, igitur, etc.

79 Item, imaginemur vacuum inter lapidem et locum suum naturalem et medium plenum aeris supra. Tunc dempto aere supra lapidem qui sit grave simplex infinite velociter moveretur lapis ad locum suum, ut patet per Aristotelem IV *Physicorum* capitulo de vacuo, et nihil mundi potest velocius moveri quam infinite velociter moveri, igitur propter pulsum medii non moveretur nec potest moveri velocius quam sine pulsu alias moveretur.

80 Item, si sic, tunc grave mixtum motum in vacuo versus locum suum naturalem numquam intenderet motum suum.

81 Item, sequitur quod grave mixtum velocius moveretur in pleno quam in vacuo et multa alia inconvenientia sequuntur.

<Quintum inconveniens>

82 Quinto. Si velocitatio motus gravis, etc., igitur gravitas accidentalis quam acquirit grave in descendendo esset causa velocitationis motus talis gravis, sicut ponit quinta secta. Sed contra. Ex isto sequitur quintum inconveniens quod probo sic.

1 quod] *om.* K | cuiuscumque] cuiuslibet R 2 alia] aliqua R | plerique] plurimi R 3 item] tunc K et R 4 tardius] in *add.* R 5 condensat] densat R 10 imaginemur] imaginetur K | vacuum…lapidem] lapidem inter vacuum *sed corr. lin.* R 11 tunc...supra] *marg.* K 12 simplex] tunc *add.* R 13 suum] naturalem *add.* K 15 infinite] infinities V | moveri] *om.* RV 16 moveretur…velocius] potest velocius moveri nec movetur V | nec] neque R | sine] sui R | sine…alias] alias sine pulsu V 19 motum suum] naturalem *add.* R 21 pleno] plano R 24 quinto si] quintum sic probatur: si V | gravitas] gravis V 26 velocitationis…talis] talis velocitationis V quinta] certa K 27 sic] *om.* R

13 ut...14 vacuo] Cf. Arist., *Phys.*, IV.8, 215b-216a. | 24 si...26 secta] Cf. Ricardus Kilvington, *op. cit.*, § 81, 85.

24 quinto] Cf. § 58.

Sit aequilibris et pondera appensa A, B aequalia et separentur A, B pondera ab aequidistantia orizontis A versus centrum mundi, B supra, et sit C aliquis situs ad quem descendet A. Tunc A est levius in isto situ quam in aliquo situ ab aequidistantia orizontis usque ad C, ut patet ex quarta propositione Jordanis *De pensis ponderibus* quae est quod pondus in quacumque parte descendat ab aequalitate secundum situm sit levius. Et si sic, igitur A | in situ C K 206vb est levius quam in aliqua parte supra et in illo eodem situ est gravius quam prius, cum continue descendendo versus locum suum naturalem acquirat gravitatem accidentalem quae est gravitas secundum situm, igitur idem | grave seipso secundum R 161rb eiusdem situm sit gravius et levius.

83 Item, sit A aliquod summe grave et moveatur ad locum suum naturalem, tunc A aut | intendit motum suum aut non; si sic, et V 127v non ab alia gravitate maiori accidentali vel per se, cum ipsum sit summe grave, igitur intensio huius motus est et non a gravitate accidentali.

84 Item, si A continue intendat motum suum, igitur gravitas A continue intenditur et ad intensionem gravitatis A sequitur remissio levitatis in A, igitur A fuit aliqualiter leve et continue minus et minus, per consequens A non fuit summe grave. Si A non intendat motum suum, hoc est contra positionem eorum et contra praedicta similiter.

85 Item, grave simplex intendens locum suum velocitat motum suum et motus est causa caloris, igitur continue calefiet et igitur continue | acquirit de levitate, et <non> simul in eodem P 33rb intenditur gravitas et levitas, igitur, etc.

1 aequilibris] aequalia? R aequilibra V | A[1]] et *add.* R | A[2]] et *add.* R 2 A] *om.* R 3 et] *om.* R 5 Jordanis] Jordani K | De…ponderibus] *om.* R 6 quod] per R 7 sit] ut R | et] A R 8 et] cum K 11 gravitas] gravis V seipso] *om.* R 12 levius] se ipse *add.* R 14 et] igitur R 15 alia] aliqua K actuali R | alia…maiori] maiori gravitate V | accidentali] *om.* R 16 et] cum K 18 igitur…intenditur] *om.* R 19 ad] *om.* K | sequitur] si *add.* K 20 in] *om.* KR 22 hoc] igitur R | eorum] *lin. et add* aeris R 23 praedicta] dicta R similiter] *om.* V 24 simplex] *om.* V | intendens] antecedens K | locum] motum V | locum suum] suum locum naturalem R 25 et[2]] *om.* R 26 non] *om.* K sic R | in…intenditur] intenditur in eodem V 27 igitur] *om.* K | etc] *om.* R

5 ut…7 levius] Jordanus de Nemore, *Elementa Jordani*, P.04, p. 156. | 24 item...26 levitate] Cf. Ricardus Kilvington, *op. cit.*, § 85.

86 Item, si sic, igitur intensio huius motus foret totaliter accidentalis, quia foret totaliter a forma gravis accidentali et non per se, igitur, etc.

<Sextum inconveniens>

87 Ad probationem sexti arguitur sic: si velocitatio motus gravis, etc., igitur appetitus esset causa velocitationis motus, sed hoc non, nam ex isto sequitur primo inconveniens sextum, quod probo sic: sit grave motum in medio ceteris paribus versus locum suum naturalem D gradu velocitatis. Tunc sic: hoc grave movetur naturaliter aliquo certo gradu velocitatis, et illo non appetit moveri, igitur, etc. Minor probatur. Nam hoc grave appetit velocius moveri quam in illo gradu quo movetur, quia ipsum appetit intendere motum suum, igitur velociori gradu appetit moveri quam illo quo movetur, igitur gradus motus quo movetur est violentus et ultra, igitur illo gradu non appetit moveri, igitur, etc.

88 Item, hoc grave non appetit intendere motum suum, igitur intensio talis motus non attenditur penes appetitum. Antecedens arguo: hoc grave in quacumque distantia ponatur a loco suo naturali appetit, ut immediate post hoc sit in suo loco naturali, igitur appetit quiescere a motu, igitur non intendit motum suum.

89 Item, hoc grave appetit infinite velociter moveri, igitur, etc. Antecedens arguitur: appetit, ut sine medio sit in loco suo naturali, igitur appetit subito moveri, igitur, etc.

90 Item, imaginatur spatium infinitum inter hoc grave et locum suum naturalem, tunc sine medio post hoc appetit hoc grave pertransire spatium infinitum, igitur infinite velociter moveri, igitur, etc.

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2 et non] conclusio K 7 igitur] sequitur quod R | velocitationis] gravis *add.* R | motus] gravis *add.* V 8 non] est *add.* R | nam] quia V primo…sextum] sextum inconveniens V 10 D] de R | tunc] arguitur *add.* R 12 etc…17 etc] *om.* R | minor...etc] *om.* (*hom.*) K 13 in] *om.* V 14 intendere…suum] moveri velocius et intendit motum V 15 illo] ille sit V | quo[1]] qui P | motus] medius V 20 arguo] quod *add.* R arguitur sic V 21 naturali[1]] *om.* K 22 a] in R | intendit] appetit intendere KRV 23 infinite…moveri] moveri infinite velociter R 24 appetit] *om.* R 25 etc] *om.* R 26 imaginatur] A *add.* KR | inter…28 infinitum] *om.* (*hom.*) R 27 tunc] *om.* V

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6 sexti] Cf. § 59.

91 Item, si sic, sequitur, sicut patet ex dictis, ut simul moveatur et quiescat.

92 Item, tale grave appetit in infinitum velocius moveri quam sufficit moveri, igitur appetitus est frustra. Antecedens patet ex dictis.

93 Item, quodlibet grave mundi appetit ita velociter moveri per medium plenum sicut per medium vacuum, ut patet ex dictis, et intensio | motus sequitur appetitum, igitur ita velociter intendit V 128r motum suum per medium plenum sicut per medium vacuum. Consequens falsum, igitur positio | ex qua sequitur. R 161va

<Ad oppositum articuli>

94 Ad oppositum arguitur ex positionibus antetactis famosis et per Aristotelem IV *Physicorum* commento 71, et per Jordanem *De pensis ponderibus*.

<Opinio auctoris ad articulum>

95 Ad istum articulum cum quaeritur, utrum velocitatio motus gravis sit ab aliqua causa certa, dico quod si iste terminus certam determinat praecisionem, ut sit sensus, aliqua est causa praecisa velocitationis gravis in descendendo, sic dico quod non. Nam velocitatio gravis versus deorsum in suo descensu est a pluribus causis, sed una sit principalior ceteris. Unde dico cum magistro Adam | de Pipewelle quod minoritas resistentiae est causa K 207ra principalis et continuatio motus, propinquitas, pulsus medii, gravitas | accidentalis, inclinatio naturalis, quae est appetitus, sunt P 33va

1 sicut patet] *om.* KR | dictis] sicut patet *add.* K | ut] patet ut *add.* R 3 appetit] ut *add.* K | in] *om.* V | in infinitum] *om.* R 4 igitur] *om.* K 6 item quodlibet] quod licet R 7 et] cum K 9 medium[2]] *om.* V 10 consequens] est *add.* R 11 ad] in R | antetactis] antedictis R 12 Aristotelem] et Commentatorem *add.* R | IV] VII KRV commento…et] *om.* R | Jordanem] Jordanum RV 13 pensis] appensis K 15 gravis] *om.* R | certam] *correxi ex* certa P certa RV 16 ut] sic R 17 velocitationis] motus *add.* R | gravis] versus deorsum *add.* V 19 sed] licet V | principalior] principium K | cum] quod R 20 de Pipewelle] *om.* R Pypewelle V | Pipewelle] Pipelvele K 21 et] cum K | motus] et *add.* V propinquitas] et *add.* V 22 quae est] et V | est] *correxi ex* et P | est appetitus] et appetitus dicitur K | sunt] omnes *add.* K dicuntur R

19 unde...21 principalis] Cf. Ricardus Kilvington, *op. cit.*, § 136. | 21 et…39,8 descensus] Cf. ibidem.

causae partiales; est enim quaecumque illarum causa partialis et coadiuvans, non tamen est causa necessario requisita ad velocitationem motus gravis, sicut satis probant argumenta. Ad hoc enim vadunt argumenta quod nulla illarum est causa praecisa aut causa necessario requisita per se in velocitatione motus, et hoc est verum. Non tamen volo dicere quod aliqua illarum est causa principalis vel secundaria in velocitatione gravis per totum tempus descensus. Sed usque continguerit locum suum naturalem et centrum mundi, nam deinceps grave continue tardat motum suum, cum continue post illud crescat resistentia sive moveatur in vacuo vel in pleno. Unde dico quod in motu gravis versus deorsum, ubi etiam cetera omnia sunt paria, minoratio resistentiae est causa principalis et penes illam principaliter attenditur velocitatio motus gravis, etc.

96 Concurrunt tamen et aliae causae partiales dictae modo ista in uno casu, modo illa in alio, sed per minorationem resistentiae est velocitatio motus gravis principaliter attendenda.

<Responsio ad primum inconveniens>

97 Et tunc ad primum in oppositum admitto casum et nego primum assumptum, scilicet quod 'Socrates, cum pervenerit ad B, est tantae potentiae ad movendum, quantae numquam prius fuit' nec hoc sequitur ex casu et causa est illa, quia ad hoc quod Socrates ulterius moveretur a B qui est terminus a quo in principio secundi saltus, non est terminus fixus nec motus iste

1 partiales] dicuntur causae partiales et coadiuvantes *add.* R | est enim] *om.* V | quaecumque…partialis] causa partialis quaelibet illarum K | illarum] est *add.* V | causa] *om.* R 3 velocitationem] velocitatem K | gravis] *om.* R ad...argumenta] *om.* (*hom.*) R 5 causa] *om.* R | per…motus] maxime ad velocitationem motus gravis KR 6 verum] notum R 7 vel] et aliqua *add.* V | velocitatione] motus *add.* R 8 tempus] eius KR *om.* V | tempus descensus] cuius descensus P | usque] quo *add.* R 9 et] cum K 10 suum] et *add.* R | crescat resistentia] crescantur resistentiae V 11 vel] sive RV 12 ubi…omnia] cetera V | cetera] cetere K | omnia] *om.* R | minoratio resistentiae] resistentia V 13 principaliter] *om.* R 14 attenditur] praecise R etc] item *add.* R 15 tamen] cum V | et] *om.* K 16 illa] *om.* R | alio] a *lin. et exp.* P | per] penes | est] et R 17 principaliter] specialiter R 21 quod] *om.* R cum V | pervenerit] venerit R 22 tantae] erit *add.* R | prius] *om.* K prius fuit] praefuit V 24 Socrates] Sequitur V

20 ad primum] Cf. § 60.

haberet aliunde terminum fixum, et tamen omne motum in motu suo necessario indiget aliquo fixo, ut patet per Philosophum in libro *Caeli et mundi*. Sequitur in hoc casu quod Socrates non movebitur ultra B et sic non sequitur inconveniens adductum.

98 Ad secundum concedo quod grave existens in concavo sphaerae ignis velocius moveretur quam in sphaera aeris et sic deinceps, ex quo non sequitur quod tale grave continue tardaret motum suum, | quia licet in motu gravis sic descendentis V 128v continue crescat resistentia partialis, minoratur tamen continue resistentia totalis quae est a sphaera ignis ad centrum mundi et penes | minorationem totalem habet velocitatio huius gravis R 161vb attendi, et sic non sequitur aliquod inconveniens.

99 Ad tertium negatur primum assumptum, quia et illud falsum est, ubi maxime cetera sunt paria, et tunc ad primum in oppositum quod fortius et velocius curreret homo super terram quam super aquam, hic dico quod cetera non sunt paria, quia resistentiae respectu cuius est motus sunt diversae speciei, et etiam quod homo scilicet sic curreret super terram velocius quam super aquam, hoc esset maxime propter terminum fixum quem vel saltem ita solidum, sicut fixum non habet in aqua, vel haberet, si moveretur in ea, et hoc maxime facit ad motum, ut dictum est supra, et sic patet quod cetera non sunt paria in illo argumento. | P 33vb

100 Ad aliud concedo quod in casu fortius sagittaret arcus in distantia maiori quam in distantia certa minori, sed in isto casu continuatio motus multum ageret ad hoc simul quod virtus

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1 aliunde] aliquem KV aliquando R | et tamen] cum V | in…suo] *om.* K 2 suo] *om.* R | indiget] in motu *om.* K eget R 3 mundi] et *add.* K 5 concavo…ignis] sphaere ignis concavo R 8 suum] *om.* K | licet] licet *lin. et add. sed exp.* habet P | gravis] continue *add.* K 10 a] *om.* R | et] igitur V 11 totalem] resistentiae totalis R totaliter V | huius] motus R 13 negatur] illud *add.* R | et] *om.* R 14 ubi maxime] et maxime ubi V | et] *om.* R et…16 paria] *om.* (*hom.*) K 16 hic] *om.* V | quod] cum R 17 resistentiae respectu] resistentia illius V | cuius] quae R | est] sit R 18 etiam] hoc V quod] *om.* P | homo] quod *add.* P | homo scilicet] *om.* V | scilicet] *om.* R 19 hoc…maxime] est R | terminum fixum] infixum V 20 vel[1]] *om.* V sicut] vel R 21 in] cum V 23 casu] aliquis V | arcus] D *add. sed exp.* P *om.* V 25 continuatio] continue R | simul] sic V

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1 omne…3 mundi] Arist., *De coelo*, I.8, 277a.

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5 ad secundum] Cf. § 61. | 13 ad tertium] Cf. § 62. | 23 ad aliud] Cf. ibid.

motiva sagittae foret maior in distantia maiori et augeretur ex continuatione motus, et sic patet quod sit tenendum in casibus motus gravis.

<Articulus secundus: Utrum velocitas motus sphaerae cuiuslibet penes punctum vel spatium aliquod attendatur>

101 Articulus Utrum velocitas motus sphaerae cuiuslibet penes punctum vel spatium aliquod attendatur.

K 207rb **102** Et arguitur primo quod non, quia si sic, sequitur | quod sphaera stellarum fixarum non moveretur velocius sphaera terrae, sed quod aequaliter movetur praecise.

103 Secundo quod sphaera A moveretur in duplo velocius B, et tamen nec potest nec sufficit moveri in duplo velocius B.

104 Tertio quod aliqua sphaera moveretur per horam latitudine motus uniformiter difformis, quae tamen per eandem horam continue uniformiter moveretur.

105 Quarto quod nulla sphaera mundi posset uniformiter circumvolvi per horam.

106 Quinto quod aliqua duo mobilia aequaliter distant nunc a terminis suis fixis et per totum tempus per quod movebuntur ad terminos suos fixos continue aequaliter distabunt ab illis, et aeque cito devenient ad terminos suos fixos, et tamen unum illorum per totum movebitur improportionaliter velocius altero.

107 Sexto quod nullum grave mundi sphaericae tamen figurae potest intendere motum suum ad terram.

<Primum inconveniens>

108 Ad probationem primi inconvenientis arguitur sic: si velocitas motus sphaerae cuiuslibet penes punctum aliquod attendatur, etc., igitur talis velocitas attendetur penes punctum infinitum,

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1 ex] in R 2 tenendum] dicendum et respondendum K dicendum seu respondendum R | casibus] causis V 9 velocius] sphaera *add.* R | et] cum K 10 nec[1]] non K | in duplo] *om.* R 12 difformis] difformiter V tamen…continue] continue per eandem horam R 14 mundi] *om.* V uniformiter] iterum *add.* K *om.* R 15 circumvolvi] revolvi KV 16 aequaliter…nunc] nunc aequaliter distant R 18 suos] *om.* K | et] *om.* R 20 movebitur] manebitur K | altero] alio R 21 tamen] *om.* R 26 punctum] vel spatium *add.* R 27 etc] *om.* KR

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25 primi inconvenientis] Cf. § 102.

sicut est positio aliquorum communis quae positio sumit exordium et colorem ab isto quod orbes stellarum errantium et etiam ipsae stellae erraticae quanto plus et distantius removentur ab orbe stellarum fixarum, tanto | V 129r velocius moventur, ut patet. Et per consequens punctus datus quiscumque in axe sphaerae mundialis quanto plus removetur a circumferentia primi orbis et maxime, tanto velocius movetur, et per consequens punctus maxime distans versus | R 162ra inferius qui est punctus infimus maxime et velocissime movetur. Et per consequens motus orbis stellarum fixarum attenditur penes istum punctum sic quod isto gradu motus quo movetur punctus infimus, eodem vel ita intenso gradu movetur totus orbis. Quod probo esse falsum sic: si enim hoc esset verum, sequitur inconveniens primo ductum, quoniam tota terra respectu orbis stellarum fixarum est quasi punctus secundum Ptolemaeum in principio *Almagesti* et est punctus ultimus mobilis saltem, ut patet. Igitur secundum illam positionem motus orbis supremi attenditur penes illum punctum et constat quod idem punctus movetur continue, ut probatum est supra, igitur quam velociter praecise movetur orbis supremus, | P 34ra tam velociter praecise movetur orbis terrae, et econtra. Ex quo sequitur inconveniens adductum et per consequens, cum motus caeli sit velocissimus et sensibilis, igitur et motus terrae esset velocissimus et sensibilis.

109 Item, si sic, cum unus et idem est punctus infimus omnium sphaerarum stellarum errantium et fixarum, igitur unus et idem et

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1 aliquorum] aliquarum K *om.* R | positio^2] ponuntur et V 2 isto] ex illo R 3 erraticae] errantes V | removentur] recedentur R 4 ut patet] *om.* R ut…consequens] *om.* V 5 punctus…7 consequens] *om.* R 6 a] *om.* V 7 velocius] plus K 8 est] *correxi ex* et P etiam R | infimus] et *add.* RK 11 motus quo] *om.* R 12 sic] quoniam *add.* V | enim] *om.* V 13 ductum] adductum KR datum V 15 Ptolemaeum] Ptholomeum K Tholomeum P principio] primo V 16 mobilis] motus V 17 illum] illud P 18 quod…movetur] ille punctus motus R | idem] iste V 21 sequitur] *om.* R | cum] quod K 22 igitur] et *add.* KR 24 si…cum] quod R | cum] *om.* K | et idem] *om.* R | infimus] *om.* R 25 sphaerarum] *om.* K | et^3…motus] esset motus et aequalis R motus aequalis erit V

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13 tota…15 Almagesti] Claudius Ptolemaeus, *Almagestum*, Venetiis 1515, I.3, f. 2v: Et quod ipsa [scil. terra] secundum magnitudinem et spatium est quasi punctum quantum ad orbem stellarum fixarum.

aequalis esset motus omnium sphaerarum stellarum errantium et fixarum, et per consequens omnes orbes et omnes stellae aeque velociter moverentur.

110 Item, si sic, cum punctus maxime infimus, centrum mundi, sit immobilis, sequitur quod penes non gradum motus attenderetur velocitas motus sphaerae et sic penes non motum attenderetur, quod est absurdissimum. Nec etiam potest dici quod velocitas motus sphaerae in circumvolutione sphaerae attenditur penes aliquem punctum inter medium punctum et punctum infimum, quia cum non sit maior ratio de uno quam de quolibet, igitur penes nullum sub medio attendetur et habetur propositum. Vel si sic, igitur penes quemlibet, et per consequens ipsa sphaera numquam uniformiter, sed difformiter circumvolvitur, et simul et semel velocius et tardius, et simul et semel infinite tarde, et multum velociter circumvolvitur et movetur; quae sunt nimis absurda in philosophia naturali concedere.

<Secundum inconveniens>

111 Secundo ad articulum arguo sic: si velocitas motus cuiuslibet
K 207va attenderetur etc. | penes punctum medium inter punctum infimum et supremum, sicut est opinio et positio magistri Ricardi de Versellys in tractatu suo *De <motu>*.

1 omnium sphaerarum] *om.* KR | stellarum] et *add.* R 2 omnes[1]…stellae] omnes orbes et omnes stellae V 4 cum] quod R | centrum] poli K caeli R centrum mundi] caeli V | mundi] mundum R 5 immobilis] mobilis R motus attenderetur] attenditur V 7 absurdissimum] absurdum RV 9 inter medium] *iter.* K | medium] *iter.* R 10 punctum] *om.* V | quia] et KR quam…igitur] puncto quam de alio gradu quolibet K 11 quolibet] alio RV | igitur] quilibet *add.* R | nullum] unum R | sub medio] *om.* V 12 si] *om.* V 13 difformiter] movetur vel *add.* V 14 circumvolvitur] circumvolveretur R | et[3]…semel] *om.* R 16 nimis] *om.* R | in philosophia] et philosophiae R 17 concedere] contraria R 20 ad] eundem *add.* KR cuiuslibet] sphaerae *add.* RV 21 etc] *om.* R igitur attenderetur *add.* V 22 infimum…supremum] supremum et punctum infimum R | et positio] *om.* V 23 Versellys] Versellis KV Uselis R | De] *om.* KR *lac.* PV

22 sicut…23 motu] Cf. Thomas Bradwardinus, *op. cit.*, 128, 95-99.

20 secundo] Cf. § 103.

112 Sed contra, ex isto sequitur secundum inconveniens, quod probo sic: circumvolvatur sphaera A ex toto centro suo fixo cuius medius punctus semidiametri sit B et punctus supremus C, et punctus centralis et infimus D. Tunc A movetur in duplo velocius B. Probo: C movetur in duplo velocius B, probo: sphaera mota orbiculariter circa centrum suum quilibet punctus remotior a centro movetur velocius. | Hoc | arguo sic: quilibet punctus R 162rb remotior a centro pertransit maius spatium in aequali tempore, ut |V 129v patet, igitur movetur velocius. Et ultra, igitur quilibet punctus in sphaera quanto plus distat a centro, tanto velocius movetur, sed C in duplo plus distat a centro quam B, igitur C in duplo velocius movetur quam B, et A movetur ita velociter sicut C vel aliquis punctus suus, quia tantam sphaeram pertransit in aequali tempore sicut C vel aliquis eius punctus, igitur A movetur ita velociter sicut C. Sed C movetur in duplo velocius B, igitur A movetur in duplo velocius B, et tamen secundum illam positionem non potest nec sufficit in duplo velocius, sed praecise aeque velociter, igitur, etc.

113 Item, tunc orbis supremus stellarum fixarum movetur aeque velociter praecise cum suo medio puncto et per consequens cum suo orbe medio, puta orbe solari vel | aliquo alio citra ipsum, et P 34rb per consequens motus suus non esset velocissimus, sed orbis Saturni vel Martis esset eo velocior contra omnes astrologos.

114 Item, si aliqua sphaera moveretur aequaliter praecise cum suo medio puncto et tota sphaera in eodem tempore pertransit spatium lineale in duplo maius quam suus medius punctus, igitur quod duplum spatium in aequali tempore pertransit, solum movetur aequaliter, consequens mere falsum, ut patet.

<Tertium inconveniens>

2 A] *om.* K | A ex] eius in suo R | suo] *om.* R 3 semidiametri…punctus] *om.* (*hom.*) K 5 C…probo] *om.* KV in R 6 punctus] punctorum V remotior] circa *add. sed exp.* R 9 ultra] igitur *add.* KRV 10 tanto] *om.* R C] D R 11 distat] velocius *add.* R | in^{2}] *iter.* R 12 C] *om.* K | C vel] *om.* R 13 punctus suus] eius punctus V | suus quia] eius quod R | tantam sphaeram] tantum spatium V 15 igitur…B] *om.* (*hom.*) R 17 etc] *om.* R 19 puncto] quanto R 21 orbis] motus K | orbis Saturni] motus Saturni orbis R 22 velocior] quod est *add.* RV 23 aequaliter] *om.* R | cum] *om.* R 26 quod] quoddam K 27 movetur aequaliter] *om.* KR | mere] est RK patet] igitur etc. *add.* V

115 Tertio ad articulum arguo sic. Si velocitas motus sphaerae attenderetur penes aliquem eius punctum et constat quod non attendetur penes infimum vel medium, igitur velocitas motus sphaerae cuiuslibet attendetur penes punctum supremum, et ista est positio magistri Thomae de Bradvardyn in tractatu suo *De proportionibus*.

116 Contra quam tamen arguo: quia ex illa sequitur tertium inconveniens adductum. Probo: circumvolvatur sphaera A circa centrum suum et sit B unum mobile quod per ymaginationem incipiat moveri a non gradu et a centro fixo et inde progrediatur intendendo motum suum sine saltu usque ad motum puncti supremi, sic videlicet quod intendendo motum suum moveatur omni gradu quo movetur aliquis punctus sphaerae citra punctum supremum antequam hoc mobile deveniat ad punctum per fluxum motus a centro usque ad circumferentiam supremam. Hoc casu supposito mobile datum et sphaera data moventur eadem latitudine vel aequali unifomiter difformi. Hoc arguo sic: latitudo motus a centro sphaerae usque ad circumferentiam est latitudo motus uniformiter difformis, quia latitudo motus cuius qulibet gradus est remississimus qui non est supra et intensissimus qui non est sub, sicut patet per motum punctorum in sphaera. Motus enim cuiuscumque puncti in sphaera in alia et alia circumferentia est | V 130r remississimus qui non est supra et intensissimus qui non est sub, igitur tota latitudo motus sphaerae est uniformiter difformiter difformis, igitur si sphaera data circumvolvatur in hora, | R 162va haec sphaera movetur latitudine motus

1 arguo] sequitur R 5 Bradvardyn] Berdvardi *et iter. marg.* Thomas Berdvardi K Bradvardin R Bradwardin V 9 B] *om.* R 10 centro] suo *add.* KR 11 sine…suum] *om.* (*hom.*) KR 12 suum] *om.* V 13 omni] cum KV cum aliquo R | gradu] A *add.* R | citra punctum] circa centrum V 14 deveniat] perveniat V 15 centro] caelo P 16 supposito] dato V 17 eadem] aequali R | aequali] eadem R | difformi] uniformi *add.* V 18 sphaerae] *om.* R | sphaerae usque] *om.* K 19 motus[1]] *om.* KR 20 remississimus] remissus R | qui…intensissimus] *om.* (*hom.*) R supra…est] *om.* (*hom.*) KV 21 punctorum] motoris *add.* K 22 enim cuiuscumque] cuiuslibet K | enim…puncti] cuiuscumque moti R | in[3]] *om.* KR 23 circumferentia] qui *add.* R 25 difformis] *om.* R | data] dupla K

4 ista…6 proportionibus] Cf. Thomas Bradwardinus, *op. cit.*, 130, 115-118.

1 tertio] Cf. § 104.

uniformiter difformis, et tamen per eandem horam solum movetur uniformiter, quia ponatur quod punctus supremus tantum pertranseat de circumferentia maxima in uno tempore, sicut in alio sibi aequali. Tunc sic: punctus supremus illius sphaerae per totam horam movebitur et movetur uniformiter, et secundum illam positionem velocitas huius sphaerae attenditur penes punctum supremum et velocissime motum; igitur haec sphaera per totam horam movebitur et movetur uniformiter, igitur aliqua sphaera movetur per horam latitudine motus uniformiter difformis quae tamen per eandem horam continue uniformiter movetur.

117 Item, si ista positio foret vera, sequitur quod aliqua sphaera tardaret continue motum suum per horam, quae tamen per eandem horam | continue uniformiter moveretur. Probatur: sit P 34va aliqua sphaera et centro fixo volvatur circa ipsum et sicut continue volvit, ita continue corrumpantur | puncta suprema K 207vb circumferentialia velocissime mota quousque totum sit sub non gradu quantitatis sphaeralis, ita quod corruptio illius sphaerae ab extremis punctis cicumferentialibus progrediendo versus centrum sine saltu punctorum; volo tamen quod nullus punctus qui movebitur intendat vel remittat motum suum dum movetur, sed semper eodem gradu moveatur quo incipit moveri donec totus punctus corrumpatur. Tunc isto casu supposito arguo sic: ista sphaera movetur alio et alio puncto continue tardiori et tardiori, quia quo magis attendet ad centrum, eo magis movebitur aliquo puncto supremo qui movebitur tardius primo puncto et tardius, et secundum illam positionem in omni motu suo motus istius attenditur penes punctum velocissime motum, sed continue alius et alius erit punctus velocissime motus continue tardior et tardior,

1 difformis] difformi V 2 punctus] *om.* R 4 sibi] vel R 5 totam] illam RK 6 attenditur] intenditur R 7 et] a R 8 totam] illam *add.* R | movetur] penes *add.* K | uniformiter] penes uniformitatem RV 10 quae] qui R 13 motum…horam] per horam suam motum R | quae] quem R 15 aliqua] A *add.* R | et[1]] in KRV | fixo] fixa R 16 ita] sic R 18 sphaeralis] pedalis KR | corruptio] centro R 19 versus] suum *add.* R | centrum] suum *add.* K 20 nullus] *om.* R 21 intendat] intenda R 24 alio[1]] alia K | et tardiori] *om.* R | tardiori[2]] ut patet ex casu *add.* V 25 quia] ex *dd.* K | quo] quanto V attendet] procedit R | centrum] caelum V 26 qui] igitur R 27 et] *om.* K 28 penes] suum *add.* R | motum…motus] non tamen R | sed…motus] *om.* K | alius…erit] erit alius et alius V 29 et[2]…ut] *om.* R

ut patet ex casu. Igitur haec sphaera continue tardabit motum suum, et tamen continue uniformiter movebitur, nam in quolibet instanti totius horae quilibet punctus qui movebitur vel movebatur ex casu uniformiter movetur, et per consequens quilibet punctus supremus velocissime motus continue uniformiter movetur. Sed motus sphaerae sequitur motum puncti supremi secundum illam positionem, sed quilibet talis punctus movetur et movebitur uniformiter, igitur et tota sphaera uniformiter movetur, igitur sequitur quod aliqua sphaera tardabit continue motum suum per horam, et tamen per eandem horam continue uniformiter movebitur; in quo etiam casu sequitur quod V 130v aliqua sphaera continue tardabit motum suum, | et tamen quilibet punctus qui movebatur, movebitur et movetur, uniformiter movebitur vel movetur.

118 Item, si positio foret vera, sequeretur quod aliqua sphaera tardabit continue motum suum per horam quae per eandem R 162vb horam continue | velocitabit eundem. Probo: remaneat casus prior per totum hoc solum excepto quod quilibet punctus de quo suppositum est prius quod uniformiter moveretur, nunc continue quamdiu est, intendat motum suum quod potest fieri, si talis sphaera simul cum hoc quod corrumpitur velocius et velocius circumvolvatur. Tunc sic: haec sphaera continue tardabit motum suum per horam, hoc probatum est supra, et tamen per eandem continue velocitabit motum suum, quoniam omnis punctus quo movebitur continue velocitabit motum suum, et per consequens omnis punctus supremus qui est vel qui erit continue velocitabit

1 ut] *om.* K 3 quilibet] *om.* K | qui] quo V | movebitur…movebatur] movebatur vel movebitur KR 4 et…6 movetur] *om.* (*hom.*) K consequens] sequitur quod *add.* R 8 et movebitur] *om.* R | et[2]] *om.* KR 9 aliqua] tota V | tardabit…suum] continue tardabit motum suum R 11 in] ex R | etiam] tamen V | casu] *om.* R 12 et] *om.* R 13 movebitur…movetur] movetur vel movebitur K movetur et movebitur R 14 movebitur…movetur] *om.* KRV 15 foret] media *add.* R 16 tardabit…suum] continue tardabit motum suum R | continue] *om.* V quae] et continue R | per[2]…continue] continue per eandem horam V 17 probo] probatio K 18 solum] solo K | quilibet] quibus R 19 suppositum] *lac.* V | nunc] tunc R | continue] *om.* R 20 est] ille R 21 cum] et R 22 circumvolvatur] econtra volvatur R | tunc] arguo *add.* R talis V 24 continue] *om.* K | continue…suum] horam velocitabit eundem R | omnis] eius V 25 velocitabit…continue] *om.* V 26 qui[2]] *om.* K velocitabit] tardabit V

motum suum et penes huius puncti motum attenditur motus sphaerae totius, igitur tota haec sphaera continue velocitabit motum suum. Ex quo sequitur, ut videtur, inconveniens adductum. In isto etiam casu sequitur quod aliqua sphaera continue movebitur tardius et tardius, et tamen quilibet punctus penes quem attendetur motus totius movetur | velocius et velocius. Multa alia possent hic argui, sed dimitto propter prolixitatem operis demittandam(*sic!*) solum tango et breviter, dans aliis materiam perscrutandi profundius et diffusius arguendi. P 34vb

<Quartum inconveniens>

119 Quarto ad articulum arguo sic: si velocitas motus sphaerae cuiuslibet penes aliquid attendatur et penes nullum punctum vel motum cuiuscumque puncti infimi, medii vel supremi attenditur, sicut patet et argutum est, igitur talis velocitas attendetur penes aliquod spatium et descriptionem alicuius spatii in tanto tempore vel in tanto.

120 Sed hoc arguo esse falsum, quia si sic, igitur velocitas talis sphaerae attendetur penes spatium corporale descriptum a tali mobili vel a tali, sicut est communis positio et vulgaris. Sed contra: ex isto sequitur quartum inconveniens adductum contra articulum. Probo: circumvolvatur aliqua sphaera, tunc sic: in ista sphaera sic mota infinitae sunt sphaerae concentricae aliae quarum aliquae infinitae pertranseunt maius spatium in aequali tempore quam una certa data et etiam quarum aliquae infinitae pertranseunt minus spatium corporale in aequali tempore una

1 puncti] h *add. sed exp.* P 2 tota] totalis R | tota…sphaera] haec sphaera tota V 4 in...casu] et in illo et in casu R | etiam] autem V 6 totius] totus R | et velocius] *om.* KR 7 alia] *o*m. KRV | sed dimitto] *om.* V 8 operis] *om.* V | et] *om.* R 9 aliis] aliquibus R | perscrutandi] perscrutandam V profundius…diffusius] diffusius et profundius R 13 et] cum K vel…supremi] *om.* R 14 infimi] vel *add.* V 15 sicut] ut R 16 tanto] vel in tanto *add. sed del.* K | tempore…tanto] vel in tanto tempore K 19 sphaerae attendetur] semper attenderetur V | penes] aliquod *add.* R 20 mobili] motu KR | positio…vulgaris] opinio divulgata R | vulgaris] vulgata K 22 tunc…sphaera] *marg.* R 23 aliae] *om.* R aliquae V 24 quarum] *iter.* V | infinitae] sunt quae infinitae K 25 quam…tempore] *om.* K | et…49,1 data] *iter.* V | etiam] *om.* V | quarum] quare R 26 minus] maius RV | corporale] *om.* V | tempore] quam *add.* V

12 quarto] Cf. § 105.

certa data, igitur in sphaera totali sunt infinitae quae moventur velocius et aliae infinitae quae moventur tardius, igitur totalis sphaera movetur difformiter et non, simul movetur uniformiter et difformiter, ut patet, igitur sequitur inconveniens adductum.

121 Item, omnis sphaera mundi in aequali tempore plus describit K 208ra de spatio | corporali et pertransit quam aliqua eius pars, quia totum describit totum et partem simul pars vero non nisi partem, R 163ra | igitur omnis sphaera mundi movetur |V 131r | velocius quam aliqua eius pars, et per consequens quaelibet pars cum toto movetur difformiter, igitur totum movetur difformiter.

122 Item, si illa positio foret vera, sequeretur quod aliqua sphaera moveretur praecise in duplo velocius alia quae tamen in octuplo velocius moveretur eadem. Probo: sit aliqua sphaera mota circa centrum suum cuius punctus supremus sit A et medius B, tunc sphaera cuius A est punctus supremus movetur praecise in duplo velocius illa sphaera cuius punctus supremus est B, ex quo praecise in duplo plus distat a non gradu motus et a centro fixo. Et tamen in octuplo velocius movetur, quia octuplum spatium corporale in aequali tempore describit et pertransit, igitur, etc.

123 Infinita alia possent adduci, sed transeo, quia reputo falsissimam illam positionem.

<Quintum inconveniens>

3 difformiter] *om.* KRV 4 ut patet] nec potest KR | sequitur] patet KR 5 mundi] quae transit *add.* K 7 simul] partes *add.* R | non] est K non…partem] non est nisi per partem R 8 movetur] velocius quam aliqua eius pars etc. *add. marg* V 9 et…pars] *om.* (*hom.*) R | quaelibet] eius *add.* K 10 igitur…difformiter] *om.* (*hom.*) KR 12 praecise] *om.* K praecise…velocius] in duplo velocius praecise V | velocius] praecise *add.* R octuplo] duplo R 13 mota] in *add.* R 14 suum] *om.* R | supremus] medius K 15 praecise] *om.* R 16 illa] secunda R 17 et] cum K | a[2]] *om.* R 18 velocius] plus R | quia] qua R 19 in…et] per inaequale tempus V 20 infinita] quasi *add.* K | transeo] pertranseo R | quia…positionem] propter brevitatem V | reputo] puto R 21 falsissimam…positionem] illam positionem falsissimam RK

124 Quinto ad articulum. Si sic, igitur talis velocitas augeretur penes spatium supersimile descriptum a mobili in tanto tempore vel in tanto, sicut ponunt alii et est secta quinta.

125 Sed contra: ex illo sequitur quintum inconveniens adductum contra articulum. Probo quod sequitur: sit A B C D unum quadratum et supponatur per totum aliqua linea A B et moveatur illla linea ab A B in C D describendo | totum quadratum sic quod P 35ra aequaliter per totum tempus motus aequaliter A C D (*sic!*), tunc capio totam illam lineam quae sit E et punctum describentem qui sit F, et sequitur quod deduxi, quoniam in principio temporis E, F mobilia aequaliter distant a terminis suis fixis et per totum tempus aequaliter distabunt in motu a suis terminis, et aeque cito devenient ad suos terminos, ut patet ex casu, et tamen E improportionaliter velocius movetur per totum tempus quam F, quia in eodem tempore improportionaliter maius spatium describit quam F, quoniam E describet totum quadratum et F solum costam in aequali tempore, igitur etc.

126 Item, eadem vel similia inconvenientia vadunt contra illam positionem sicut contra quartam arguendo in circulis, sicut est argutum in sphaeris, quam etiam positionem reputo esse falsam.

<Sextum inconveniens>

127 Sexto ad articulum. Si sic, igitur velocitas motus sphaerae cuiuslibet maxime motae circa centrum suum attenderetur penes spatium lineale a puncto velocissime moto descripto vel penes spatia linealia a punctis velocissime motis in eodem tempore vel

1 quinto] *om.* K | articulum] arguitur sic, quia *add.* V | augeretur] attenderetur K 2 supersimile] supersphaerale K sphaerali R 3 secta quinta] quinta opinio V 4 adductum] quod *add.* R 6 supponatur] super ponatur V 7 illa…sic] ita KR 8 totum] *om.* V | aequaliter^{2}] aliqualiter K aequaliter^{2}…D] aliqualiter A C A R 9 capio] accipio KV actio R 10 sequitur] ad *add.* V | quoniam] quod V | E] est R 13 et] cum K 14 velocius] *om.* K 16 quoniam] 4 R 18 similia] consimilia R | contra] versus R 19 quartam] positionem *add.* R | sicut^{2}…argutum] ut argutum est R 20 etiam positionem] opinionem V 24 motae] mota V

1 talis…3 quinta] Cf. Thomas Bradwardinus, *op. cit.*, 128.

1 quinto] Cf. § 106. | 23 sexto] Cf. § 107.

aequali descripta, sicut tenet una opinio et est positio magistri Thomae de Bradvardyn quam positionem reputo necessariam et veracem. Concordat tamen illa positio cum tertia quas ceteris abiectis arbitror esse finaliter sustinendas.

128 Contra quam tamen nihilominus arguo: si illa positio foret vera, sequitur primo sextum inconveniens ductum contra R 163rb articulum. Probo: sit A aliquod grave simplex | sphaericae tamen V 131v figurae | positum extra locum suum naturalem in medio uniformiter resistente per totum non impeditum et moveatur ad locum suum naturalem. Isto casu supposito si positio esset vera, sequitur quod hoc grave non potest intendere motum suum, quod arguo sic: in motu descensus huius gravis omnis eius revolutio semper erit per spatium lineale aequale vel idem, quia per circumferentiam propriam et eandem, ut patet, et velocitas K 208rb motus huius descensus attenditur penes huius spatium | lineale descriptum a puncto velocissime moto. Sed in omni tali revolutione punctus velocissime motus describit continue eandem circumferentiam vel aequalem, igitur hoc grave datum non intendit motum suum nec intendere potest, quia quantumcumque velociter moveretur, semper describeret sphaeras vel circulos quorum diameter gravis esset eadem vel aequalis in omnibus diametris spatiorum descriptorum ab A gravi et per consequens per Euclidem in tertio *Geometriae* suae,

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1 sicut] ponit et *add.* K | opinio…est] *om.* K | positio] *om.* R positio…Bradvardyn] Thomae de Brawandyn magistri V 2 Bradvardyn] Bardvardi K Usulis R | positionem] opinionem V | necessariam] intraneam V | et] *om.* R 3 veracem] et *add.* K | tamen] cum K in R positio cum] *om.* R | cum] et K 4 arbitror…finaliter] reputo finaliter esse V 5 tamen] *om.* K | nihilominus] per ordinem R | positio] opinio V 6 primo] *om.* R | ductum] adductum RV 7 tamen] *om.* V 8 figurae] *om.* R positum] ponatur K 9 per…naturalem] *om.* R 10 supposito] posito R positio] data *add.* K 12 huius] *om.* V | omnis] *om.* K 13 semper] *om.* V semper erit] supervenit K | erit] est R | lineale] eius K | quia] *om.* V 15 huius[2]] huiusmodi V 19 nec] neque R 21 gravis esset] essent V gravis…diametris] *om.* (*hom.*) R 22 spatiorum] sphaerarum V | spatiorum descriptorum] sphaerarum descriptarum K | gravi] gradui R *om.* V 23 in] *om.* V

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1 sicut...3 veracem] Thomas Bradwardinus, *op. cit.*, 130. | 23 per[2]…suae] Campanus de Novara, *Elementa*, in: *Campanus of Novara and Euclid's Elements*, ed. by H. L. L. Busard, Franz Steiner Verlag 2005, vol. I, lib. III, 108: Quorum diametri sunt aequales, ipsos circulos aequales esse (...).

continue describeret spatia quorum circumferentiae maxime essent aequales et per consequens qualitercumque moveretur hoc grave, numquam potest intendere motum suum.

129 Secundo. Si positio esset vera, sequitur quod aliqua sphaera moveretur in duplo praecise velocius alia, et tamen motus illius ad | motum alterius esset multo minor quam duplus. Probo: sit A P 35rb
aliqua sphaera et in sphaera A signo aliquam sphaeram concentricam per B cuius diameter sit praecise duplus ad diametrum A sphaerae. Deinde signo punctum velocissime motum in A per C et punctum velocissime motum in B per D in eodem semidiametro et circumvolvantur A, B in aequali tempore vel eodem. Tunc sic: A et B circumvolvuntur circa idem centrum et A describit duplum spatium lineale in aequali tempore vel eodem tempore praecise, igitur A in duplo velocius movetur quam B praecise. Consequentia patet et antecedens similiter, cum aequale spatium maximum lineale pertransietur in aequali tempore ab A B et C D, sed C pertransiet duplum spatium lineale in eodem tempore vel aequali ad D, igitur et A pertransiet duplum spatium lineale etc. ad B, igitur A praecise in duplo velocius movebitur B in eodem tempore vel aequali, et tamen motus istius A erit multo minor quam duplus ad motum B. Quod arguo sic: motus ipsius C erit multo minor quam duplus ad motum D, sed motus A et B sunt aequales motibus C et D iuxta illam positionem, igitur motus illius A est multo minor quam duplus ad motum illius B. Assumptum arguo sic, scilicet quod 'motus ipsius C', etc., nam arguo sic: A sphaera est praecise dupla ad B sphaeram, ut

2 qualitercumque] quantocumque V 4 secundo] item K *om.* R 5 moveretur] continue *add.* R | moveretur…praecise] praecise moveretur in duplo velocius K | praecise] *om.* R | illius] unius K 7 sphaera A] ista V aliquam] aliamV 8 concentricam] *om.* R 9 signo…motum] signato puncto velocissime moto V 11 circumvolvantur] circummoventur R aequali…eodem] eodem tempore vel aequali V 12 tunc...eodem] *iter.* K sic…B] A B V | circumvolvuntur] circummoventur R 13 lineale] *correxi ex* linealem P | vel…tempore] *om.* V 14 tempore] *om.* KR | in…praecise] in duplo movebitur velocius praecise quam B *et iter.* in duplo movebitur velocius praecise R 15 cum aequale] tamen K 16 aequale] *om.* R aequaliter V | in…tempore] *om.* R 17 tempore] *om.* V 18 et] cum K 19 lineale etc] *om.* V | etc] *om.* R | praecise…velocius] in duplo velocius praecise R 20 B] *om.* K | istius] *om.* V 21 B] D KRV | quod...D[1]] *om.* KRV 23 A…B] D et A V | et[2]] *om.* K 24 illius] *om.* V 25 assumptum] antecedens R 26 ut probabo] non probando K *om.* R

probabo, igitur diameter A ad dametrum B est multo minor quam dupla. Consequentia patet, quia proportio sphaerarum est V 132r proportio diametrorum triplicata ut patet | XII E<uclidis>, et per consequens diameter A ad diametrum B est multo minor quam dupla, ut patet, et per consequens maxima circumferentia A R 163va ad maximam circumferentiam B erit multo minor quam dupla, | sed talis erit proportio motus A ad motum B, qualis est proportio maximae circumferentiae A ad maximam circumferentiam B ex positione data, sed ista proportio circumferentiarum est multo minor quam dupla, ut dictum est, igitur motus A ad motum B erit multo minor quam duplus. Nunc quod A sphaera sit praecise dupla ad B sphaeram arguo sic: A secundum omnem dimensionem, secundum longitudinem, latitudinem et profunditatem est praecise duplum ad B, igitur A est praecise duplum ad B.

130 Item, maxima dimensio A secundum longitudinem, latitudinem et profunditatem est praecise dupla ad maximam dimensionem B secundum longitudinem, latitudinem et profunditatem, igitur A est praecise duplum B, utraque consequentia est satis nota et antecedens similiter notum, quia maximae dimensiones A vel B assignantur per diamteros earundem, ut patet. Sequitur ex casu: diameter A ad diametrum B se habet in proportione dupla, igitur etc., et sic sequitur P35va inconveniens adductum.|

131 Item, si positio foret vera, sequitur hoc inconveniens quod A et B sunt duo mobilia quae moventur aequaliter in hora, cuius

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2 dupla] duplum V | consequentia patet] *om.* V 3 ut patet] *om.* V | XII Euclidis] *om.* R | Euclidis] E E (*sic!*) P Euclidis et proportio sphaerarum K 5 ut…dupla] *om.* (*hom.*) V 6 erit…8 B] *om.* (*hom*) K 7 talis] circumferentia *add.* R | A] *om.* V 9 positione] iam *add.* R 10 erit] est V 12 dupla] *om.* K 13 secundum] scilicet KR | longitudinem] et *add.* K 14 duplum] dupla V ad…17 dupla] *om.* V | B] *lac.* K 15 duplum] ad *add.* KR 16 longitudinem] et *add.* K 19 duplum] ad *add.* K 20 est…nota] satis patet K patet V notum] est *add.* V 21 dimensiones] illius *add.* KR istius *add.* V 22 earundem] eorundem V | sequitur] sed RV | A] *om.* R 23 sic] *om.* KR 26 in] per R

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2 quia…3 Euclidis] Campanus de Novara; *op. cit.* lib. XII.15: Omnium duarum sphaerarum est proportio alterius ad alteram tamquam suae diametri ad diametrum alterius proportio triplicata. Cf. Bradwardinus, *op. cit.*, 124.

ultimum instans est D, et tamen nec ante D instans nec in D instanti, nec post D instans erunt motus A et B aequales. Probo: sint A et C duae tabulae rotundae et circularis figurae et plane eiusdem quantitatis praecise et C tabula fixa, moveatur et circumvolvatur A tabula supra eam, ita tamen quod quilibet eius punctus uniformiter circumvolvatur, vel sit casus de duabus molaribus in molandina, ubi unum molare suprapositum alteri circumferetur et supponantur omnia quae de istis tabulis supponuntur. | K 208va Signetur punctus in maxima circumferentia A, puta molaris superioris, per B et situs ipsius B in lapide molari inferiori, puta C, signetur per E a quo situ incipiet revolutio tam A quam B et hora in qua fiet ista revolutio signetur per K. Tunc sic: maxima circumferentia ipsius C quae est maximum spatium lineale ipsius C pertransiretur ab A et B mobilibus in K hora, ut patet ex casu, igitur A et B mobilia movebuntur aequaliter in K hora. Consequentia patet ex positione data et antecedens est verum ex casu, igitur consequens, et tamen motus A et B numquam erunt aequales, quia nec in fine horae, quia tunc cessabit motus, nec post finem horae, per consequens nec ante finem horae. Quod probo: per totam horam per quam A et B moventur A in aequali parte horae pertransiet maius spatium lineale quam B, igitur per totam horam movetur A velocius quam B, igitur, etc. Probatio antecedentis: A | R 163vb per totam horam pertransiet tantum quantum pertransiet in tota hora, quia per totam horam pertransiet circumferentiam | V 132v ipsius B et nec minus nec maius pertransiet in tota hora, igitur in omni parte horae qua B pertransiet aliquid de maxima circumferentia C, in eadem hora A pertransiet totam illam circumferentiam, igitur per totam

1 ante…instanti] in D instanti, nec ante D instans V 2 motus…B] A et B motus K 3 et[1]] *om.* V | plane] et *add.* V 4 et circumvolvatur] *om.* KRV 6 vel] ut V 7 molaribus…molandina] molentibus in molendino R molandina] molendino KV 8 circumferetur] circumferatur R 9 supponuntur] supposita sunt V 10 molaris superioris] molare superius K 12 in] *om.* KR | sic] si R 13 est] et R 14 K] *lin.* K 15 ex…aequaliter] *om.* R 17 igitur] et *add.* RV 18 nec] non R 19 horae] et *add.* R *add. sed. del.* V 20 probo] arguo sic R 21 parte horae] *om.* R 23 igitur etc] *om.* R probatio antecedentis] probo antecedens V | A…horam] per totam horam a R 24 tantum] spatium *add.* V | quantum] B *add.* R | hora] B *add.* V quia] puncta(?) *add.* K punctus *add.* R 25 pertransiet] transit R | ipsius] *om.* KR | nec] non R 26 maius] magis KV 27 aliquid] *om.* KR 28 A] *om.* R

horam et in omni parte horae movebitur A velocius B, igitur numquam erunt motus A et B aequales ante finem horae. Et quod in omni parte K horae erit maxima circumferentia C tota simul pertransita ab A; probo: nam omnis punctus illius circumferentiae erit pertransitus simul ab A in omni parte horae, cum ita erit quod quilibet punctus istius A superponitur alicui puncto maximae circumferentiae C mutabit situm suum in omni parte horae, igitur, etc.

132 In quo etiam casu sequitur aliquid aliud inconveniens quod B punctus usque ad finem horae continue movebitur uniformiter, et tamen ante finem horae intendit motum suum. Probo: nam prima pars inconvenientis sequitur ex casu, et probo secundam sequi, nam sequitur: in omni parte horae movebitur A velocius B, P 35vb igitur in omni | parte horae gradus motus quo movebitur B erit tardior et remissior gradu motus quo in eadem parte movebitur A, igitur in omni parte horae gradus motus quo movebitur B distat a gradu motus quo in eadem parte horae movebitur A, et illum tamen gradum habebit B in hora, igitur ante finem horae B intendet motum suum, igitur, etc. In isto etiam casu sequitur quod A et B incipiunt aequaliter moveri, A tamen sine omni proportione velocius, patet satis consideranti positionem et casum et multa alia sequuntur in casu isto, et multa possent hic argui, sed dimitto ne fastidium generem intuenti.

<Ad oppositum articuli>

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1 et] *om.* R | omni] tempore horae *add.* K tempore horae et *add.* R 2 et quod] cum K 3 parte…erit] horae K parte R | erit] et K 4 pertransita] pertansitur R | probo…A] *om.* (*hom.*) K 5 A] et *add.* V | cum…7 horae] *om.* (*hom.*) KR 9 aliquid] *om.* V 10 usque ad] versus K 11 horae] K *add.* R | probo] probatio KV 12 et] sed R | secundam] illam K 13 parte] illius KR 15 in…movebitur] movebitur in eadem parte KR 16 horae] *om.* R 17 parte horae] hora R 18 illum] nullum KR | horae] *om.* K 19 igitur etc] *om.* V 20 A tamen] actu R 22 et[1]] *om.* V | sequuntur…argui] possent argui in casu isto et sequuntur V 23 dimitto…intuenti] dico propter brevitatem R | ne…intuenti] quam breviter K

133 Ad oppositum est magister in tractatu suo *De proportionibus* capitulo 3 et 4, ubi dicit quod velocitas cuiuslibet sphaerae motae saltem orbiculariter attenditur penes punctum velocissime motum et motus quarumlibet duarum sphaerarum in eodem tempore vel aequali circumvolutarum attendetur penes maxima spatia linealia in eodem tempore vel aequali descripta[rum]. Quod sic intendit quod quam velociter movetur punctus supremus qui inter omnia puncta sphaerae maxime distat a centro sphaerae, tam velociter movetur tota sphaera, ita quod motus totius denominatur a motu istius puncti, et motus quarumlibet duarum sphaerarum est secundum maxima spatia linealia a suis punctis velocissimis in eodem vel aequali tempore descripta[rum], quod sic intendit: acceptis duabus sphaeris in eodem tempore vel aequali circumvolutis qualis erit proportio | maximae R 164ra circumferentiae unius descriptae a suo puncto extremo et supremo | ad cicumferentiam alterius in eodem tempore vel V 133r aequali descriptam a suo puncto extremo et supremo, talis erit proportio velocitatis unius ad velocitatem alterius. Unde si a puncto velocissime moto describatur in eodem vel aequali dupla circumferentia ad aliam, motus istius erit duplus ad alium, si aequalis circumferentia, aequalis motus, si minor circumferentia, minor motus, et hoc loquendo semper de maxima | K 208vb circumferentia sphaerae. Et illud reputo ab eo infallibiliter demonstratum et ideo per illam partem non arguo, cum ab ipso sint ista demonstrative arguta.

1 ad] in V | oppositum] huius *add.* R | magister] *om.* K Thomas de Bardvardin R magistri V 2 velocitas] motus *add.* R 4 motum] punctum K | quarumlibet] cuiuslibet V | duarum] *om.* K 5 circumvolutarum…aequali] *om.* (*hom.*) KR 7 intendit] intelligit V | quam velociter] quanta velocitate R 8 sphaerae[2]] *om.* V 9 tam velociter] tanta velocitate R | totius] sphaerae *add.* V 10 denominatur] denotatur R 12 velocissimis…descriptarum] in eodem tempore vel aequali descripta R 13 intendit] intelligit VK 16 ad…supremo] *om.* (*hom.*) R 17 suo] *om.* V 19 describatur…aequali] in eodem tempore vel aequali describatur R eodem] tempore *add.* K 20 istius] ille K | alium] istum V 21 circumferentia[1]…motus] motus erit aequalis R 22 semper de] a parte V 24 et] *om.* R | per illam] pro ista V | ab…arguta] per ipsum sit demonstrative argutum V

1 magister...6 descriptarum] Thomas Bradwardinus, *op.cit.*, cap. IV, 128-130.

<Opinio auctoris ad articulum>

134 Unde per hoc dico ad articulum concedendo articulum et dico quod velocitas cuiuslibet sphaerae motae circa centrum suum attenditur penes punctum et penes punctum suum velocissime motum, ita quod tota sphaera movetur ita velociter sicut ille punctus et non velocius, et denominatio totius motus P 36ra erit a denominatione motus istius puncti. Consimiliter dico | de duabus sphaeris motis in eodem tempore vel aequali uniformiter revolutis, quod qualis fuerit proportio maximarum circumferentiarum et etiam talis erit proportio motus sphaerarum.

135 Unde abiectis prima, secunda, quarta et quinta opinionibus tamquam falsis, sextam et tertiam sustineo tamquam veras.

136 Apparentia vero et color adductus pro prima positione non vadunt ad propositum nec contra propositum, quia aliter est de sphaeris in eodem tempore revolutis et versus eandem differentiam positionis quam de sphaeris quarum una movetur versus unam differentiam positionis, alia versus aliam differentiam positionis et una complet cursum suum omni die, ut sphaera stellarum fixarum, alia in mense, ut Luna, alia in anno, ut Sol, alia in triginta annis, ut Saturnus. Sphaera etiam stellarum fixarum movetur ab oriente in occidentem, econtra aliae sphaere infra ipsam ab occidente in orientem econtra revolvuntur.

1 unde] *om.* V | articulum^{2}] ipsum R 3 et…punctum] *om.* (*hom.*) V suum^{2}] summum R 6 motus] *om.* R | Consimiliter dico] similiter dico quod R 7 motis] *om.* KR 9 et] *om.* V | et etiam] etc K *om.* R | proportio] *om.* R | proportio…sphaerarum] motus sphaerarum proportio V 11 prima] et *add.* V | secunda] quarta *add.* KR tertia quarta *add.* V 12 sextam…tertiam] tertiam quartam a R | sustineo] astruo K 13 positione] opinione RV 14 vadunt] vadit V 15 in] *om.* V | et] semper *add.* R | versus…movetur] *om.* R 16 quam…18 positionis] alia vero secundum aliam differentiam positionis quam de sphaeris quarum una movetur versus unam differentiam positionis et alia versus aliam differentiam positionis K 17 positionis] differentiam et *add.* R 18 differentiam positionis] *om.* R | complet] appetit R | ut…Luna] *om.* V 21 occidentem] occidens K | econtra] sed R 22 infra ipsam] infernae moventur R | orientem] oriens K | econtra revolvuntur] *om.* R revolvuntur] revolvantur K

11 prima] Cf. § 108. | secunda] Cf. §§ 111--114. | quarta] Cf. §§ 119--123. | quinta] Cf. §§ 124--126. | 12 sextam] Cf. §§ 127--132. | tertiam] Cf. §§ 115--118. | 13 pro prima] Cf. §§ 108--110.

137 Dico etiam, ut ille magister Ricardus de Versellys demonstrat, quod velocitas motus sphaere attenditur penes punctum medium nec hoc tenendum est. Sed forte videatur dicere quod tota latitudo motus localis correspondeat suo medio gradui, sicut consequenter conceditur; nec hoc repugnat huic quod motus localis attenditur penes punctum velocissime motum, unde stant simul quod in omni motu sphaerali vel locali quocumque motus istius attendatur penes punctum velocissime motum, et tamen in intensione motus, ubi partes motus non remanent, quod tota latitudo motus correspondeat suo gradui medio, sed hoc non oportet, ubi motus extenditur et [per] partes motus remanent in actu, sicut patet in motu sphaerae, et illud videtur dicere magister Guilelmus Hentisberus in tractatu suo *De motu*, ista tamen materia tractabitur in articulo proximo.

<Responsio ad tertium inconveniens>

138 Unde illis quattuor | opinionibus habitis tamquam falsis et R 164rb
erroneis, dico ad primum factum contra tertiam positionem | V 133v
quod aliqua sphaera moveretur per horam latitudine motus uniformiter difformis, et tamen per eandem horam uniformiter moveretur, conceditur tamquam possibilis, et in casu sumpto vera est et dico quod possibile est quod aliquid moveatur motu uniformiter difformi et tamen uniformiter, sicut demonstrative

1 ut] quod nec K nec R | ut ille] quod nec iste V | Ricardus…Versellys] Thomas de Verseli V | Versellys] Verselis K Uselis R 2 medium] *om.* R 3 sed] licet V | forte] *om.* R 4 motus] *om.* KR | suo] sui R 5 huic quod] quia K | quod] quia R 8 istius] velocitatis *add.* R | istius attendatur] iste attenditur K 10 motus] *om.* R 11 oportet] videlicet R | extenditur] attenditur R | per] super R *om.* V 12 illud] idem R | magister Guilelmus] *om.* R 13 Guilelmus] Willhelmus P Guilelmus *et add. sed del.* in tractatu K Guilelmus Hentisberus] Hechybyry P Wilhelmus Hetisberi V Hentisberus] Hentiberi K Zeberus R | tractatu suo] *om.* R ista…proximo] *om.* R 14 in] *om.* K 17 quattuor] *om.* R | habitis] abiectis K praehabitis V | falsis…erroneis] *om.* V | et] in R 18 erroneis] scilicet quattuor *add.* R | primum] argumentum *add.* R 19 moveretur] movetur V moveretur…horam] per horam movetur R 20 difformis] difformi K 21 sumpto] supposito RK fundato V 23 difformi] difformiter K

12 videtur…13 motu] Guilelmus Hentisberus, *De motu locali*, § 26.

1 ut…demonstrat] Cf. §§ 111--114. | 18 primum…positionem] Cf. §§ 115--116.

probatur ex casu. Et est verum de omni sphaera sic mota: unde motus talis sphaerae non attenditur penes latitudinem motus a centro sphaere ad circumferentiam quae est uniformiter difformis, sed penes gradum quo movetur punctus velocissime motus qui in casu sumpto manebit continue uniformis. Nec hoc repugnat huic, quod dictum est superius, quod tota latitudo motus | P 36rb uniformiter difformis suo medio gradui corresponderet, quia in motu sphaere extenso correspondet suo gradui ultimo et supremo ubi vero motus continue intenditur in extenso, ibi habet opinio illa locum.

139 Ad secundum cum arguitur quod aliqua sphaera tardaret continue motum suum per horam quae tamen per eandem horam continue uniformiter moveretur, dico quod hoc non sequitur ex casu nec est verum in casu supposito | K 209ra quod A continue tardat motum suum, quia post primum instans corrumpitur A nec A manet, per consequens nec A tardat motum suum, quia nec potest tardare motum suum nisi esset.

140 Sed hoc non videtur solvere argumentum, quia ponatur quod A continue condensetur versus centrum suum, ita quod condensatio incipiat<ur> a partibus circumferentialibus extremis, et sequitur inconveniens prius adductum, quoniam A manebit continue per totum tempus usque ad finem horae et continue movebitur per circumferentiam minorem et minorem, igitur continue tardabit motum suum, et tamen continue uniformiter movebitur. Ad illud dico adhuc quod non sequitur, sed bene

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1 ex] in R 3 sphaere] usque *add.* V 4 gradum] motum V 5 sumpto] suppostio RK 6 dictum…quod] superius dictum est, quia V 8 quia…correspondet] *om.* (*hom.*) K | quia…supremo] *om.* R | et] in KV 9 extenso] (*corr. ad sensum*) extensus KPV extensio R 10 opinio…locum] locum ista opinio V 11 cum] quod R 12 quae] qui V | per^2…horam] *om.* V 14 quod] sed ego concedo quod nihil est penes quod talis velocitas attenditur, quoniam in tali corruptione vel condensatione nullus umquam erit punctus velocissime motus, in tali enim motu describetur una linea girativa, ut patet intuenti. Haec Donatus de Monte. *marg.* K | continue] *om.* R | tardat] tardet K 15 primum…manet] A corrumpitur neque A manet et R 16 manet] et *add.* K | per consequens] *om.* V | nec A] A non R | A] *om.* V | tardat] intendit K | quia…suum] *om.* (*hom.*) KR | nec^2] non V 17 nisi] ubi K 20 a] ex V 21 prius adductum] antedictum V | adductum] deductum R | quoniam] nam R 25 ad...movebitur] *om.* R

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11 ad secundum] Cf. § 117.

sequitur quod motus A continue erit tardior et tardior, et tamen continue uniformiter movebitur.

141 Contra, sequitur motus A continue erit tardior, igitur A continue tardabit motum suum. Dico quod non sequitur et nego consequentiam, quia illae propositiones diversa signant nec sunt eaedem. Per illam enim 'motus A continue erit tardior et tardior', signatur solum quod motus A erit continue per circumferentiam aliam et aliam minorem et minorem, quod sufficit ad hoc quod motus A dicatur continue tardior et tardior, quod non significatur per aliam. Unde ad hoc quod illa sunt vera vel quod ita sit quod 'A continue tardet motum suum', requiritur quod non in aliis et aliis circumferentiis in aequali tempore pertranseat minus de spatio lineali, sed quod continue in eadem circumferentia in tempore similiter | V 134r aequali continue minus pertranseat de eadem circumferentia et de eodem spatio lineali. Sed quia non est sic in casu supposito, ideo non oportet nec est verum quod A continue tardat motum suum, unde per illud patet quid | R 164va sit dicendum de alia. Nam communiter respondendo et meo iudicio probatur et verum est et concedendum quod motus alicuius sphaerae continue erit tardior et tardior, et tamen quilibet qui movebatur movebitur vel movetur, movebatur, movebitur vel movetur uniformiter continue.

142 Nec sequitur tertium inconveniens nec aliquod, sed hoc bene sequitur in casu quod motus alicuius sphaerae continue erit tardior et tardior, et hoc per aliquam certam horam, quae tamen per eandem horam velocitabit continue motum suum sumpta condensatione | P 36va sphaerae, et per illud patet ad ultimum ibi ductum.

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3 contra] sed contra quia R | erit] est et R | tardior] et tardior *add.* V 4 continue] *om.* R 7 solum] *om.* R 9 non] *om.* R 10 sunt] sit KRV quod[2]] *om.* K 11 continue] *om.* R 13 quod] *om.* KR 14 similiter] sibi K aequali…pertranseat] in certo(?) tempore aequali pertranseat minus R 16 oportet nec] *om.* V 17 unde] *om.* KR 18 nam] omnia V | nam...et[2]] instantia quia R | et[1]…probatur] in illo probatione K 20 quilibet] punctus *add.* R 21 movebitur[1]…movetur] movetur vel movebitur R movetur vel V | movebatur…uniformiter] in tempore R 23 nec…sed] de R 26 eandem…suum] illam continue velocitabit suum motum R | sumpta] supposita K 27 ibi] inconveniens R

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23 tertium inconveniens] Cf. § 118.

<Responsio ad sextum inconveniens>

143 Et ad quartum et primum contra sextam positionem, quia istae positiones sunt eaedem vel similes, dico quod conclusio adducta non sequitur. Ad hoc enim quod sequeretur, requireretur quod non solum in omni revolutione sua describerent spatium lineale aequale, sed quod in omni revolutione in aequali tempore vel maiori describerent spatium lineale aequale, qualiter non erit in proposito, quia in prima parte proportionali horae mensurantis illam revolutionem describet aliquod spatium lineale et aequale vel idem describet in secunda parte proportionali, et aequale vel idem describet in tertia. Et per consequens continue velocius movebitur, ex quo continue in minori tempore pertransiet spatium lineale aequale et sic non concludit argumentum aliquod contra me, ut probat propositum, immo oppositum sequitur in casu.

144 Ad quintum quomodocumque sit de conclusione, dico quod ipsa non sequitur ex casu et nego quod A est praecise duplum ad B. Et tunc cum arguitur contra: 'A est duplum ad B praecise secundum omnem dimensionem, secundum longitudinem, latitudinem, profunditatem, concedo igitur A est praecise duplum ad B'; patet quod non sequitur, sicut superius est ostensum quaestione de augmentatione ubi fit consimile argumentum de corpore cubico; et patet etiam de quadrato aliquo totali diviso in quattuor quadrata aequalia, totum enim quadratum ad quodlibet
K 209rb illorum parvorum quadratorum | quo ad dimensiones est praecise duplum, igitur totum quadratum est praecise duplum ad quodlibet illorum, patet quod non sequitur, quia totum

2 et] *om.* KRV | sextam] secundam R | positionem] opinionem V quia…positiones] quod illa paene R 3 similes] similia R 4 enim] *om.* V quod] ut R 5 describerent…revolutione] *om.* (*hom.*) KR 8 horae] *om.* R 10 proportionali] *om.* R | et] vel V 13 aliquod…15 casu] *om.* V 14 ut] nec aliquid R | propositum…sequitur] immo sequitur oppositum R 16 de] illa *add.* R 17 A] *om.* RV 18 contra] *om.* KR | A…20 igitur] *om.* R | est] praecise *add.* V | praecise…dimensionem] *om.* V 19 dimensionem] divisionem P | secundum[2]] scilicet K 20 latitudinem] et *add.* KV | igitur] quod *add.* K 21 B] praecise *add.* K 23 quadrato…totali] quadrata aliquo K | aliquo] *om.* V 24 enim] *om.* KR 26 duplum[1]] igitur *add.* KR 27 quod…patet] *om.* (*hom.*) R

2 ad…positionem] Cf. §§ 127--128. | 16 ad quintum] Cf. §§ 129--130.

quadratum ad quodlibet illorum est quadruplum, et sic patet ad illud.

145 Ad sextum et ultimum dico quod conclusio in casu est possibilis nec inconveniens ex casu tamen iam supposito non sequitur. Et tunc ad argumentum in oppositum admitto casum, et tunc ad punctum argumenti dico quod A et B aequaliter moventur in toto tempore et per totum tempus. Et tunc cum arguitur contra: ‘per totam horam per quam A et B movebuntur, A pertransit maius spatium lineale quam B, igitur velocius movetur, etc.’, dicitur negando consequentiam, quia hic motus est motus circularis non rectus;| V 134v et est illa descriptio secundum diversam mutationem situs punctorum omnium simul penes quam et qualem descriptionem non debet attendi velocitas A, sed penes illud quod punctus velocissime describit in tota hora. Aliter potest | R 164vb dici negando antecedens istius consequentiae, videlicet quod ‘in omni parte temporis qua B describit aliquid de suo spatio lineali, de eodem in eodem tempore pertransit A maius’; et negando similiter quod ‘in omni parte temporis A pertransit totum spatium lineale et totam circumferentiam suppositam simul’, quia ad hoc quod illam pertranseat proprie et complete non requiritur nec sufficit quod quilibet punctus mutet situm suum, sed quod fiat completa revolutio A per B, a C | P 36vb puncto in idem C punctum, quae revolutio non complebitur nisi in fine horae. Et per consequens nec proprie nec complete erit totum pertransitum ab A ante finem horae et sic non sequitur inconveniens adductum, et per istud clare patet quod in eodem casu ad alia sic dicendum.

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3 et] ad *add.* R | in…supposito] *om.* KK 8 A^{1}…B] aequaliter *add. sed exp.* P 9 movetur] *om.* K 10 etc] *om.* R | dicitur] uno modo *add.* KRV 11 circularis] et *add.* V | illa] secunda R | secundum] per KR 12 omnium] sunt R | et] in R 14 illud] hoc R | velocissime] motus *add.* V | in…hora] per totam horam V | aliter] et ita KR 15 videlicet] scilicet R 17 in eodem] *om.* R | maius] minus R 18 omni] *om.* R 19 totum…lineale] spatium lineale totum R | suppositam] *om.* R 20 ad] *iter sed exp.* R | illam] *om.* KR 21 quod…punctus] in quibus punctis R 23 in] prope R 24 nec complete] om. KR 26 istud clare] idem R | in…alia] ad alia in eodem casu R 27 ad] de V

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3 ad...ultimum] Cf. §§ 131--132.

<Articulus tertius: Utrum velocitas omnis motus localis uniformiter difformis incipiens a non gradu sit aequalis suo medio gradui>

146 Articulus Utrum velocitas omnis motus localis uniformiter difformis incipiens a non gradu sit aequalis suo medio gradui.

147 Et arguitur primo quod non, quia si sic, sequitur primo quod A et B sunt duo motus aequales praecise, et tamen A est in infinitum intensior B.

148 Secundo quod aliquis motus remittetur per horam et in tali remissione ante finem horae deperdet gradum duplum, immo plus quam duplum et plus quam triplum, et tamen in fine horae erit praecise in duplo remissior quam in principio.

149 Tertio quod aliquod mobile simul et semel movetur et quiescit.

150 Quarto quod A est unum tale quod distat a B et C extremis a quibus nec distat nec aequaliter nec inaequaliter.

151 Quinto quod Socrates et Plato aequaliter iam moventur, et Plato post hoc movebitur per horam et solum per horam in qua solum movebitur ita velociter, sicut nunc movetur, et Socrates per eandem horam continue movebitur velocius et velocius Platone et numquam tardius eo per illam horam, et tamen in fine horae non movebitur Socrates velocius quam nunc movetur, sed praecise aequaliter.

152 Sexto et ultimo pro hac vice quod A et B moventur aequaliter in C hora, et tamen per totam C horam inaequaliter moventur.

<Primum inconveniens>

153 Probatio primi inconvenientis. Incipiat Socrates a non gradu motus localis moveri localiter uniformiter difformiter quousque fuerit sub C gradu et signetur talis latitudo uniformiter difformis
R 165ra per A et eius gradus medius | per B. Tunc sic: A motus est latitudo uniformiter difformis incipiens a non gradu et eius

1 uniformiter] indeformiter R 3 non] sit *add.* R | si…sequitur] sequiretur R 7 ante…horae] *om.* R 12 C] *om.* K | extremis a] mediis R 13 nec distat] tamen distat nunc V | nec^2] *om.* K 16 solum…velociter] ita velociter movebitur R | ita velociter] in velocitate K 17 et velocius] *om.* K 18 eo] *om.* KRV 19 movetur] Plato *add.* V 21 pro…vice] *om.* K 22 in] per R 28 A^2] *om.* R 29 uniformiter difformis] difformiter difformis R

25 primi inconvenientis] Cf. § 147.

medius gradus | est B, igitur motus A, B sunt aequales et ultra A V 135r continet ultra B infinitos gradus quorum quilibet est intensior B, igitur, etc.

<Secundum inconveniens>

154 Probatio secundi inconvenientis: sit A latitudo motus localis uniformiter difformis incipiens a non gradu et in extremo intensiori terminata ad B gradum et signetur medius gradus A per D, et medius gradus inter D et non gradum per C, et sic deinceps. Tunc volo quod A remittatur secundum extremum intensius usque ad D gradum exclusive per horam. Tunc sic: A motus remittetur per horam et in tali remissione ante finem horae deperdet gradum duplum ad C, immo plus quam | duplum et P 37ra plus quam triplum, quia in tali remissione deperdet B gradum qui est magis quam duplus et magis quam triplus, ut patet, ad gradum sub quo erit A motus intensius in fine horae, puta C, et tamen in fine horae remittetur A ad unam latitudinem cuius medius gradus erit C praecise subduplus ad istum gradum cui primo A fuit aequalis. Et tunc cum omnis latitudo in A terminata ad non gradum sit aequalis suo medio gradui, igitur A motus in fine horae erit praecise in duplo remissior | quam in principio, et K 209va tamen ante finem horae deperdet gradum magis quam duplum et magis quam triplum ad istum gradum sub quo erit A motus intensius in fine horae.

155 Et ex isto sequitur unum aliud inconveniens, quod A motus deperdet aliquem gradum motus quem non habuit nec habebit, nec habere potest. Probatio illius: ponatur quod A remittatur ultra uniformiter difformiter usque ad non gradum per tempus post hanc horam et sit tempus remissionis totalis a B usque ad non gradum G, et sit D gradus medius inter C et non gradum, et E

1 medius] *om.* K | A^{1}] et *add.* R | A^{2}] *om.* K 3 igitur etc] *om.* KR 6 probatio] secundi *add.* KRV 7 uniformiter difformis] difformiter difformis R 9 D^{1}] B PRV | D^{2}] B PRV 10 extremum] sui *add.* R 11 D] B *lin. add.* K B PV C R | tunc...horam] *om.* (*hom.*) K 15 magis^{2}] *om.* K 16 motus] gradus K | intensius] intensissimus R | tamen] tunc R 18 C praecise] *om.* R | praecise] *om.* KV | fuit] primo *add.* R 21 praecise] *om.* R 22 ante] in R 23 motus] *om.* R 27 illius] huius R 30 G^{1}] A V

6 secundi inconvenientis] Cf. § 148.

gradus medius inter D et non gradum, et sic deinceps. Tunc sic: A motus cum fuerit remissius ad B gradum deperdet gradum quadruplum, et tunc non erit nisi praecise in duplo remissior quam in principio, et cum A fuerit remissius ad C, depredet A octuplum gradum ad istum sub quo erit A tunc intensius, et tamen tunc non erit, nisi praecise in quadruplo remissior quam R 165rb fuerit in principio et sic deinceps | usque ad non gradum, igitur A motus continue deperdet gradum intensiorem quam ipsum remittetur ad aliquem. Sed A remittetur ad gradum subduplum, subquadruplum, suboctuplum, et sic in infinitum, igitur A in tali remissione deperdet gradum quadruplum, octuplum, et sic in infinitum, igitur gradum infinitum et talem non habuit nec habebit, nec habere potest, igitur, etc. Et consequentia patet, quia V 135v quantum decrescit proportio remissionis A, tantum | et amplius crescit proportio deperditionis graduum A motus.

<Tertium inconveniens>

156 Probatio tertii inconvenientis: sit A sicut prius latitudo motus localis, etc. cuius medius gradus sit C et gradus terminans extremum sui intensius sit B, et moveatur Socrates illa latitudine remittendo motum suum continue uniformiter difformiter a B usque ad non gradum. Et arguo tunc sic: B et C iam distant a non gradu et C in subduplo minus distat a non gradu quam B, et utrumque uniformiter difformiter remittetur ad non gradum, P 37rb igitur C in duplo citius erit sub non | gradu quam B. Sed quam cito erit C sub non gradu, tam cito erit A sub non gradu ex quo A et C sunt aequales, igitur totum A erit in duplo citius sub non gradu quam B, igitur Socratis motus illa latitudine in duplo citius quiescet quam ipsemet quiescet, et per consequens in instanti quietis simul movebitur et quiescet. Et confirmo hoc sic: quam diu remanet aliquis gradus A, tam diu movebitur Socrates, ut

1 tunc] et V 2 remissius] remissus R 3 erit nisi] *om.* R | praecise] *om.* K 4 quam] erat *add.* K | A[1]] *lin.* R 6 tamen] *om.* R 9 gradum] *om.* R 11 quadruplum] et *add.* V 13 etc Et] *om.* R 14 amplius] plus R 20 illa] in K 21 uniformiter] indifformiter (?) R 22 et] *om.* R 23 C] A PV subduplo] duplo R | quam B] *om.* V 25 quam[2]] quanto R 26 C] B V | tam cito] *om.* R | quo] *om.* K 27 C] B R 28 citius] *om.* K | citius quiescet] quiescetur R 30 confirmo] confirmatur V 31 tam] tamen R

18 tertii inconvenientis] Cf. § 149.

constat et clarum est, sed postquam tota latitudo A fuerit sub non gradu remanebit aliquis gradus A distans a non gradu per certam latitudinem, igitur postquam Socrates fuerit sub non gradu, adhuc pro tunc movebitur Socrates, igitur in isto instanti in quo erit Socrates sub non gradu motus erit, ita quod Socrates simul et semel movetur et quiescit, igitur, etc.

<Quartum inconveniens>

157 Probatio quarti inconvenientis: sit F aliqua latitudo uniformiter difformis terminata ad non gradum motus in uno extremo et in alio ad B gradum et sit A gradus medius distans a suis extremis, ab B et C non gradu motus gratia exempli, tunc A distat ab B et C, et tamen nec aequaliter nec inaequaliter. Non inaequaliter, quia tunc alius foret gradus medius inter B et C et alius quam A. Consequens falsum.

158 Item, cum medium omne aequaliter distet ab extremis et A sit medium inter B et C extrema, igitur A aequaliter distat a B et C nec tamen aequaliter, quia cum A solum distet a B et C, ab B per latitudinem ab A ad B, a C per latitudinem ab A ad C, igitur A B latitudo foret aequalis A C latitudini. Consequens falsum, quoniam istae latitudines nec sunt aequales | intensive nec R 165va extensive; non intensive, quia A B latitudo continet infinitos gradus quorum quilibet est intensior A C latitudine, igitur A B est intensior B C, igitur, etc.

159 Item, triplum spatium pertransiretur per A B latitudinem in aequali ad illud quod pertransietur per A C latitudinem in eodem tempore vel aequali, igitur latitudo motus A B est in triplo intensior, igitur etc. Et patet antecedens, quia in prima medietate

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1 et] ut R | est] *om.* K | tota] *om.* KR 2 remanebit…gradu^{2}] *om.* (*hom.*) V 12 ab] *om.* R 13 ab] a R 14 alius…medius] foret gradus alius medius gradus V | et^{1}] *om.* K | et alius] *om.* V 15 consequens] est *add.* R quod est V 16 cum…omne] quod medium omne R | extremis] suis *add.* V 17 extrema…C] *om.* (*hom.*) KR | B^{2}…C] C et B V 18 tamen] etiam V | et] A *add.* KR | ab B] *om.* KR 19 A^{1}] *iter. sed exp.* R | a C] *om.* V | a^{2}…per] per C K | ab^{2}…C] ad A et C V 20 consequens] est *add.*R 21 quoniam] quia R nec^{1}] non RV | intensive…extensive] nec extensive, nec intensive V 22 latitudo] *om.* R 23 A^{1}…intensior] *om.* (*hom.*) V 24 etc] *om.* R 25 in…latitudinem] *om.* (*hom.*) R 27 igitur] A B *add.* K 28 et] *om.* V et...antecedens] antecedens patet R

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9 quarti inconvenientis] Cf. § 150.

V 136r temporis mensurantis talem motum Socrates incipiens moveri | ab C in B solum pertransiret unam quartam in F et in secunda medietate tres quartas sui spatii pertransiret, igitur, etc. Ex quo sequitur quod A B et A C latitudines non sunt aequales extensive, cum A B latitudo extenditur per triplum subiectum in aequali tempore ad B C, et per consequens A B et A C latitudines nec sunt K 209vb aequales intensive nec extensive, igitur, etc. |

<Quintum inconveniens>

160 Probatio quinti inconvenientis: si articulus foret verus, P 37va sequeretur quod talis latitudo motus foret aequalis suo | gradui medio sic quod, si Socrates moveretur tali latitudine uniformiter difformi, tantum spatium pertransiret in tempore, quantum Plato, si per idem tempus moveretur uniformiter suo gradu medio, et econverso: aequale spatium pertransiret Plato in aliquo tempore modo unico gradu uniformi et gradu medio latitudinis motus Socratis, quantum in eodem tempore vel aequali pertransiret Socrates et praecise tantum. Sed contra. Ex isto sequitur quintum inconveniens quod arguo sic: sit tunc Socrates qui incipiat moveri aliqua latitudine uniformiter difformi et per idem tempus moveatur Plato gradu uniformi aequali gradui medio latitudinis motus Socratis uniformiter difformis, et manifestum est tunc quod Socrates in prima medietate pertransiet solum unam quartam et Plato duas, in secunda pertransiet Socrates tres et Plato non nisi duas. Tunc sic: Socrates per totam primam medietatem temporis movebitur [Socrates] tardius Platone et per totam secundam medietatem temporis movebitur Socrates

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1 talem] illum R 2 ab] a R 3 tres] duas KR 4 B…latitudines] A et B latitudo et A C R 5 cum] tamen K | A] et *add.* R 6 A[2]] K | nec] *corrup.* R non V 12 si] *om.* K | uniformiter] indifformiter R 13 difformi] et *add.* K 14 si] *om.* KR | tempus] scilicet *add.* R 16 modo] modico V | unico] in aliquo R | uniformi] uniformiter K difformi V | uniformi et] uniformiter cum R 19 tunc] nunc K 20 difformi] difformiter K | per…tempus] *om.* R 21 uniformi] uniformiter K | latitudinis] latitudini K 22 Socratis] Socrati R 23 pertransiet solum] solum pertranseat V 24 Plato] in *add.* K duas] et V | pertransiet] et *add.* R 25 duas] duo V | tunc] arguitur *add.* R Socrates] semper R *om.* V | totam] *om.* R | totam…medietatem] primam medietatem totam K 26 et…68,1 Platone] *om.* (*hom.*) KR

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10 quinti inconvenientis] Cf. § 151.

velocius Platone, igitur in instanti medio copulante illas duas medietates temporis Socrates et Plato movebuntur aequaliter. Consequentia patet, cum quilibet prius movebitur aequaliter quam velocius et prius deveniet ad aequale quam ad excessum, sit igitur nunc medium temporis in quo sic Socrates et Plato moventur aequaliter. Et sequitur tunc inconveniens adductum, nam Socrates et Plato aequaliter iam moventur et Plato post hoc movebitur per horam sic ita et solum per horam in qua solum movebitur ita velociter, sicut nunc movetur, et Socrates per eandem horam continue velocius et velocius movebitur Platone et numquam tardius eo, sicut patet ex casu. Ex casu enim sequuntur omnes istae particulae, sicut clarum est, et tamen in fine horae non movebitur Socrates | velocius Platone, sed R 165vb praecise aequaliter, quod arguo sic: motus Socratis et Platonis erunt aequales et solum in fine horae, quia solum in fine horae et non citius erunt aequalia spatia pertransita ab utroque, igitur in fine temporis movebitur vel erit motus Socratis aequalis motui Platonis et tunc | non erit intensior quam nunc est per casum, V 136v sed aequalis praecise, igitur in fine horae erit motus Socratis aequalis illi qui est in hoc instanti et ultra, igitur Socrates in fine horae non movebitur velocius quam nunc movetur, sed praecise aequaliter, ex quo sequitur inconveniens adductum.

<Sextum inconveniens>

161 Sextum inconveniens sequitur in hoc casu et eodem, nam Socrates et Plato in eodem tempore motus sui pertransient spatia aequalia in aequali tempore, igitur in toto tempore movebuntur aequaliter, et tamen per totum tempus inaequaliter movebuntur, quia per totum primae medietatis temporis et per totum secundae medietatis temporis, igitur Socrates et Plato per totum tempus

1 copulante] copulanti K 2 medietates] s *add. sed. del.* K 3 cum] quia R 4 sit…nunc] signetur tunc R 6 tunc] *om.* K | adductum] *om.* K 7 nam] tunc *add.* V | aequaliter…moventur] moventur aequaliter V 8 solum^{2}] Plato R 11 sicut] ut R | ex…est] et manifestum est ex casu R | ex...enim] unde K *om.* V 14 sic] *om.* K | et] motus *add.* V 15 et^{2}…citius] *om.* K 22 adductum] ante dictum K 25 sequitur] probatur KR | in…eodem] in eodem casu R | et] in *add.* V 27 in^{2}…tempore] per totum tempus V 29 quia…69,1 movebuntur] *om.* (*hom.*) K 30 temporis] *om.* R

25 sextum inconveniens] Cf. § 152.

P 37vb inaequaliter movebuntur. Nec potest dici quod | argumentum non valet, quia in instanti medio aequaliter moventur, quia hoc falsum est, nam per magnum tempus post illud instans erunt motus Socratis et Platonis aequales, et per consequens non in isto instanti medio erunt motus illi aequales.

162 Item, in hoc eodem casu sequitur quod Socrates movebitur aliqua latitudine motus uniformiter difformi terminata usque ad duos gradus aequales. Consequens impossibile et hoc sequitur ex probatione quinti inconvenientis.

163 Et multa alia possent duci quae propter brevitatem dimitto, solum tango illa dans aliis materiam diffusius arguendi et defendendi se. Propter illa igitur et similia quae possent fieri, dicitur a quibusdam quod in latitudine motus localis terminata ad non gradum tota latitudo motus non est aequalis suo gradui medio nec sibi correspondet, sed solum gradui intensissimo sic quod denominatio latitudinis totius sit a denominatione gradus intensissimi in illa latitudine, et proportio motuum secundum proportionem graduum intensissimorum illorum motuum. Sed hoc totum falsum, sicut arguetur in arguendo ad oppositum articuli, quia improbare articulum est improbare illam positionem et econtra improbare articulum, ideo contra utrumque arguetur simul.

<Ad oppositum articuli>

164 Ad oppositum istius articuli arguitur et probatur quod in
K 210ra omni motu | locali uniformiter difformi incipiente a non gradu

4 Socratis…Platonis] illi K | aequales] aequalis R | et[2]…aequales] *om.* (*hom.*) K | isto…aequales] instanti erunt motus aequales R 7 aliqua] eadem K | difformi] difformis K | usque] *om.* KRV 8 consequens] est *add.* R 11 et] vel R | duci] adduci RV | dimitto] obmitto K 12 tango] tangentes R | illa] et *add.*R | illa dans] *om.* K | dans…materiam] aliis materiam dans V | diffusius…se] defendendi et arguendi et in consimilibus R 13 defendendi] *correxi ex* diffundendi P | se propter] reliquens K illa…similia] igitur illa et multa alia R | similia] consimilia K 17 quod] solum *add.*R | gradus] gradui V 20 totum] est R 21 articuli] et *add.*R improbare[1]] improbando K improbo R | est…articulum] *om.* KR 22 et…articulum] *om.* V | econtra] *lin.* P | arguetur simul] *inv.* R 24 istius articuli] huius R

9 probatione…inconvenientis] Cf. § 160.

tota latitudo motus sit aequalis suo medio gradui et solum illi, quia signetur talis latitudo cuius medius gradus sit B et illa tota latitudo A. Tunc arguo sic: in A latitudine est aliquis | gradus R 166ra medius, puta B, quo est alia latitudo intensior et quo est alia latitudo remissior, igitur in A latitudine est aliqua latitudo sibi aequalis et nulla nisi tota, igitur tota est sibi aequalis. Consequentia ultima est necessaria sicut patet, et prima similiter est formalis, ut patet per Commentatorem II *Ethicorum* commento 10, ubi dicit quod in omni continuo et divisibili ubicumque est invenire maius et minus, ibi est invenire aequale.| V 137r Et assumptum primum patet, quia latitudo ab B ad extremum sui intensius est intensior B gradu et latitudo ab B usque ad non gradum est remissior B gradu, ut patet satis, et sic sequitur quod in A sit aliqua sibi aequalis et nulla nisi tota, quia accepta quacumque latitudine quae est pars illius latitudinis totius, illa vel est intensior B, vel remissior B, et per consequens sola latitudo totalis est sibi aequalis, igitur, etc. Ex quo sequitur ultra etiam quod tota latitudo nec est aequalis nec correspondet suo gradui intensissimo.

165 Secundo arguitur sic: sit Socrates qui incipiat a non gradu uniformiter difformiter intendere | motum suum usque ad B P 38ra gradum et Plato ab eodem gradu vel sibi aequali incipiat remittere motum suum uniformiter difformiter ad non gradum, tunc istae latitudines motuum Socratis et Platonis sunt aequales et aliquibus gradibus sunt aequales, et non extremis gradibus suis, ut patet,

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1 medio gradui] *inv.* R 2 latitudo] motus *add.* V | B] c K | et illa] *om.* V et…tota] tota illa K 3 sic] sed *add.* R | in] si K 4 quo[1]…est[3]] tunc R 6 nulla…tota] aliqua non V | tota[1]] latitudo *add.* R | tota[2]] latitudo *add.* K 7 patet et] *om.* R | similiter…formalis] est formalis similiter K 10 ubicumque] ubi K | est invenire[1]] enim reperitur R | invenire[1]] reperire K | maius] magis V | ibi…invenire] *om.* V | invenire[2]] reperire KR 11 assumptum] argumentum R | primum] *om.* V | ab] a V | B] usque *add.* V 12 ab] a KR 13 patet satis] ex casu *add.* R | satis] patet *add.* K 14 sibi aequalis] similis K | nulla] non R 15 totius] A *add.* R 16 sola…totalis] tota latitudo V 17 etc] *om.* R | etiam] *om.* RV 20 gradu] gradum R 22 sibi] *om.* K | sibi aequali] consimili V 24 motuum] *om.* K | et[2]] cum K 25 suis] sunt aequales *add.* R *om.* V

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8 patet…10 aequale] Cf. Arist., *Eth. Nicom.* II.6, 1106a; Aver., *Moral. Nic. Expos.* Venetiis apud Iunctas 1592, 24vb.

igitur mediis, quia non videtur quibus aliis corresponderent aut forent aequales, igitur, etc.

166 Item, sit aliqua latitudo motus localis uniformiter difformis incipiens a non gradu et signata per A et eius medius gradus per B, et incipiat Socrates moveri A latitudine, et incipiat Plato moveri B gradu medio eiusdem. Tunc sic: Plato pertransiens aliquod spatium B gradu in aliquo tempore tantum spatium praecise pertransiet, quantum pertransiet Socrates tota A latitudine in eodem tempore vel aequali, igitur A latitudo est aequalis B gradui. Consequentia est manifesta et probo antecedens ad cuius probationem sumo quod gradus terminans A in extremo suo intensiori sit C signatus per 8, gradus medius inter B et C sit D signatus per 6, B vero gradus medius latitudinis totalis per 4, gradus medius inter B et non gradum E signatus per 2, et sit F tempus in quo Socrates pertransiet aliquod spatium, quod sit G. Et arguo tunc sic: per totam primam medietatem F temporis Socrates et Plato movebuntur, Socrates continue tardius Platone, igitur Plato continue velocius Socrate, sic igitur quod in duplo velocius et quod Socrates per latitudinem quae est a non gradu
R 166rb usque ad gradum medium exclusive qua solum | movebitur prima medietate temporis, pertranseat unam quartam tantum. Et sequitur: tunc in prima medietate F temporis pertranseat Socrates unam quartam de spatio suo et Plato per idem tempus movebitur in duplo velocius, igitur Plato in eodem tempore pertransiet duas quartas de spatio suo, igitur in prima medietate F temporis Socrates pertransiet solum unam quartam et Plato solum duas. Et
V 137v tunc ultra sic: in prima medietate F temporis | Socrates pertransiet unam quartam et in secunda medietate F temporis movebitur in triplo velocius, igitur in secunda medietate F temporis pertransiet Socrates tres quartas, igitur in toto F tempore pertransiet quattuor quartas et tantum praecise in eodem
K 210rb tempore erit pertransitum a Platone, | quia in prima medietate F

4 et[1]] *om.* V 6 medio] motus R 7 spatium[2]] gradum R 10 et] *om.* R 11 sumo] suppono K | A] *om.* KR | suo] *om.* K 12 medius] *om.* K 13 6] etc. *add.* R | latitudinis] inter latitudines V 14 gradus] *om.* R | medius] *om.* V | gradum] sit *add.* KRV | F] *om.* RK 15 tempus…quo] spatium per quod V | in quo] per quod R | G] F R 16 et...tunc] tunc arguo R 17 movebuntur] et *add.* R 18 igitur[1]] et V 20 movebitur] in *add.* V 27 temporis] solum *add.* V 28 F] *om.* RV 29 in[1]] igitur V 30 Socrates] *om.* R | igitur] et K 32 prima] ipsa R | medietate] secunda *add.* R

temporis pertransiet Plato duas quartas et per totum F tempus movebitur uniformiter, igitur in secunda medietate F temporis tantum pertransiet, quantum in prima, sed duas pertransivit in prima medietate, igitur et alias duas pertransivit in secunda, igitur quattuor in toto, igitur tantum quantum Socrates. Et quod Socrates in secunda | medietate F temporis movebitur in triplo P 38rb velocius quam in prima; probo: signetur latitudo motus qua movebitur Socrates in secunda medietate F temporis quae erit latitudo terminata ad B et C gradus cuius etiam gradus medius est D, sicut patet. Tunc sic: Socrates movebitur velocius Platone per totam secundam medietatem F temporis vel igitur secundum proportionem graduum mediorum illarum latitudinum, quae est a B ad C et a B ad non gradum, vel secundum proportionem graduum extremorum latitudinum earundem, quia non videtur penes quos alios debeat attendi velocitates Socratis in secunda medietate F temporis supra velocitatem eiusdem in prima medietate F temporis; si penes proportionem graduum mediorum, cum gradus medius acquisitus in secunda medietate sit triplus praecise ad gradum medium acquisitum in prima medietate, quia D ad E est proportio tripla, ut patet; igitur Socrates movebitur in triplo velocius in secunda medietate quam in prima, et habetur propositum. Si detur alia pars quod in secunda medietate F temporis movebitur Socrates non secundum proportionem graduum mediorum, sed secundum proportionem graduum extremorum, cum unus sit praecise duplus ad alium, ut est C ad B, igitur solum pertransiret Socrates in secunda medietate temporis duas quartas; et sic in toto F tempore plus pertransiretur a Platone quam a Socrate, quia in toto tempore pertransirentur a Platone quattuor quartae et a Socrate non nisi tres, quod falsum

1 pertransiet…temporis] *om.* (*hom.*) KR 3 quantum] pertransiet *add.* K pertransivit *add.* R | in^{1}] ipsam *add.* K | duas] duo V 4 medietate] *om.* V secunda] medietate *add.* R | igitur^{2}] duo *add. sed. del.* R 7 probo] probatur K | motus] temporis R 8 F] *lin.* P | quae] qua RK 9 etiam…medius] esset medius gradus R | gradus^{2}] *om.* V | gradus medius] motus gradus K 10 movebitur] tunc *add.* R 11 secundam] *om.* KR | secundum] in *add.* R 12 a] ab V 13 secundum] quattuor divisionem et *add.* R 15 debeat attendi] attendatur V 17 si] sed R 18 gradus medius] medius gradus motus K | acquisitus] *om.* V 22 habetur] sic sequitur R | alia] secunda R pars] scilicet *add.* R 26 pertransiret] pertransiet R | medietate] F *add.* RV

est nec concedet adversarius. Reliquitur igitur primum quod R 166va velocitas motus Socratis in secunda medietate F temporis supra | velocitatem eiusdem in prima medietate F temporis attenditur penes proportionem graduum mediorum latitudinum praedictarum. Et quod in prima medietate F temporis Plato movebitur in duplo velocius ipso Socrate, probatio: latitudo motus Socratis qua Socrates movebitur in prima medietate F temporis - et illa velocitas motus Socratis - aut attenditur penes gradum medium eiusdem latitudinis, puta penes E gradum, aut penes gradum extremum, puta B. Si penes gradum medium, cum omnis gradus medius per totum tempus istius intensionis usque V 138r ad B gradum sit praecise subduplus | ad gradum extremum, ut patet de E et B, igitur per totam primam medietatem F temporis movetur Socrates praecise in subduplo velocius Platone, et per consequens Plato per totam primam medietatem F temporis movebitur in duplo velocius Socrate, et habetur propositum et intentum. Si dicatur quod velocitas motus Socratis etc. attenditur penes suum gradum extremum, puta penes <C>; contra, igitur in P 38va prima medietate temporis tantum foret pertransitum | a Socrate quantum a Platone, et per consequens per totam primam medietatem F temporis ita velociter moveretur Socrates sicut Plato. Consequens falsum et consequentia patet, quia Socrates per totam primam medietatem F temporis moveretur a latitudine motus terminata ad aliquem gradum et ad gradum aequalem gradui motus quo movetur Plato, penes quem gradum attenderetur praecise velocitas motus Socratis, igitur etc.

167 Per illam demonstrationem cogor firmissime concedere et tenere quod in motu locali, ubi a non gradu incipit talis motus uniformiter difformiter intendi, tota latitudo motus illius

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1 nec concedet] quod non concederet K | concedet adversarius] concedendum ab adversario R 2 supra] super R 6 ipso] *om.* V 9 penes] *om.* K 10 gradum^{1}] *om.* V | B] item *add.* R | si] item K | cum omnis] tunc communis R 12 ad^{2}…medietatem] *om.* K | ad^{2}…14 Platone] tunc movetur Plato in duplo velocius R 15 Plato] *om.* RK | primam] *om.* V 16 propositum et] *om.* RV 18 suum] *om.* V | C] B V | contra] intensissimum R 19 foret] erit K 22 consequens] est *add.* R | Socrates] B K *om.* R F V 23 a] aliqua KRV 24 gradum^{2}…gradui] aequalem gradum gradui motus R 25 gradui] motus *add.* V 26 etc] etV 28 a…incipit] incipit a non gradu V 29 difformiter] difformis V | illius] *om.* R

correspondet, vel est aequalis suo gradui medio, sic quod tantum praecise erit pertransitum ab isto qui moveretur gradu medio istius latitudinis per horam, quantum foret pertransitum ab eodem, si per eandem horam moveretur illa latitudine. Ex quo sequitur tunc ultra quod talis latitudo motus localis vel velocitas talis motus non attenditur penes gradum intensissimum talis motus; et hoc arguo modo in sphaerali ex illo impossibilia deducendo.

168 Quarto. Si velocitas omnis motus localis etc. non sit aequalis suo gradui medio, sed gradui intensissimo, contra, retento casu proximi | argumenti et tertii, arguo sic: Socrates in prima K 210va medietate F temporis pertransiet unam quartam et per totam secundam medietatem F temporis movetur uniformiter difformiter continue intendendo motum suum, igitur in medio instanti secundae medietatis F temporis erit Socrates sub medio gradu inter B et C qui est D, et tunc intensio istius motus attenderetur penes istum gradum, sed tunc D gradus se habebit ad B gradum in proportione | sexquialtera. Igitur Socrates in R 166vb medio instanti secundae medietatis F temporis movebitur praecise in sexquialtera proportione velocius quam ipsemet movetur in medio instanti totius temporis, igitur Socrates in prima medietate F temporis acquiret sexquialterum spatium ad illud quod pertransivit vel acquisivit in prima; sed in prima solum acquisivit unam quartam, igitur in prima medietate secundae medietatis F temporis solum pertransiet quartam et dimidiam et non maius spatium, sed praecise tantum acquiret in secunda medietate secundae medietatis F temporis, ut patet ex casu, igitur Socrates ita velociter movebitur in prima medietate secundae medietatis F temporis, sicut in secunda medietate secundae medietatis F temporis et ultra, igitur per totam secundam

1 suo] *om.* V 3 istius] totius K 7 in] *om.* R | sphaerali] et *add.* V | ex illo] *om.* K 8 deducendo] deduco V 9 sit] si R 11 et] scilicet R | sic] in forma *add.*V 13 F temporis] *om.* R | temporis] *om.* V | movetur] movebitur K 14 continue] nunc *add.* R 15 secundae…gradu] *om.* R 18 Socrates] scilicet V 20 ipsemet] ipsummet V 22 prima] secunda R | F temporis] *om.* R 23 in^{2}…solum] solum in prima R 24 igitur…quartam] *om.* (*hom.*) KRV 25 et dimidiam] cum dimidio R 26 maius] magis V 27 secundae] *om.* KRV | ut…30 temporis] *om.* R | casu] secunda V 29 medietate] *om.* K 30 F…medietatem] *om.* (*hom.*) V

11 proximi…tertii] Cf. § 166.

V 138v medietatem | F temporis movetur Socrates uniformiter. Et ex isto sequitur hoc impossibile quod Socrates movetur per aliquod tempus continue intendendo motum suum in isto tempore, et tamen per idem tempus continue uniformiter movebitur, et patet demonstrative quod sequitur.

169 Ad idem. In eodem casu arguo sic: si intensio motus Socratis sit secundum gradum intensissimum sui motus, cum gradus intensissimus sui motus in fine temporis sit praecise duplus ad P 38vb gradum intensissimum sui motus | in medio instanti, igitur totus motus Socratis in fine temporis erit praecise duplus ad totum motum Socratis in fine primae medietatis istius temporis. Et si sic, igitur Socrates praecise duplum pertransiret in secunda medietate temporis ad illud quod pertransiret in prima medietate et in prima Socrates non pertansiet nisi unam quartam, igitur in secunda non pertransiet nisi duas, vel igitur in prima medietate secundae medietatis pertransiet unam quartam praecise vel non. Si quartam praecise, igitur per totum illud tempus non intendebat motum suum quod est contra casum, si minus illa quarta et per totum illud tempus movebatur velocius quam in prima, igitur Socrates minus pertransiit de spatio quando velocius movetur et cetera alia fuerunt paria, et hoc est impossibile, igitur, etc. Si maius quarta pertransietur in prima medietate secundae medietatis, et per totam secundam medietatem secundae medietatis movebitur velocius quam in prima medietate secundae medietatis, igitur plus quam duae quartae sunt pretransitae a Socrate in secunda medietate F temporis, cuius oppositum est deductum.

170 Quinto. Si ex opposito articuli velocitas motus localis non attenditur penes gradum medium, sed penes gradum intensissimum sui motus, contra: sit igitur aliqua potentia motiva quae sit aequalis potentiae suae resistentiae et crescat potentia motiva continue movendo resistentiam suam gradu intensissimo sui signato per 6, et sit quod sua potentia resistiva resistat sibi

1 et...isto] ex quo R 7 gradum] suum *add.* R 8 motus…sui] *om.* R in…motus] *om.* (*hom.*) K 9 igitur] arguo sic R 11 Socratis] sed R 12 pertransiret] pertransiet K 15 duas] quartas *add.* V 17 si] unam *add.* K totum illud] idem R 18 quarta] pertransita K 21 alia fuerunt] sunt V hoc] non *add.* K | etc] *om.* R 22 maius] minus RV 26 in] igitur V 28 si] sic R | articuli] arguitur K *lin.* R 30 motiva] activa R 31 potentiae] *om.* VK 33 sibi] similiter R

gradu intensissimo resistentiae suae signato per 4 qui sit D. Tunc sic: si ista potentia motiva movebit suam resistentiam intensissimo gradu sui signato per 6 qui est C, et velocitas vel intensio huius motus attenditur penes intensissimum gradum | R 167ra suum, igitur duplato C gradu ad duplum praecise, illa potentia motiva movebit praecise in duplo velocius quam nunc movet. Intendatur igitur illa potentia ad duplum, puta ad gradum signatum per 12, et sequitur tunc quod ista potentia movebit suam praecise in duplo velocius. Unde sic: ita accipio igitur aliam potentiam motivam quae modo |sit aequalis praecise potentiae V 139r resistivae primae ipsius A et volo quod ista potentia motiva intendatur post hoc uniformiter difformiter continue, quousque fuerit sub gradu signato per 9 qui sit B, et potentia illa resistiva signata per 4 sit D, et gradus quo prima potentia motiva movebit D in prima medietate temporis sit C signata ut prius per 6, quia ponatur quod in principio potentia motiva C et sua resistentia fuissent aequales et quod C cresceret in potentia sua continue intendendo motum suum sic quod in prima medietate temporis sui motus intendat ad gradum signatum per 6. | Et arguo tunc P 39ra sic: qualis est proportio B ad C, | K 210vb talis est proportio C ad D, igitur proportio B ad D est dupla ad proportionem C ad D et motus intensionis D sequitur illam proportionem, ut constat; igitur B movebit D praecise in duplo velocius quam C, et duplato C ad gradum signatum per 12 potentia cuius C est gradus intesissimus, non movebitur D nisi praecise in duplo velocius, ut deductum est, igitur hic sunt duae potentiae motivae quae movebunt eandem resistentiam vel aequalem, et hoc aequaliter, et tamen una a proportione maiori quae est a 12 ad 4 et alia a proportione minori

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1 intensissimo] et *add.* V 2 si] *om.* VK 3 gradu sui] suo gradu V | et] *iter. sed del.* R 5 suum] sui K | igitur] latitudo *add.* R | gradu] movebitur *add.* R illa…praecise] *om.* (*hom.*) KR 6 motiva] *om.* V 7 duplum] gradum *add.* R 9 suam] resistentiam *add.* RV | praecise] *om.* K | unde] *om.* RK | aliam] aliquam R 10 modo] non R mere V | potentiae resistivae] *om.* R potentiae…primae] resistentiae K 12 post hoc] *om.* V | continue] *om.* KR 13 B] C V | illa] illius R 14 signata] *om.* K | motiva] *om.* K | movebit] movebat R 15 signata] signat a R 16 ponatur] ponitur R 17 in…continue] continue in potentia sua K 19 sui] *om.* R | intendat] motum *add.* V 20 igitur…D^2] *om.* R 21 C] *lin. et add.* B P | et] *om.* V 22 D] *correxi ex* B 23 duplato] duplico R 24 cuius] ipsius R 25 non] qui R | D nisi] *om.* R 26 hic] haec R 27 et^2] cum K 28 ad] et R

quae est a 9 ad 4, quia una est proportio tripla et alia duplicata sexquialtera. Sequitur etiam in hoc casu quod duae potentiae inaequales, puta quarum una se haberet ad aliam in proportione sexquialtera, aequaliter moverent eandem resistentiam vel aequalem. Consequens impossibile, igitur, etc.

171 Sexto. Si per oppositum articuli intensio talis motus localis, etc. non sit aequalis vel <non> correspondeat suo gradui medio, sed gradui suo intensissimo, contra: accipio igitur Socratem qui a non gradu incipiat moveri localiter continue intendendo motum et hoc latitudine motus localis uniformiter difformis. Tunc sic: Socrates continue intendet motum suum et hoc latitudine motus R 167rb uniformiter | difformi terminata ad non gradum, igitur Socrates continue movetur aliqua latitudine cuius gradus intensissimus continue est praecise duplus ad suum medium gradum et intensio huius motus attenditur penes proportionem intensissimi gradus sui ad suum gradum medium, igitur Socrates continue intendet motum suum a proportione dupla, ex quo sequitur ultra: Socrates V 139v continue intendet motum suum a proportione aequali, | igitur continue intendet motum et sequitur etiam Socrates continue intendet motum suum et hoc a proportione aequali, igitur motus Socratis non plus intendetur in uno tempore quam in altero sibi aequali quocumque, et ultra, igitur Socrates non continue intendit motum suum, igitur si Socrates continue intendet motum suum, quod est contradictio, et per consequens impossibile. Ista sunt impossibilia et ista sequuntur ex ista positione, igitur positio impossibilis.

<Opinio auctoris ad articulum>

172 Ad articulum igitur, cum quaeritur ‘utrum velocitas, etc.’, dico quod sic et concedo quod in motu locali uniformiter

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1 una] alia V | alia] dul (*sic!*) *add.* R | duplicata] *om.* K 2 sexquialtera] *correxi ex* sextriquarta P | etiam] *om.* RK 5 consequens] est *add.* KRV etc] *om.* R 6 sexto si] igitur K | si] *om.* R 8 sed] suo R | igitur] *om.* V 9 localiter] *om.* V | motum] suum *add.* RV 10 difformis] difformi V tunc...difformi] *om.* (*hom.*) KV | sic] si *add.* R 12 igitur…16 medium] *iter.* V 15 proportionem] intensionem K huius *add.* V 16 suum] dictum R 18 suum] *om.* K 19 intendet] suum *add.* R | motum] suum *add.* V 21 non…tempore] in uno tempore non plus intenditur R | in[1]] *om.* K altero] alioV 23 igitur…suum] *om.* (*hom.*) R 24 et…26 impossibilis] *om.* R 25 ista[1]] *om.* V | positio] est *add.* V 27 igitur] *om.* K 28 in] uno *add.* R

difformi incipiente a non gradu tota latitudo motus est aequalis suo medio gradui et hoc loquendo de motu qui continue est in intendi. Et hoc totum sic intelligo: in omni motu uniformiter difformi incipiente a non gradu qui continue est in intendi tantum pertransitur de spatio in aliquo tempore, quantum in eodem tempore vel aequali pertransitur gradu | suo medio et P 39rb econverso. Et signanter loquor de motu qui continue est in intendi cuius nullus gradus acquisitus manet similis cum alio et per tempus, quod dico pro tanto, quia in motu extenso sphaerae uniformiter revolutae cuius quilibet gradus motus manet cum alio, in tali motu extenso tota latitudo motus correspondet gradui intensissimo et extremo, sed in motu continue intenso et non extenso non oportet nec est verum.

<Responsio ad primum inconveniens>

173 Et tunc ad primum: cum arguitur, quod tunc sequitur quod 'A et B sunt duo motus aequales praecise, et tamen A est in infinitum intensior B', dico quod hoc non sequitur, et tunc ad eius probationem admitto casum. Et ultra cum arguitur: 'A motus est latitudo uniformiter difformis incipiens a non gradu, cuius medius gradus est B, igitur motus A et B sunt aequales', conceditur consequens ad intellectum datum, videlicet quod aequalia spatia in aequali tempore sunt pertransita per A et B. Et tunc ultra, et 'A ultra B continet infinitos gradus quorum | R 167va quilibet est intensior B, igitur A est in infinitum intensior B', non sequitur, quoniam si haec formula valeret, sequitur quod A sit etiam in infinitum remissior B, quia A continet B et praeter B continet infinitos gradus quorum quilibet est remissior B, igitur A est in infinitum remissior B. Patet quod | neutrum argumentum K 211ra

2 hoc] non K 3 et...intelligo] videtur hoc totum sic intelligere R 4 in] *om.* R 6 et] vel V 7 signanter] *om.* R | in] *om.* R 9 in] omni *add.* V | extenso] intenso K *om.* R | sphaerae] intenso *add.* R 12 intensissimo et] intenso in R | non extenso] continue non intenso R 19 et] tunc *add.* V | ultra] tunc R 22 ad] et R | videlicet] scilicet R | quod] in *add.* R 23 spatia] quae *add.* KV | B et] *om.* V 24 et…B] A B R 25 A] 6 V | in] *om.* V | B[2]] et *add.* K 26 quoniam] nam R | sequitur[2]] etiam *add.* R 27 etiam] *om.* V | remissior] intensior R | A] *om.* R | praeter] prima V 28 infinitos] infinita c R remissior B] remissionis R | A] *om.* R 29 patet] oportet R | argumentum] *om.* V

16 ad primum] Cf. §§ 147, 153.

valet; sed si argueretur sic, 'A continet B et praeter B continet infinitos gradus motus quorum quilibet est intensior B, et nullum gradum remissiorem B continet, igitur A est in infinitum intensior B', forma esset magis apparens, sed tunc esset neganda ultima particula antecedentis, consequentia tamen non esset formalis.

<Responsio ad secundum inconveniens>

174 Ad secundum cum arguitur quod tunc 'aliquis motus remittetur | [V 140r] per horam et in tali remissione ante finem horae deperdet gradum, immo plus quam duplum et plus quam triplum, et tamen in fine horae erit praecise in duplo remissior quam in principio', dico quod in casu sumpto non est inconveniens, sed est verum et sequitur ex casu. Nec ex isto sequitur quod 'A motus deperdet aliquem gradum motus, quem non habuit nec habebit, nec habere potest'. Et cum arguitur quod sic: 'quia A motus cum fuerit remissus ad B gradum, deperdet gradum quadruplum, et tunc non erit nisi praecise in duplo remissior quam in principio, et A similiter cum fuerit remissus ad C, deperdet A gradum octuplum ad istum gradum sub quo erit tunc A intensus, et tunc non erit nisi praecise in quadruplo remissior quam in principio et sic deinceps usque ad non gradum, igitur A motus continue deperdet gradum intensiorem quam ipsum remittetur ad aliquem'; patet quod non sequitur, quia ad omnem gradum quem A deperdet, remittitur A. Etiam non sequitur in simili argumento: 'prima pars proportionalis est | [P 39va] dupla ad secundam, vel gradus terminans primam partem proportionalem illius latitudinis uniformiter difformis est duplus ad gradum terminantem primam partem proportionalem, et triplus ad triplum, et quadruplus ad quadruplum, et sic in infinitum, igitur gradus terminans primam partem proportionalem est infinitus'; non sequitur, sicut patet,

1 praeter] prima V | continet[2]] 1 *add sed exp.* P 2 et…4 B] *om.* (*hom.*) R 3 B] *om.* V 8 cum] quando R 9 tali] aequali R | horae] *lin.* R 10 gradum] duplum *add.* R | immo] primo K | triplum] duplum R 12 sumpto] supposito KR 13 motus] gradus V 15 cum[1]] tamen K | arguitur] quod *add.* KV 16 quadruplum] duplum R 17 remissior] velocior V | quam] fuit *add.* V 19 erit] est R 20 remissior] intensior R | quam] quia R 21 continue] perdet *add. sed del.* K 23 quem] quam V 24 etiam] et KR 27 primam] secundam R 28 et triplus] duplus R 29 quadruplum] *del.* duplum *add.* K | et sic] nec K | gradus] *om.* V

8 ad secundum] Cf. § 154. | 13 nec...sequitur[2]] Cf. § 155.

sed si foret aliquis gradus certus ad quem aliquis gradus certus, puta A, foret duplus, triplus, quadruplus, et sic in infinitum, tunc iste gradus A foret infinitus, sed sic non est in proposito nec etiam hoc arguitur, et ideo non sequitur illud correspondentum adductum ad secundum inconveniens recitatum.

<Responsio ad tertium inconveniens>

175 Ad tertium: cum arguitur 'B et C iam distant a non gradu, C in duplo plus quam B, et utrumque uniformiter difformiter remittitur ad non gradum, igitur B erit in duplo citius sub non gradu quam C', nego consequentiam, sed solum sequitur: 'igitur B in duplo velocius remittetur quam C et etiam quam tota latitudo A', et | hoc concedo nec est inconveniens aliquod, sed est verum R 167vb et sequens ex casu supra dato et ex isto patet ultra ad argumentum.

<Responsio ad quartum inconveniens>

176 Ad quartum potest dici quod non sequitur et tunc ad argumentum: 'A distat a B et C', conceditur, 'et tamen nec aequaliter nec inaequaliter', potest dici quod aequaliter. Et tunc ad argumentum cum arguitur 'si sic cum A solum distet a B et C, a B per latitudinem A B, et a C per latitudinem A C, igitur A B latitudo foret aequalis A C latitudini', concedo. Contra 'illae latitudines nec sunt aequales intensive, nec extensive', dico quod sunt aequales extensive, non quod istae latitudines actu super

1 foret] forte R | certus[1]] esset R | certus[2]] esset duplus *add.* R *om.* V 2 A] *om.* R | duplus triplus] *inv.* V | quadruplus] *om.* R | tunc…infinitus] *om.* R 3 nec etiam] contra K | nec…hoc] et sic hic R 4 et] *om.* R | illud] hoc K ad R 5 adductum ad] *om.* R | adductum…recitatum] ad secundum inconveniens adductum V | recitatum] recitatam R 8 iam] *om.* RV | C[2]] *correxi ex* B P 9 B] *correxi ex* C P 10 B] *om.* R 11 C] *correxi ex* B P *om.* R B] C R 12 C] E R 13 nec] hoc *add.* RK | est[1]] *om.* V | aliquod] *om.* R 14 et[1]] *om.* K | et sequens] consequens K | ex[1]…dato] posito illo R supra dato] posito K 19 nec] *om.* R 20 nec inaequaliter] *lin.* R 21 cum[1]] quando V | cum[2]] quod R 22 per[2]…C] *om.* R | A B[2]] *om.* KR 23 A C] *om.* KR 24 aequales] nec *add.* RV | intensive…extensive] nec extensive nec intensive V | nec[2]] videlicet R | dico…extensive] *om.* (*hom.*) KRV

8 ad tertium] Cf. § 156. | 18 ad quartum] Cf. §§ 157--159.

aliqua spatia extenduntur, sed quod per imaginationem illae latitudines prout sunt abstractae a spatiis et a sensu et solum in V 140v intellectu copulantur ad aliquem gradum medium in | eadem latitudine imaginata, sicut posita latitudine motus uniformiter difformi semidiametri in circumvolatione sphaerae medietates istius latitudinis sunt aequales extensive, non quod per aequalia K 211rb spatia extendantur, sed quia latitudo a gradu medio ad gradum | supremum in eodem semidiametro, non in spatio, quod describit, est aequalis alteri latitudini extensive quae est a medio gradu usque ad non gradum in eodem semidiametro, ita quod illa aequalitas secundum extensionem respicit spatia recta, non obliqua. Et sic dico in proposito quod in continua intensione motus medietates motus sunt aequales extensive in se imaginatae et non in spatiis quibuscumque obliquis vel rectis. Facile est illud videre, si ponas illam aequalitatem extensionis in imaginatione vel intellectu et non in actu. Potest tamen aliter responderi et meo iudicio hoc est probabile quod in latitudine motus uniformiter P 39vb difformis incipientis a non gradu et qui est | continue in intendi, nullus est gradus medius, accipiendo stricte gradum medium pro isto qui aequaliter distat ab extremis, ut apparenter probatur argumentum. Et tunc si tu quaeras, quid ego voco gradum medium cui tota latitudo est aequalis, dico quod si aliqua latitudo motus incipiat intendi a non gradum ad certum gradum et hoc per aliquod tempus, tunc in medio instanti illius temporis acquiretur certus gradus cui tota latitudo motus est aequalis, et istum voco gradum medium qui aequaliter secundum tempus distat a terminis temporis et a terminis latitudinis. Dico secundum tempus, quia in aequali tempore deveniet ad gradum duplum sicut ad gradum subduplum.

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1 aliqua…extenduntur] aliquod spatium intenditur R | quod per] secundum K | per imaginationem] *om.* R 2 et solum] *om.* V | in intellectu] intellectu comprehenduntur R 3 eadem] aliqua K 4 imaginata] *om.* V posita] *om.* R ponitur in K 5 difformi] difformiter R difformis V | in] et R 6 istius] illae K 7 quia] quod R | latitudo] o *add.* R 8 supremum] summum R | in[1]] *lac.* V 11 aequalitas] inaequalitas R | spatia…obliqua] spatiam rectam non obliquam K 12 dico] quod *add.* R 14 et…spatiis] non in spatio V | quibuscumque] quibus ex R | est] *om.* K 15 vel] in *add.* K 17 quod] quia V 18 incipientis] incipiente R | et] *om.* VK | et qui] quod R | in] *om.* R 19 stricte…medium] gradum medium stricte VK 20 probatur] probat K 21 ego] *om.* R 24 aliquod] *om.* V 27 et…terminis] *om.* V 28 quia…aequali] quod in eodem R 29 ad] et R

<Responsio ad quintum inconveniens>

177 Ad quintum dico quod conclusio illa non sequitur et nego quod in instanti medio illius | temporis erit haec vera: ‘Socrates R 168ra et Plato moventur aequaliter’. Et cum arguitur contra: ‘Socrates per totam primam medietatem temporis movebitur tardius Platone et per totam secundam medietatem movebitur velocius Platone, igitur in instanti medio temporis Socrates et Plato moventur aequaliter’; nego consequentiam, sed sequitur: igitur in toto tempore moventur aequaliter.

<Responsio ad sextum inconveniens>

178 Et per illud patet responsio ad sextum quod non est inconveniens, sed possibile et sequens in casu supposito quo aliqui duo per totum tempus moventur inaequaliter, et tamen in toto tempore movebuntur aequaliter, quia aliud dicit ly ‘per’ et aliud ly ‘in’, et aliter dat intelligere et stat ista propositio ‘per’ et aliter haec propositio ‘in’, quia haec propositio ‘per’ facit illum terminum ‘totum’ stare syncathegorematice et dat intelligere quod per omnem partem temporis moventur | inaequaliter, et hoc est V 141r verum, sic ly ‘in’ facit istum terminum ‘toto’ stare cathegorematice et dat intelligere quod in toto tempore collective sumpto ly ‘toto’ Socrates et Plato aequaliter moventur, et hoc est verum, et sic patet ad articulum tertium et ultimum; sic igitur expeditis istis articulis expediamus et breviter quaestionem.

<Opinio auctoris ad quaestionem>

4 quod] *om.* R | illius] *om.* V | erit…vera] hic vero R 6 totam] c R 8 temporis] *om.* R 13 et] *om.* RK | per…responsio] *iter.* K | illud] idem R responsio] solutio R | sextum] *om.* R 14 et sequens] *om.* V 15 tamen] in cuicumque *add. sed del.* K 16 quia] ad R 17 ly…aliter] *om.* K | aliter…et] et dat intelligere et aliter V | et^{2}] enim R | ista] suppositio *add.* R | et^{3}] *om.* R | et^{3}…in] ly aliter ista propositio K 18 haec propositio^{1}] quam R ista V haec^{2}] ista VK 20 moventur] inaequaliter et hoc etc *marg.* V 21 sic] *corr. ex* si KPR sed V | in] per R | istum] *om. R* | toto] totum V *om.* K 23 toto] *om.* K | aequaliter] et equaliter *add. sed del.* R 24 tertium] articulum *add.* K 25 istis] tribus R | expediamus…quaestionem] explicamus et pro brevitate quaestionem (*sic!*) K | et] *om.* R | quaestionem] illam *add.* R

3 ad quintum] Cf. § 160. | 13 ad sextum] Cf. § 161.

179 Ad quaestionem illam cum quaeritur, utrum in motu locali, etc., dicitur quod sic et cum arguitur quod 'non, quia tunc sequitur quod talis velocitas', etc., concedo disiunctionem pro ultima sui parte, videlicet quod in motu locali attenditur velocitas penes proportionem proportionum potentiarum moventium ad potentias resistitivas.

<Responsio ad argumenta pro inconvenientibus ad tertiam opinionem>

180 Et cum arguitur quod non, quoniam tunc sequitur primo quod 'A et B sunt duo gravia cuius proportio gravitatis, etc.', dico quod conclusio non est inconveniens, sed possibilis et vera in casu supposito et causa est, quia cetera non sunt paria. Nam licet A et B aequaliter componantur ex gravi et levi, tamen inaequaliter disponuntur et etiam inaequaliter situantur, modo et situs et P 40ra dispositio bene iuvant ad motum. Si | enim grave simplex poneretur in vacuo imaginato circa centrum mundi cuius quaelibet pars esset extra centrum mundi vel non moveretur, vel si moveretur, moveretur velocitate infinita. Sed si idem grave in vacuo imaginato circa centrum mundi sic situaretur, ut minor pars eius foret sub centro, maior vero supra, idem grave tunc moveretur et hoc velocitate finita. Patet etiam in casibus communibus quod dispositio bene iuvat ad motum, qualiter in R 168rb casu supposito A habet quaedam promoventia | motum suum K 211va quae nec aequalia habet B, et iam cetera non sunt paria in casu, | ideo conclusio proposita non est inconveniens, sed est vera.

1 ad...illam] *om.* R | illam] *om.* V 3 concedo] dist (*sic!*) *add.* R disiunctionem…parte] pro ultima sui parte disiunctionem V 4 videlicet] scilicet R 5 proportionum] *om.* VK 6 potentias resistitivas] suas resistentias K 11 cum] tamen o | tunc] *om.* R | primo] *om.* V 13 possibilis…vera] possibile et verum R 16 et[1]] *om.* R | etiam] *om.* V et[2]] *om.* KRV 17 bene] *om.* V 18 poneretur] moveretur K 19 extra] *om.* K 20 moveretur[2]] *om.* K maneretur R | idem] illud V | in…imaginato] imaginatum in vacuum R 21 situaretur] situatum R 22 supra] centrum *add.* V 23 et hoc] *om.* R 24 qualiter] quare V 25 supposito] supposita R 26 quae…aequalia] qualia non V | aequalia] qualia P | iam] sic R | casu] et *add.* K 27 conclusio] ibi *add.* V | est[2]] *om.* V

11 cum arguitur] Cf. §§ 3, 9.

181 Aliter potest responderi et probabilius dicendo quod nec A nec B movebitur in hoc casu. Et cum arguitur quod sic de A: tota levitas in A ultra centrum appetit ascendere et tota gravitas in A citra centrum appetit contingi cum centro mundi, igitur omnia promoventia A quantum ad motum erunt sua gravitas citra centrum et levitas ultra centrum; negatur consequentia propter implicationem falsi, quia implicat quod A habeat aliqua promoventia ipsum ad motum suum. Sed adhuc ultra, cum dicitur: et nihil est impediens nisi solum levitas citra centrum, hoc negatur simpliciter, immo impeditur a toto et a proportione totius gravitatis A ad totam suam levitatem, a qua non potest esse motus. Unde licet pars inferior A appetat motum et pars superior similiter appetat moveri, ut satis probatur in argumento, tamen ipsum totum appetit quiescere nec moveri; nec moveretur a se, nisi a porportione maioris inaequalitatis totius gravitatis ad totalem levitatem eiusdem, qualiter non est in proposito.

182 Et per illud patet | responsio ad secundum quod est simile V 141v
huic per totum et ideo similis huic detur responsio et ex toto.

183 Ad tertium dicitur quod conclusio non sequitur et admittatur casus, deinde cum arguitur 'G movebit D ex se quousque sit in loco suo naturali, ultra et antequam idem G erit in loco suo naturali, idem G habebit aliquam resistentiam quae erit maior quam sua potentia motiva', hoc nego simpliciter. Tunc ad eius probationem: cum dicitur 'tota potentia motiva G antequam idem G erit in loco suo naturali excedit suam resistentiam intrinsecam per minus quam per duo - esto - et plus quam per unitatem', hoc

1 et] *om.* KR 2 nec] et R | B] non *add.* R | A] quia *add.* R 3 ascendere] quiescere V | gravitas] gravitate K 4 contingi] contiguari KPV continguari R | omnia] omnis R 5 quantum] quo R 6 et] sua *add.* K 7 implicationem falsi] impossibilitatem simili R | falsi] *om.* V 8 adhuc ultra] *om.* R | ultra] *om.* K 9 dicitur] arguitur R | et] *om.* K quod V 11 gravitatis] levitatis *lin.* gravitatis K 12 motum] moveri KV 13 similiter] simpliciter K 14 moveretur] a *add.* V 15 nisi] neque R nec K porportione] totius *add.* K 16 totalem] totam R | qualiter non] quare ideo V 17 illud] idem R | quod] quam R 18 ex] per R 19 conclusio] *om.* V 20 G] A V 21 et] *om.* V | idem] *om.* R 22 G] B R 23 nego] negatur KV simpliciter] similiter K et *add.* RV 24 cum] quando V | tota] eius *add.* K 25 intrinsecam] *om.* RV 26 esto] *om.* R | et] quod V | plus…unitatem] *om.* R

2 hoc casu] Cf. § 9. | 17 ad secundum] Cf. §§ 4, 10--11. | 19 ad tertium] Cf. §§ 5, 12.

nego. Immo, si G debeat moveri sic quod per talem motum sit in suo loco naturali, tunc G non excedit suam resistentiam
P 40rb intrinsecam nisi solum per medietatem unitatits vel per tertiam | partem unitatis, vel per quartam et sic de singulis, ita quod non per unitatem. Et si tu vis ponere quod sic, tunc dico quod numquam erit G in suo loco naturali, vel dicam quod casus non est admittendus, quia partes casus repugnant.

184 Ad quartum dicitur quod conclusio non sequitur nec casus ibi suppositus est possibilis, quia impossibile est quod aliqua
R 168va potentia intendatur et hoc per uniformem acquisitionem | potentiae, quia ex illo sequitur impossibile et utraque pars contradictionis, quia sequitur ista potentia intenditur, igitur intenditur. Et sequitur etiam ista potentia intenditur et hoc per uniformem acquisitionem potentiae, igitur illa potentia non plus acquirit de potentia in una parte temporis quam in alia sibi aequali, igitur illa potentia non intenditur, igitur simul illa potentia intenditur et non intenditur. Et ideo dico quod impossibile est quod aliquid mundi uniformiter intendatur vel uniformiter remittatur. Unde illud quod continue intenditur vel remittitur, non uniformiter intenditur vel remittitur, sed uniformiter difformiter intenditur vel remittitur. Unde, si aliqua potentia signata per 2 deberet continue intendi ad potentiam signatam per 8 in aliquo tempore, tunc in medio instanti totius temporis acquireret potentiam signatam per 4 et in medio instanti secundae medietatis temporis acquireret potentiam signatam per 6, et in fine potentiam signatam per 8, ex quo patet quod non uniformiter acquireretur potentia signata per 8. Sed contra forte arguitur sic: per primam medietatem secundae medietatis

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3 solum…medietatem] per medietatem solum R | unitatits] virtutis K tertiam] triplam K 4 unitatis] virtutis K | quartam] partem virtutis *add.* K unitatis *add.*R | et…singulis] *om.* R 5 per] *lac.* R | unitatem] virtutem K tunc] *om.* R 6 numquam…G] G numquam erit R | dicam] *om.* R dico V 9 suppositus] positus R | quia impossibile] impossibile enim V 13 etiam ista] haec R 15 sibi…igitur] et sic R 17 et] *om.* R | est] *om.* R 18 vel] et V 19 intenditur…remittitur] remittitur vel intenditur R 20 sed…remittitur] *om.* (*hom.*) RV 23 totius] illius K 24 acquireret] acquiret K | et] *om.* K 25 acquireret] acquiret K 26 ex…8] *om.*(*hom.*) KR non] *om.* V 27 forte] *om.* KR 28 sic] *om.* V

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8 ad quartum] Cf. §§ 6, 13.

temporis non acquiritur nisi dualitas quae est inter 4 et 6 et in secunda medietate secundae medietatis temporis non acquiritur nisi alia dualitas quae est inter 6 et 8, et sic de duabus | acquisitis V 142r in prima medietate temporis, igitur, ut videtur, haec est uniformis acquisitio potentiae, et tamen illa potentia continue intenditur per medietatem, igitur, etc. Ad illam respondeo negando quod in prima medietate secundae medietatis temporis non acquiretur nisi dualitas, immo dico quod numerus senarius acquiritur vel potentia signata per 6, unde licet in prima medietate secundae medietatis temporis acquiretur dualitas | ultra quaternitatem; K 211vb tamen simul cum hoc acquirit et de novo numerum senarium vel potentiam signatam per 6. Et consimiliter dico quod licet in secunda medietate secundae medietatis temporis acquiretur dualitas ultra numerum senarium, tamen simul cum hoc isti potentiae acquiretur numerus octonarius seu potentia assignata per 8 et sic non procedit in aliquo argumentum.

185 Ad quintum dicitur quod conclusio est satis possibilis et vera in casu supposito. Nam licet eadem sit proportio potentiarum moventium ad suas potentias resistivas, tamen non est aequalis proportio inter ipsas potentias nec istae potentiae aequaliter excedunt suas resistentias |. Potentia enim assignata per 9 multo P 40va maior est potentia assignata per 6, etiam per plus potentia assignata per 9 excedit suam resistentiam signatam per 3 quam potentia signata per 6 excedat suam resistentiam signatam per 2. Et ex quo plus excedit potest potentia maior moveri cum maiori resistentia, et per consequens 9 possunt moveri maiorem

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1 est] *lin.* V | et 6] 1 V | 6] *correxi ex* 7 PK duo R | in] *om.* K 2 secunda] prima R 3 8] *correxi ex* 9 PK 7 V 4 videtur haec] habetur hic R 6 medietatem] me V | etc] *om.* R 8 immo] conclusio R 9 unde] tamen *add.* K | prima] secunda R 10 temporis] *om.* RK | quaternitatem] quartam medietatem K 11 cum] et R | et] simul *add.* R | vel] per V 12 consimiliter] ultra V cum sic K 14 tamen simul] *om.* R 15 assignata] signata K 17 dicitur] dico K 18 casu supposito] proposito casu R | nam] unde V 19 ad…proportio] *om.* R | potentias resistivas] resistentias KV 20 ipsas potentias] se R | potentias] resistentias K 21 resistentias] tamen non est eadem proportio potentiarum moventium ut istae aequaliter excedentes sua (*sic!*) resistentias R | assignata] signata R | 9] 8 V 23 per[1]] *lac.* R 24 signata] assignata KV | excedat] excedit KRV | resistentiam] potentiam R | resistentiam signatam] assignatam K 25 et] igitur *add.* R moveri] maiori K 26 9] *om.* R

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17 ad quintum] Cf. §§ 7, 14.

resistentiam quam 6, et tamen eadem est proportio in principio istius potentiae ad suam resistentiam et alterius potentiae ad suam resistentiam. Sed quod eadem sit proportio aliquarum
R 168vb potentiarum moventium ad suas resistentias | et etiam potentiae motivae inter se sint aequales, et quod convenit addere uni potentiae certam resistentiam cum qua una potentia sufficit moveri et alia non, hoc reputo simpliciter impossibile. Sed ubi potentiae sunt inaequales inter se, licet aequalis sit proportio inter eas et suas resistentias, non est hoc impossibile, sed possibile et verum et sequens in casu supposito. Et per illud patet responsio ad sextum quod huic argumento simile est in toto.

<Responsio ad argumenta pro inconvenientibus ad secundam opinionem>

186 Ad secundum principale et primo ad primum dico quod conclusio ibi ducta non sequitur ex casu supposito nec est vera propter duas causas. Primo, quia cetera non sunt paria, quia ascensus medii istius A bene iuvat ad motum istius A, quod iuvamentum non habet B, et tamen hoc fuit assumptum in conclusione, puta quod cetera fuissent paria. Secundo, quia A
V 142v dividit medium suum a maiori proportione | quam dupla, quia A dividit medium suum non solum a proportione istius A ad suum medium, sed ab illa proportione cum iuvato ascensus sui medii, quod non facit B, et ideo falsum est nec probatur quod dicitur quod A et B continue ab eadem proportione dividunt sua media. Unde in isto casu dicendum est quod A in duplo velocius movetur quam B.

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1 quam…resistentiam] *om.* (*hom.*) K | 6…et^{2}] *om.* R 4 moventium] *om.* KR 5 quod] hoc R *om.* V | convenit] concludit K 9 non] tamen R 10 sequens] sequitur KR | supposito] posito V | responsio] respondeo V 17 ibi ducta] inducta KR | ex casu] *om.* V | est] etiam *add.* R 18 quia^{1}] quod R 19 medii…A] medius V | iuvat] ipsius A bene iuvat *add.* V istius^{2}] ipsius V 21 puta] videlicet V | quia] quod R 22 medium suum] suum medium non solum R | maiori] *om.* R 23 istius] *om.* V 24 illa] alia K | iuvato] iuvamento V 25 B] *om.* K | et] *om.* R 27 isto] hoc V 28 quam] *om.* R

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11 ad sextum] Cf. §§ 8, 15. | 16 ad primum] Cf. §§ 17, 23--25.

187 Ad secundum dico quod conclusio ex casu non sequitur nec est vera, et tunc ad eius probationem potest primo dici quod casus non est possibilis, et hoc pro eo quod cum aliis supponitur quod B medium ascendat per illam secundam medietatem horae velocius et a maiori proportione quam potentia A augeatur vel saltem quam potentia A augmentata dividat suum medium, quoniam aeque primo dividit A suum medium complete et cessabit completus ascensus sui medii, et econtra. Et tamen admisso casu tamquam possibili, adhuc non sequeretur quod C in secunda medietate illius horae movebitur tardius quam prius. Et cum arguitur quod sic, nam prius movebatur tardius quam si medium quiesceret, hoc nego, immo velocius. Contra, ascensus medii aliqualiter impedit descensum C. Concedo sic quod C propter ascensum sui medii non moveretur omni gradu velocitatis a gradu velociori in quo C | nunc movetur, si medium P 40vb istius C existeret minus dense. Sed ascensus medii illius C non impedit motum C, quin ipsum C propter ascensum medii movetur velocius quam si medium quiesceret. Nec sequitur quod tunc medium quantumcumque densum non impediret grave quantum ad motum descensus, quia impedit ne ipsum grave ita velociter moveatur, sicut moveretur, si medium | foret minus K 212ra densum aut | magis subtiliatum. R 169ra

188 Et per illud patet ad tertium argumentum dicendo quod casus ibidem non est bene possibilis propter causam consimilem allegatam in argumento priori, et secundo quod admisso casu

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1 ex…sequitur] non sequitur ex casu KR 2 potest…dici] sic primo dico V 3 hoc] *om.* K | hoc pro] cum V | cum aliis] alius R 4 secundam] *om.* R 5 et] cum K 6 saltem…A] quam V | A] *om.* R | augmentata] aucta K 7 quoniam] quia R | quoniam…medium] *om.* (*hom.*) V | dividit…medium] A dividet medium suum K 8 completus] complete K 9 possibili] *iter. sed del.* R | sequeretur] sequitur V 10 et cum] etc. tunc V 11 cum] tunc K quando R | quam] *om.* V 12 contra] *om.* K quam R 13 C^1] et *add.* K 14 moveretur omni] motu et R | omni] *om.* K 16 C^1] ex *add.* R | dense] densum R denso V | illius] ipsius R 17 motum] ipsius *add.* V 18 nec] non KR tunc *add.* V 20 ne] *om.* R 23 et] propter R | illud] idem V 24 bene] *om.* R | consimilem] assignatam vel *add.* K | consimilem allegatam] assignatam vel allegatam R 25 argumento] consimili *add.* V | et secundo] dico etiam R

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1 ad secundum] Cf. §§ 18, 26--27. | 23 ad tertium] Cf. §§ 19, 28.

tamquam possibili gratia argumenti, adhuc non sequitur conclusio ibi ducta, quod patet ex responsione priori.

189 Ad quartum: nec conclusio sequitur nec est vera nec aliquis est gradus certus quo E grave movebitur quousque fuerit in loco suo naturali. Et hoc, si cetera debent esse paria, quia si cetera sint paria, per magnum tempus ante non gradum motus continue intendet motum suum et per magnum tempus ante non gradum motus continue remittet motum suum, ita quod nullus est gradus quo praecise movebitur E grave versus locum suum naturalem, vel quo appetat moveri. Et sic consequenter est concedendum quod ceteris paribus quodlibet grave appetit moveri versus locum suum naturalem, et tamen nullus est gradus quo appetit moveri V 143r versus locum suum naturalem, vel quo movetur | ad locum suum naturalem et hoc quo gradu praecise movetur per totum tempus descensus.

190 Ad quintum dico quod nec conclusio sequitur nec est vera in casu supposito, et tunc ad eius probationem admitto casum quod F, B, C sint aequalis potentiae quantum ad hoc quod nullum agat in reliquum, et admitto ultra quod educatur caliditas de C et inducatur frigiditas tanta sicut est humiditas praecise, sed tunc cum dicitur quod 'adhuc F est fortissimum, quod non sufficit agere nec in B, nec in C', hoc nego simpliciter. Immo modo F non est fortissimum, quod non sufficit agere in C, quoniam non quodlibet fortius F sufficit agere in C, stante illa ultima dispositione C. Et cum arguitur quod sic, quia 'F adhuc non sufficit agere in C', concedo. Et 'per quantumcumque foret maior, sufficeret', hoc nego, quia licet frigiditas eius nunc sit tanta, quanta praefuit caliditas eiusdem. Humiditas tamen est maior quam siccitas illius F, quia ad hoc quod F, B, C sint aequalis potentiae in resistendo, oportet quod frigiditas B sit aequalis caliditati F et humiditas B aequalis siccitati illius F et quod

1 gratia argumenti] *om.* V 2 ducta] deducta R 3 nec[3]] est *add.* V 4 E] est R 5 quia…paria] *om.* (*hom.*) K | quia…7 et] *om.* R 8 motus] *om.* K remittet] intendit R | suum] *om.* V 10 vel] in KR | et sic] nec K 12 et…naturalem] *om.* (*hom.*) KV | et…14 naturalem] *om.* R 20 tunc] *om.* R 22 nec[1]] neque R | nec[2]] neque R 23 agere] nunc *add.* R 25 C…cum] et tunc R | et] tunc *add.* V 27 nunc sit] non R 29 aequalis] aequales R 31 illius] ipsius R

3 ad quartum] Cf. §§ 20, 29--31. | 16 ad quintum] Cf. §§ 21, 32--33.

humiditas C sit tanta quanta sunt simul frigiditas et humiditas illius B vel caliditas et siccitas illius F. Et per consequens C per frigiditatem nunc sibi additam magis resistat quam prius et <sit> inaequalis | potentiae tam cum B quam cum F, et tunc cum C non P 41ra indifferenter plus resistat quam prius vel quam B, sequitur quod sit dare aliquid quod sit minus C sic disposito et minus F quod adhuc non sufficit agere in C. Et per consequens F non est fortissimum quod non sufficit agere in C et per illud patet quod sit dicendum ultra, cum dicitur 'frigiditas in ipso C est tanta sicut praefuit caliditas, praefuit et humiditas C sicut siccitas', vel patet ex casu quod secunda pars copulativae debet negari.

191 Ad sextum et ultimum | secundi principalis dico sicut ad alia R 169rb quod nec conclusio est vera nec sequitur in casu supposito. Contra: ponatur quod caliditas G ignis sit signata per 4 et humiditas B aeris per 2, admitto, et sit quod caliditas B sit tanta in principio sicut humiditas eiusdem et quod postea inducatur figiditas tanta in B, sicut praefuit caliditas quae successive per tempus corrumpat caliditatem B. Tunc sequitur quod G est una potentia quae iam sufficit agere in B, concedo, et continue resistentia B intenditur, concedo, et tamen in fine intensionis sufficit agere velocius in B quam prius vel saltem aeque velociter; hoc nego, quia | B resistebat in principio ipsi G secundum suam K 212rb humiditatem et non secundum suam caliditatem. Et per consequens, cum ipsum B nunc de novo resistat per frigiditatem aequalem | siccitati priori, minoratur proportio G ad B vel etiam V 143v dato quod frigiditas inducatur a minori proportione quam dupla, adhuc continue minoratur proportio G ad B, et per consequens tardius moveret suam resistentiam quam prius. Sed contra

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1 quanta sunt] similiter sicut K sicut R 2 illius[1]] ipsius V | B] *lin.* R illius[2]] ipsius V 4 cum[1]] *om.* KRV | cum[2]] *om.* RV 5 indifferenter] indivisibiliter KV 7 et...C] *om.* (*hom.*) K | per…et] *om.* (*hom.*) R | non[2]] vere V 8 illud] idem V | quod[2]] quid R 9 ultra] quod *add.* R 10 praefuit[2]] *om.* R | C] et R | vel] et V 11 quod] quia R 12 et] *om.* K dico] dici R 13 est…sequitur] sequitur nec est vera. K | supposito] posito R 14 G] *om.* K | sit signata] se habeat KV 15 humiditas…aeris] B aeris humiditas R | et] quod V | in principio] *om.* V 18 caliditatem] in *add.* V 19 agere] *om.* R | et…concedo] *om.* (*hom.*) R 21 sufficit…velocius] velocius sufficit agere R | aeque] ita R 23 secundum] *om.* R 24 cum] quod R | novo] nove V | per] ipsam *add.* V 25 vel…27 B] *om.* (*hom.*) KR

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12 ad...ultimum] Cf. §§ 22, 34.

arguitur sic: resistentia addita B solum assignatur per 2, sit ita, et per aequale praecise assignabatur caliditas B cum qua sufficiet G moveri continue a proportione dupla, igitur et cum ista resistentia sufficit continue moveri et a proportione dupla, non sequitur in aliquo, sicut patet.

<Responsio ad argumenta pro inconvenientibus ad primam opinionem>

192 Ad tertium principale et primo ad primum illius; ad primum dico quod conclusio non sequitur nec est vera. Et tunc ad eius probationem admitto casum ibi suppositum, et tunc cum arguitur ‘A continue intendetur per tempus, igitur A velocitabit motum suum per tempus’, concedo consequentiam et consequens. Et cum ultra arguitur: ‘A velocitabit motum suum per tempus et solum a proportione potentiae motivae ad suam resistentiam, sed inter illa est proportio aequalitatis’, hoc nego, quia licet A ad B quae est resistentia extrinseca partialis sit proportio aequalitatis, non tamen inter A et potentiam suam resistivam aliquam ad B quae forte erit sua resistentia intrinseca est proportio aequalitatis, sed proportio maioris inaequalitatis. Unde A non intenderet a proportione A ad B, sed a proportione potentiae acquisitae in posteriori instanti ad potentiam habitam in priori instanti quae est
P 41rb proportio intrinseca | inaequalitatis maioris.

193 Ad secundum dico concedendo quod nullum grave mundi mixtum vel simplex movendum ad locum suum naturalem et tandem ibi locatum potest continue intendere motum usque in finem motus exclusive, quin, quamvis idem grave per magnum tempus ante finem motus intendat motum suum, necessario

1 sic] si R | addita] divisa A V | et] *om.* V 2 assignabatur] assignabitur R *om.* V | cum] sub R | G] B K 3 igitur…dupla] *om.* (*hom.*) R 10 illius…primum] *om.* (*hom.*) R | ad primum[2]] *om.* K 12 ibi suppositum] et eius finalem R 13 intendetur…velocitabit] per tempus G movebitur K 15 cum ultra] ulterius et R | arguitur] ultra *add.* R | velocitabit] movebitur K 18 partialis] quae licet *add. sed del.* K | sit] *om.* K sicut R 19 suam] *om.* K | aliquam ad] aliam a K | ad] *correxi ex* a PR 24 maioris] *om.* KR 25 mundi] *om.* V 26 movendum] motum KRV 27 tandem] tamen R ibi] continue *add.* K illud V 28 quin] *om.* R

10 ad primum[2]] Cf. §§ 36, 42. | 25 ad secundum] Cf. §§ 37, 43--45.

tamen remittet motum per tempus antequam sit in loco suo naturali totaliter. Unde remota tota terra a centro mundi et aere subsequente et occupante locum terrae, si cetera sint paria, A continue intendet | R 169va motum suum usque ad instans contactum centri exclusive et deinde continue remittet motum usque ad finem motus propter maiorationem resistentiae intrinsecae. Et inde est quod imaginato vacuo circa centrum mundi et posito in isto corpore gravi simplici, ita quod ex una parte centri esset plus quam medietas istius corporis et ex alia parte centri minor quam medietas eiusdem corporis simplicis illud grave simplex moveretur successive in illo vacuo quousque centrum istius esset cum centro mundi.

194 Ad tertium dicitur quod conclusio non sequitur nec est | V 144r vera in casu supposito et ad eius probationem admitto casum, et tunc ultra dico quod C continue velocius et velocius aget in B quam A egit in B, et nego quod infinite velociter A egit in B. Et cum arguitur contra: ‘aliquando maxima resistentia A fuit aliqualiter magna et aliquando in duplo minor, et aliquando in triplo minor, et ipsamet potentia non debilitata continue egit secundum ultimum sui, igitur infinite velociter A egit in B’, nego consequentiam. Quia per idem argumentum sequitur quod quodlibet calidum in summo approximatum ceteris paribus cuicumque passo frigido uniformiter difformi[s] secundum extremum sui intensius, in quod deberet agere successive assimilando sibi passum sive illud passum esset maioris resistentiae, sive minoris, semper aequaliter ageret, quia infinite in utrumque, ut patet, et ideo consequentiam illam nego.

195 Ad quartum dico quod nec conclusio sequitur nec est vera in casu supposito. Et ad eius probationem admittatur casus gratia

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2 aere] aeris K 4 instans] *om.* R | contactum] contactus K tactum V 5 et] *om.* V | motum] suum *add.* V 6 propter] per R | maiorationem] maioritatem K 9 istius] *om.* V | istius…medietas] *om.* KR 10 simplex] si KR 11 istius] ipsius V 13 dicitur] dico V | sequitur…vera] est vera neque sequitur R 15 et velocius] *om.* KRV 17 contra] et *add.* K aliquando] aliqua V | aliquando…minor] *om.* K | maxima…aliquando[2]] *om.* (*hom.*) R 18 aliqualiter] aequaliter | duplo…triplo] triplo minor et in duplo V | in triplo] intensio K 19 non] *om.* KR 20 igitur] *om.* K 21 quod] *om.* K 23 passo] *om.* R | difformis] difformi V 25 sive] suum K illud] idem V 26 quia] in *add.* R | in] *om.* K 27 patet] in K | et] *om.* R

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13 ad tertium] Cf. §§ 38, 46. | 28 ad quartum] Cf. §§ 39, 47--48.

argumenti et admisso casu concedo quod A continue velocius et velocius aget in B quam ipsum incipit agere in B. Et nego quod in infinitum velociter A incipit agere in B. Et cum arguitur quod sic, quia ‘B secundum extremum sui intensius secundum nullum gradum resistentiae resisteret ipsi A’, hoc nego. Contra: ad idem extremum terminatur aliqua frigiditas aliqualiter resistens et aliqua in duplo minus resistens, | K 212va et aliqua in triplo minus resistens, et sic in infinitum, et cum ibi nulla sit resistentia nisi frigiditas, igitur B secundum nullum gradum | P 41va resistentiae secundum extremum sui intensius resistit ipsi A. Nego consequentiam, non enim sequitur, ut patet, ‘B per suum extremum intensius, non per ita remissum gradum frigiditatis in isto extremo resistit ipsi A, quin per minorationem in duplo, in triplo et sic in infinitum resistit ipsi A, igitur B secundum nullum gradum, etc. resistit ipsi A’ et sic patet quod prior consequentia est neganda.

196 Ad quintum et sextum dico quod neutra conclusio est vera neque sequens ex casu, et tunc ad earum probationem admitto casum et casu admisso, nego quod A continue movebitur velocius B, vel quod conus umbrae C continue movebitur velocius cono umbrae D: utrumque nego. Et cum arguitur quod sic, nam ‘umbra C propter maiorationem et continue velocius corrumpetur’, immo aliquando velocius et aliquando | R 169vb tardius et non continue velocius nec aliud sequitur, vel probatur in illo casu. Unde in isto casu A incipit moveri velocius B in primo instanti, et tamen post primum instans in omni tempore | V 144v non terminato ad primum instans B movetur velocius A, ut demonstrative probat sextum argumentum. In toto tamen tempore terminato ad primum instans et ultimum moventur aequaliter A et B, per totum

2 quam…B] *om.* (*hom.*) R | in infinitum] infinite KR 4 B] G V | sui] suum V 5 hoc] ego *add.* K 6 terminatur] *iter. sed del.* K | resistens] remissa K 7 resistens[1]] *om.* R | aliqua] alia KR | minus resistens[2]] *om.* R 8 cum] quod R | ibi] B V 10 non enim] nam non R 11 ita] in *lin.* K *om.* R 12 in] *om.* R | ipsi…sic] *om.* K | ipsi…in[3]] *om.* R 13 duplo] et V | in infinitum] infinite K | infinitum] infinite R 14 gradum] igitur *add.* K resistit…neganda] *om.* KR 17 neque] nec tamen K | sequens] sequitur R 18 continue] *om.* R 21 propter] per K | et] C V 22 corrumpetur] negatur *add.* R 23 aliud] *correxi ex* aliquod P 24 unde] *om.* R | velocius] unde in illo casu A incipit moveri velocius *add.* R 25 tamen] *om.* R 26 primum] *om.* V | A] *om.* K

16 ad...sextum] Cf. §§ 40--41,49--50.

tamen tempus moventur inaequaliter, nec sequitur: ‘A et B per totum tempus moventur inaequaliter, igitur in toto tempore moventur inaequaliter’. Non sequitur, sicut patet in quadam responsione ad sextum argumentum tertii articuli huius quaestionis, quia ly ‘totum’ in accusativo casu stat sincathegorematice et divise et dat intelligere quod per quamlibet partem temporis moventur A et B inaequaliter, et hoc est verum, in ablativo vero casu stat cathegorematice et collective et dat intelligere quod in tempore resultante similiter ex omnibus partibus temporis moventur inaequaliter, et hoc falsum est, quia in isto tempore tantum praecise pertransitur de spatio lineali ab A sicut a B. Et sic patet ad utrumque, ad quintum videlicet et ad sextum. Et est finis quartae quaestionis quae est de proportione velocitatum in motu locali.

1 nec…inaequaliter] *om.* (*hom.*) V | A…tempus] per totum tempus A et B R 3 moventur] *om.* V | non] ut R 5 quaestionis] patet quia *add.* V | ly totum] *om.* KR 6 divise] divisive V | per…9 quod] *om.* (*hom.*) R 7 inaequaliter] aequaliter K | et[2]…verum] *om.* K 8 vero] *om.* V 9 resultante…omnibus] risultantibus simul omnibus R | ex] *om.* K 10 partibus] *om.* V 11 tempore] spatio K | de] illo *add.* K 12 patet] responsio *add.* R | ad[1]] *om.* KV A *add.* R | videlicet] licet K *om.* V | videlicet et] scilicet R 13 sextum] patet quod sic dicendum *add.* V | et] sic *add.* K 14 locali] Deo altissimo refferens gratias. Anno Christi 1404 die 18 Octobris in die beati Lucae, dum magna regnaret guera inter dominium Venetiarum et dominum Francesscum [de] Chraria [i.e. Carrara] dominum Padue et Marchionem Extenssem [Nicolaum III], (?) dominum Ferarriae, complevi hoc scribere ego Donatus de Monte. *add.* K Deo gratias, Amen. Explicit tractatus *De sex inconvenientibus*, finito libro, sit laus et gloria Christo; dabitur pro penna scriptori pulchra puella. *add.* R etc. Et est finis operis, mercedem posco laboris *add.* V

Johannes Dumbleton

De motu locali

Summa logicae et philosophiae naturalis
pars III, cap. 1, 5-12

Ms. Oxfrod, Magdalen College 32, ff. 48v-58v

JOHANNES DUMBLETON
<SUMMA LOGICAE ET PHILOSOPHIAE NATURALIS>
<PARS III: DE MOTU LOCALI>

<Capitulum 1>

1 Quia singulorum notitia motu tanquam signo naturali nobis primo inest super aliquod de eodem dicere et de eiusdem principiis tractare cum eisdem ignotis tamquam sine fundamento perit scientia subsequentium quia ignoratis principiis ignorantur ea quae sequntur.

2 In quattuor praedicamentis conceditur a Philosopho esse motus, scilicet qualitate, quantitate et ubi. In substantia motus improprie dictus cum ipsa sola substantia, quia subiectum est aliis predicamentis, vere movetur et illa tria, puta qualitas, quantitas, et locus, et singula predicamenta alia per accidens moventur solum, quod sic est intelligendum.

3 Probatio patet. Solum est unum praedicamentum scilicet substantia quod aliis substat et omnia tamquam sua accidentia eidem insunt. Sed subiectum in motu quod recipit utrumque extremum et contraria vere movetur, igitur substantia inter cetera praedicamenta per susceptionem aliorum vere movetur. Consequentia tenet et antecedens patet. Maior patet per Philosophum et Commentatorem VII *Metaphysicae*, commento 2 et eundem textu dicentem quod hoc nomen ens dicitur principaliter de ente per se, scilicet substantia, et de aliis secundarie. Et reddit causam, quia substantia est subiectum illis, et dicit quod alia non dicuntur entia nisi quia accidunt enti per se, scilicet substantiae. Minor patet per Commentatorem I *Physicorum*, commento 60 dicentem quod sperma habet subiectum

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6 conceditur…10 solum] *Cf.* Arist., *Phys.*, V.1, 225b6–9, (213vb–214ra): Si igitur praedicamenta divisa sunt substantia, et qualitate, et ubi, et quando, et ad aliquid, et quantitate, et ipso agere, et pati, necesse est tres esse motus, eum qui quantitatis, et eum, qui qualitatis, et eum qui secundum locum est. | 18 Philosophum…23 substantiae] *Cf.* Aver., *In Metaph.*, VII, com. 2: (153rb): Hoc nomen ens principaliter et simpliciter significat praedicamentum substantiae et secundario et determinate, sc. relative ad substantiam, alia praedicamenta accidentium. | 23 Commentatorem…4,2 spermatis] *Cf.* Aver., *In Phys.*, I, com. 60 (36ra): In spermate enim quod fit homo, necesse est ut fit aliquid & est illud per quod dicitur sperma; ratio enim hominis qui fit est opposita rationi spermatis.

quod recipit formam hominis et in eius adventu corrumpitur forma spermatis. Et sequitur in IV in rei veritate: illud in quo invenitur motus est subiectum, et debet esse in subiecto. Et impossibile est quod sit motus in illo cuius partes succesive destruuntur, et iste motus est in subiecto. Et si non esset subiectum, non esset aliquod differens motum et sic nihil diceretur moveri. Haec sunt dicta Commentatoris. Cum igitur substantia solum recipit generationem et corruptionem aliorum praedicamentorum omnium, et nullum praedicamentum sive alia res in aliquo praedicamento a substantia recipit corruptionem substantiae, igitur sola substantia per aliqua praedicamenta vere movetur et aliqua non, quia partes qualitatis, quantitatis et ubi sunt talia quorum partes succesive destruuntur et generantur, igitur illa non vere moventur. Sed subiectum, quod est substantia, vere movetur per adquisitionem et deperditionem succesivam in illis.

4 Item, Aristoteles V *Physicorum* textu commenti 3 et 5: species et locus et passiones in qua moventur illa quae moventur immobilia sunt, et ponit exemplum ut scientia et color, super quo dicit Commentator. Habentur in textu Aristotelis exposito ab eodem Commentatore quod idem quod movetur est diversum ab illo a quo est motus et ab illo ad quod est motus, ut lignum est aliud a calido et frigido. Et primum est illud quod est motum, scilicet lignum, et secundum a quo, et tertium ad quod est motus. Et

11 sola] *marg.*

2 illud…3 subiecto] *Cf. ibid.* (36ra–b): Sed ille motus est in subiecto & si non esset subiectum, non esset hic aliquod deferens istum motum nec aliquid quod diceretur moveri neque aliquid quod esset in potentia aliquid. | 17 Aristoteles…19 color] *Cf.* Arist, *Phys.*, V.1, 224b10–13 (208va–b): Quid igitur motus sit dictum est prius. Formae autem, & passiones, & locus, ad qua moventur ea, quae moventur, immobilia sunt, ut scientia, & calor. | 21 idem…24 motus] Cf. Aver., *In Phys.*, V, com. 3 (208ra): motum enim primo est aliquid, &c, id est & illud quod movetur per se, manifestum est ipsum esse aliud ab illo, ad quod movetur, et ista alietas(*sic!*) est ei primo, & essentialiter. Per accidens vero non est remotum(*sic!*), ut illud, quod movetur, et illud ad quod est motus, sint idem & similiter est de illo, ex quo est motus, scilicet quia etiam est aliud a moto. D.d. verbi gratia quantum lignum, &c in quantum lignum est illud, quod movetur in alteratione, & illud, ad quod est alteratio, est calidum, & frigidum, & lignum est aliud a calido, aut frigido.

dicit Aristoteles: manifestum est quod motus est in ligno et non in forma. Et Commentator in commento dicit: quod dignius quod est in motu est subiectum, scilicet res mota.

5 Item, Avicenna in I *Physicorum* in libro *Sufficientia* dicit quod omnis motus non sunt nisi in subiecto secundum hoc quod est subiectum tantum, similiter idem corpus aliquando sursum aliquando deorsum per hoc quod recipit sursum et deorsum movetur localiter sine fine et quod deorsum movetur localiter nec proprie.

6 Item, corpus sit calidum et frigidum, similiter corpus augmentatum movetur et non quantitas, sic satis monstratur solum substantiam vere moveri per susceptionem aliarum rerum in aliis praedicamentis.

<Capitulum 5>

7 Sequitur inquirere qualiter velocitas in motibus veris et succesivis producitur et causatur. Et primo de motu locali arguatur tangendo opiniones quasdam, ut per congregationem falsi tanquam per divisionem veritatem attingamus.

8 In ista materia tres sunt opiniones.

9 Prima dicit quod motus sequitur excessum potentiae motivae super resistentiam, ut ita velociter agunt 4 in 2 precise sicut 3 in 1, et hoc est quia eadem est proportio arithmetica.

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2 dicit quod] *marg.* 19 Prima dicit] *marg.*

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1 manifestum…2 forma] *Cf.* Arist., *Phys.*, V.1, 224b3–6 (207va–b): Quoniam autem est quidem aliquod movens primum, est autem aliquid, quod movetur, ad haec in quo tempus, & praeter haec ex quo, & in quod (omnis enim motus ex quodam, & in quiddam alterum enim est quod primum movetur, & in quodam movetur, & ex quo, ut lignum, et calidum, & frigidum. horum autem, aliud quidem quod, aliud vero in quod, alid autem ex quo.) motus manifestum est quod in ligno est, non in forma. neque enim movet, neque moveretur forma, aut locus, aut ipsum tantum. | 2 Commentator…3 mota] Cf. Aver., *In Phys.*, V, com. 3 (208rb): Primum igitur istorum, & dignius in motu est res mota, scilicet subiectum motus. | 4 Avicenna…9 proprie] *Cf.* Avicenna, *Sufficientia,* 2.2, in: *Liber primus naturalium.* Tractatus secundus *De motu et de consimilibus*, Édition critique par S. Van Riet (†), J. Janssens, A. Allard. Introduction doctrinale par G. Verbeke. Académie Royale de Belgique, Brussels 2006, 178: Omnes enim motus non sunt nisi in substantia secundum quod est subiectum tantum.

10 Secunda opinio dicit quod motus sequitur proportionem geometricam, ita quod si sit tanta proportio inter agens et passum, sive fuerit in motu locali sive in motu alterationis, quod tanta velocitas correspondet necessario tantae proportioni precise, ita quod omnia agentia in eadem specie motus secundum eandem proportionem aeque velociter agunt. Tamen illa positio dicit quod intensio motus sequitur augmentationem proportionalem agentis, ut si agens fiat duplum, velocitas duplabitur, et si in sesquialtera proportione vel in alia rationali et irrationali fiat potentia agentis maior, ceteris paribus, secundum illam proportionem augeabitur motus in velocitate.

11 Tertia opinio, Aristotelis et Commentatoris, quae tenenda est, talis est quod motus sequitur proportionem geometricam et remittitur et intenditur secundum hoc quod proportio maior est vel minor ad proportionem priorem, ita quod motus excedit alium motum secundum quod una proportio se habet ad aliam proportionem, secundum quas proportiones fiunt ille motus inaequales.

12 Contra primam arguitur: ex eadem sequitur quod omnis motus est infinitae velocitatis vel quilibet infinitae tarditatis. Quod sic arguitur: ponatur 4 agere in 2 et minoretur 2 in infinitum et minoretur 4 sic tamen quod continue excedat per 2, et in fine sit agens ut 2, tunc in quolibet instanti illius horae illud agens excedit passum per aequalem excessum, igitur aequaliter movebitur secundum illum excessum, igitur infiniter velocius movebitur, quia non secundum maiorem excessum supra resistentiam movet hoc agens.

13 Item sequitur: quod nullum agens finitae potentiae potest infinite velociter movere, nec potest plus excedere suam resistentiam quam ipsum est, sequitur quod nullum agens potest infinite velociter movere in medio finito, quod est impossibile.

14 Item, sequitur ex hanc opinione quod simplex movebitur in vacuo. Ponatur A esse unum grave signatum per 2 in vacuo, et aliud grave sicut 4 ponatur in medio sicut 2. Ista duo gravia moventur aeque velociter, quia non per maius excedunt 2 vacuum quam 4 excedunt 2. Sed cum motus sequitur excessum motoris super resistentiam, igitur aeque velociter moventur 4 in

14 hoc] *marg.* 23 tunc] *marg.*

pleno sicut 2 moventur in vacuo. Consequens falsum et contra Philosophum et Commentatorem IV *Physicorum* textu commenti 71 per magnum processum.

15 Item, hac positione sequitur quod 6 velocius moverentur in 2 quam 2 in vacuo, nam plus excedit 6 2 quam 2 excedunt vacuum.

16 Item, haec positio dicit opinionem Avempace quam probat IV *Physicorum* Commentator commento 71: Avempace ponit motum in vacuo maxime naturalem, quia in vacuo non habet corpus impedimentum a motu sicut habet in pleno. Et ideo posuit Avempace illum gradum motus in vacuo maxime naturalem
51v corporis. |

17 Item, habet haec positio ponere, nam si 4 agant in medio assignato per 2 et remittatur resistentia quousque tota corrumpatur, in fine illud corpus assignatum per 4 movetur velocius, quia non habet impedimentum. Igitur ille motus est sibi maxime naturalis, quia iste motus prout habet impedimentum resistentiae est sibi innaturalis. Et sic haec positio incidit in opinionem Avempace.

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16 quia…innaturalis] *marg. a.m.*

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2 Philosophum…3 71] *Cf.* Arist., *Phys.*, IV.8, 215b4–24, (158ra): Sit enim B quidem aqua D vero aer. quanto ergo subtilior est aer aqua & minus corporeus, tanto citius A per D movebitur quam per B. Habeat ergo eadem rationem, secundum quam distat aer ab aqua, velocitas ad velocitatem quare si in duplo subtilius est, in duplo tempore lineam quae est ipsum B transibit quam quae est ipsum D. & erit tempus, in quo est C duplum eius, in quo est E, atque semper quanto fuerit minus corporeum, minusque impeditivum, ac melius divisibile id per quod fertur, citius movebitur. Vacuum autem nullam habet rationem qua exceditur a corpore sicut neque ipsum nihil ad numerum (...) Similiter autem & vacuum ad plenum nullam possibile est habere rationem: quare neque motum. | 7 Avempace…11 corporis] *Cf.* Aver., *In Phys.*, IV, com. 71 (161vb): Ex hoc igitur dicit Aristoteles quod, si haec corpora moverentur in vacuo, sequeretur ut moverentur motu indivisibili & in tempore indivisibili: ponendo quod haec corpora moveri in medio est naturale eis & causa est in hoc, quod moveantur , non sicut existimatur, quod sunt mota ex se non motu naturali, sicut existimavit Avempace. Avempace igitur fecit dubitare in hoc sermone in duobus locis. Unum eorum est, in quo dicit, quod proportio motus ad motum non est sicut proportio spissitudinis medij ad spissitudinem medij. Secundus autem est, quod isti motus sunt impediti ex medio, & non naturales.

18 Item, sequitur quod infinitus motus postest fieri in vacuo, quia 4 velocius movetur in vacuo quam 2 quia plus excedit vacuum quam 2 et 4 velocius 1, et sic in infinitum, quia plus videtur secundum illam positionem motum magis naturalem fieri in vacuo quam in pleno. Consequens contra processum Philosophi in IV *Physicorum* textu commenti 71, ubi probat quod motus non posset esse in vacuo nisi infinitae velocitatis.

<Capitulum 6>

19 Secunda opinio convenit cum tertia, eo quod dicit omnia moventia secundum eandem proportionem aeque velociter moveri in eodem medio, tamen non dicit motum attendi nisi secundum proportionem agentium, et dicit primo repugnans dicto primo.

20 Ista positio fundatur super illa ratione: agant tria in duo et alia tria in alia duo utrumque secundum proportionem sesquialteram. Utrumque illorum moventium inducitur certam latitudinem caliditatis sive motus localis. Si igitur fiat unum agens ex istis duobus et applicetur post uni passo istorum, cum unum illorum agens non impedit reliquum in motu sed potius ipsum iuvat, sequitur quod illa duo agunt duplum ad prius actum, et per consequens in duplo velocius.

21 Primo arguitur contra istam positionem: in se et in solutione obiectionum solvitur fundamenti ratio illius positionis. Primo sequitur ex ista opinione quod motus non potest tardari in infinitum, et hoc in movente per se quod alterius mobilis medietas non est pars <eius>. Quod sic arguitur. Ponamus unum motorem, scilicet Socratem, sicut 4 et medium sicut 2, remittatur potentia motoris quousque fuerit aequalis medio. Sequitur ex posito quod iste motor nunquam movebitur in duplo tardius, quod sic arguitur. Si sic, detur instans et sit medium instans illius temporis, contra, in medio instanti iste Socrates erit maioris potentiae quam medium erit pro illo instanti, igitur duplum ad illum motorem pro illo instanti est maior potentia quam 4. Sit

2 quia…2] *marg.* 9 aeque velociter] *lin.* 10 medio] *marg.* | non] *marg.* motum] non *add. sed exp.* 17 illorum] *marg.* 24 mobilis medietas] *marg.*

5 Philosophi…7 velocitatis] *Cf.* Arist., *Phys.*, IV.8, 215b4–24, (158ra).

igitur dupla potentia sicut 5, tunc sic: 4 movetur nunc in duplo velocius quam 5 movebitur in medio instanti horae. Sed si 5 moverent precise 2 in duplo velocius quam tunc movebuntur 4, sequitur quod idem motus provenit ex 5 in 2 et ex 4 in eisdem. Consequens impossibile.

22 Ad argumentum sic respondetur quod nihil potest remittere motum suum in infinitum nisi voluntarium movens, ut homo potest movere se vel aliquid extraneum mobile infinita tarditate, et hoc non solum quia habet talem proportionem, sed quia habet voluntatem.

23 Contra illud arguitur sic. Ponatur quod Socrates pellat A terram, illa terra non solum pellitur a Socrate, quia Socrates solum vult eam pellere, sed oportet quod nitatur ad pellendum. Capiatur nisus Socratis, et proportio Socratis secundum quam Socrates pellit A terram. Ponatur ignis B, sit istius tantum nisus precise et secundum tantam proportionem precise remittatur sicut Socrates pellat A terram. Et remittatur aequaliter nisus Socratis cum potentia B ignis ita, quod pro quolibet instanti B et Socrates aequaliter nituntur precise agere. Et quod hoc sit possibile probatur, quia aliqua est maxima actio quam Socrates producit isto nisu et quod non maior detur ignis qui tantam faceret actionem precise in aequalem resistentiam sicut ille B. Isto posito sequitur quod pro quolibet instanti Socrates et B movebunt suas potentias aequaliter, quia secundum eandem proportionem ut ponatur et Socrates remittet motum suum usque ad quietem, igitur et B ignis remittet motum suum usque ad quietem.

24 Item, sic movetur circulus cuius centrum quiescat in aere circumdante moto, capiatur pars mota circuli in 100 tardius quam est motus proveniens ex proportione dupla <.?.> ex proportione minori proportione dupla. Ponatur quod aliquod sic movens suam resistentiam solum secundum eandem proportionem, sequitur ex ista positione, quod ita velociter movebitur illud per
52r se motum sicut illa pars circuli. | Nec potest dici quod illa pars data movetur secundum eandem proportionem secundum quam movetur punctus extremus, quia ex hoc sequitur quod omnia

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1 movetur…horae] *marg.* 3 precise] *marg.* 4 in[1]…4] *iter.* 8 mobile] *marg.* 21 quod] *marg.* 24 potentias] resistentias *sed exp. et corr. marg.* 30 dupla] illa pars movetur a motore *add. sed. exp.*

moventur secundum eandem proportionem. Et sic non esset causa quare aliquis punctus alio moveretur tardius, quod impossibile videtur.

25 Item sic, agant 4 in unum, iuxta positionem datam sequitur quod 8 agant in duplo velocius in unum quam 4, quia potentia dupla 4 ad 8 ad 4. Capiatur agens quod se habet ad 4 in proportione sesquialtera et in medietate sesquialterae proportionis, et sit A. Tunc sic, ex proportione 8 ad 1 provenit motus in duplo velocior quam motus factus ex proportione 4 ad unum. Sed cum proportio A ad 4 se habet ad proportionem 6 ad 4 sicut proportio 8 ad 1 se habet ad proportionem 4 ad 1, sequitur quod si 8 moveant unum in duplo velocius quam 4, sequitur quod A movebit 4 in duplo velocius quam 6 4. Antecedens patet, quia proportio 8 ad 1 est composita ex proportione quadrupla et eius medietate, ideo sicut octo faciunt in unum motum duplum ad 4, ita A faciet duplum motum in 4 quam 6, quia proportio A ad 6 habet proportionem sesquialteram et eius medietatem. Sed cum A sit minus 4 sequitur ex positione quod minus 9 movebit 4 in duplo velocius ad motum quo movent 6 quattuor. Quod A sit minus 9 patet, nam 9 ad 4 est proportio sesquialtera duplicata, sed cum proportio A ad 4 non sit nisi ex proportione sesquialtera et eius medietate, igitur proportio A ad 4 minor est proportione 9 ad 4. Quare sequitur 'A esse minus quam 9' per unam consequentiam in V Euclidis qui dicit: si 2 ad 3 se habent inaequaliter in proportione maioris inaequalitatis sequitur, quod maius est, in maiori proportione se habet.

26 Item, ex ista positione sequitur quod Socrates nunc non sufficit facere A difficultatem, immo maxima quam Socrates sufficit sive minima quam non sufficit distat ab A actione per magnam latitudinem. Et immediate post hoc sufficiet Socrates facere A difficultatem per partibilem augmentationem potentiae. Quod sic arguitur: sit potentia Socratis sicut 4 et resistentia sicut

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24 V…27 habet] *Cf.* Euclides, *Elementa*, lib. V, prop. 8: Si due quantitates inequales ad unam quantitatem proportionentur, maior quidem maiorem, minor vero minorem obtinebit proportionem. Illius vero ad illas ad minorem quidem proportio maior, ad maiorem vero minor erit. [Hubertus L.L. Busard, *Campanus of Novara and Euclid Elements*, vol. I, Franz Steiner Verlag, Stuttgart 2005, 181-182].

4, quia per istam positonem motus non potest tardari in infinitum ab agente irrationali, si igitur Socrates intendetur in potentia, sequitur quod immediate post hoc movebit 4 velociori gradu proveniente ex proportione dupla, ut dictum est prius. Sit igitur C gradus subduplus ad gradum provenientem ex proportione dupla; capiatur una actio quae precise erit sicut movere C gradu in 4 datis, <tunc Socrates nunc non sufficit movere cum C gradu in 4 datis> nec tantam difficultatem facere. Et si intendatur potentia Socratis partibiliter, immediate post hoc movebitur velocius quam C gradu, quare sequitur conclusio dicta impossibilis.

27 Item, sequitur quod si potentia intendatur in infinitum, quod <in> infinitum velocius intendetur latitudo motus quam latitudo proportionis. Consequens falsum et consequentiam eandem ponam cum solum dicit motum ita velocem quia secundum tantam proportionem sit. Et quod antecedens sequitur, arguitur: agant 4 in unum, tunc duplata potentia duplus motus sequitur; sed non sequitur dupla proportio, quia 8 sunt proportio dupla ad 4, et 16 sunt quae se habent in proportione ad 1 respectu proportionis 4 ad 1, quia eam continet, et sic in infinitum duplatur potentia. Et sic sequitur quod <in> infinitum maior erit latitudo motus adquisita pro eodem instanti quam latitudo proportionis. Consequens falsum, quia si aliqua latitudo motus alicuius fiat secundum tres proportiones duplas, si uni parti certae corespondet latitudo proportionum a proportione dupla usque ad proportionem aequalitatis, sequitur quod tanta pars eiusdem latitudinis motus alteri latitudini tantae proportionis correspondet.

28 Item, moveatur terra assignata per 4 in aere asignato per 2, et diminuatur terra usque ad aequalitatem aeris. Et sit A gradus pro hoc instanti proveniens a proportione dupla inter A et aerem, et sit B gradus subduplus ad A. Tunc tota latitudo motus inter B et A correspondet toti latitudini proportionis inter proportionem duplam et proportionem aequalitatis, igitur nulla proportio minor dupla correspondet gradui remissiori B nec maior dupla. Igitur magna erit latitudo motus, scilicet a B ad non gradum, cui nulla
52v proportio nec eiusdem aliqua latitudo correspondet, | quod est impossibile.

29 Item, ista posito non potest assignare causam a qua proportione proveniet motus medius inter A B vel a proportione medietatis duplae proportionis vel ab aliqua, cum non uniformiter deperdentur istae duae latitudines, scilicet latitudo motus et latitudo proportionis. Ista patent in duabus lineis scilicet ABQ et CD. Ponatur quod latitudo proportionum sit linea CD, et sit agens 4, passum sive medium 2. Et sit latitudo motus AQ, et medietas latitudinis AB, ita quod terra data non remittat motum suum ultra B gradum per deperditionem latitudinis proportionis inter proportionem duplam et aequalitatem ut inter 4 et 2, sicut CD, ut dicit positio. Ex isto patet quod latitudo motus BQ, quae terminatur ad quietem, secundum nullam proportionem potest fieri nec aliquis gradus in ea. Hoc sic probatur: <sit> C agens quid facit A motum in medio dato sicut 2. Quia igitur motus non potest tardari in infinitum nisi in moto per se, ut argutum est, patet quod exclusive desinet idem C agens ad B gradum movere, et sic sequitur quod non gradum BQ latitudinis habebit C per motum.

30 Item, agant 3 in 2, sit B medium, sicut dicit positio, 6 agent in duplo velocius in 2 quia duplata est potentia. Tunc sic se habet proportio tripla ad E medium quae proveniret ex proportione inter 6 et B, sicut se habet sesquialtera ad D motum provenientem ex proportione sesquialtera. Et quod E motus fit secundum proportionem tripla ad sesquialtera, sic D motus, igitur permutatim, sicut se habet proportio tripla ad sesquialteram, sic E motus ad D motum. Consequens falsum, quia unum duplum est ad aliud puta E motus ad D motum, et non sic erit una proportio ad alteram proportionem. Exemplum patet in linea EDQB, sit Q quies sive non gradus motus, et hoc improbat opinionem per se sine aliqua alia evidentia.

31 Item, sequitur ex positione quod 9 agant ita velociter in 4 sicut 12 in eisdem, nam 9 agunt in 4 secundum duas proportiones sesquialteras sive secundum proportionem sesquialteram duplicatam, ergo 9 agunt duplum in 4 quae faciunt 6 in 4 quae 6 solum agunt secundum unicam proportionem sesquialteram precise. Consequntia tenet per fundamentum istius positionis, quia una proportio coniuncta cum alia non impedit quod minus

28 linea] 6 3 2 et in linea *add. sed exp.*

fiat per aliam, sicut nec unum agens impedit aliud sibi coniunctum. Sed cum 12 sint potentia dupla ad 6, ideo dicit positio quod 12 in duplo plus agunt in 4 quam faciunt 6, ex quo sequitur 9 et 6 in 4 aequaliter agere. Consequentia tenet, quia 12 et 9 agunt duplum ad idem actum quod agunt 6 in 4. Hoc sic patet, quia 6 agunt secundum proportionem sesquialteram certam actionem in 4, prout medietatem latitudinis caliditatis; ergo si aliud agens agat secundum duas proportiones sesquialteras duplum in 4 inducit, quia utrique proportioni sesquialteri tantum correspondet. Cum igitur 9 agunt secundum duas proportiones sesquialteras in 4, igitur 9 agunt duplum in 4 quam agunt 6 in 4; et 12 agunt precise duplum in 4 quam 6 in 4, igitur 12 et 9 in 4 agent aequaliter. Consequens impossibile.

32 Item ad idem, agant 3, ut Socrates, in 2, et agant 4, ut Plato, in alia 2 distincta secundum proportionem numeri ad numerum. Ista duo agentia, tria et quattuor, tantum agant precise sicut unum quod agit secundum proportionem duplam et sesquialteram secundum quas proportiones agunt ista. Sed cum 12 agunt in 4 quantum ista agunt, scilicet Socrates et Plato, secundum proportionem duplam et sesquialteram, igitur 12 tantum agunt in 4 quantum ista duo agunt, scilicet Socrates et Plato, in ista duo passa. Sed cum illi plus agunt quam duplum ad illud quod fit ex duobus, sequitur quod 12 agunt plus quam duplum ad idem, fit ex proportione sesquialtera, et per consequens plus quam duplum ad actum inter 6 et 4, cuius contrarium dicit positio.

33 Item, agant 3 in 2 A motum et 8 in 4 B motum, isti motus proveniunt ex proportionibus inaequalibus vel sunt inaequales,
53r puta AB. Vel igitur secundum proportionem | agentis ad agens, vel secundum proportionem secundum quam proportio sesquialtera est minor proportione dupla. Non primo modo, ut probatum erit, quia cum infinitum agens contingit agere secundum proportionem duplam, sequitur quod motus infiniter velocior A ex proportione dupla proveniret. Consequens impossibile. Si igitur secundo modo, sequitur quod motus sequitur proportionem proportionum et non proportionem agentium, ut dicit positio. Vel sic, A motus tantum precise distat a

14 ut Socrates] *marg.* | ut Plato] *marg.* 23 ex] sex in *add. sed exp.* 24 duplum ad] *marg.*

quiete quantum distat proportio sesquialtera a proportione aequalitatis, et sicut B motus precise tantum distat a quiete quantum distat proportio dupla a proportione aequalitatis, igitur tota latitudo motus inter A et B correspondet precise latitudini proportionum inter proportionem sesquialteram et duplam. Si igitur latitudines motus et proportionis sibi invicem correspondeant, igitur secundum quod augmentatur latitudo motus, sic augmentatur latitudo proportionis et per consequens diversitas motuum sequitur diversitatem proportionum secundum quas fiunt.

34 Item, si 8 agerent in duplo velocius in unum quam 4 in unum, sequitur quod tantum agerent 8 in 4 sicut 4 in unum. Consequens falsum et contra positionem in se. Et consequentia tenet sic: ponatur quod tota actio ex 4 in unum sit A, et tota actio ex 8 in 4 sit B, tunc A B sunt aequalia per positionem, cum proportio 8 ad 1 non est maior proportione 4 ad unum nisi per porportionem duplam, et A correspondet proportioni inter 4 et 1. Sequitur quod B actum correspondet precise proportioni inter 8 et 4, ergo tantum agunt 8 in 4 in hora quantum B est. Sed cum B est aequale A, quod fit ex 4 in unum, sequitur quod tantum agunt 8 in 4 sicut 4 in 1, quod est probandum.

35 Hic erit notandum quod positio quae dicit, quod diversitas motuum respectu eiusdem medii fit secundum proportionem agentium ad invicem necessario habet dicere quod magis agens movens secundum aequalem proportionem secundum quam agit minus agens tardius agit agens debilius movens quam fortius, ut patet per argumentum factum. Nam cum duo agentia per horam uniformiter in diversis passis secundum proportionem quae est 4 ad 1, in duplo plus agunt ista duo agentia in ista duo passa quam unum illorum per se in suum passum. Sed cum 8 non agunt in unum secundum proportionem quae continet bis quadruplam, non potest assignari alia causa quare 8 agunt duplum ad 4 in unum nisi propter maioritatem potentiae, et per consequens non est motus ita velox propter proportionem.

36 Item sic, si ex proportione dupla proveniat precise A motus et ex minori non potest, sequitur quod motus duplus ad A non fit nisi ex duabus proportionibus duplis, scilicet ex proportione

24 ad invicem] *marg.*

<composita ex duabus duplis et motus subduplus ad A precise ex proportione> quae precise [valet] medietatem duplae proportionis continet, quia non duplatur effectus nisi sit duplata causa. Ex quo sequitur quod sicut motus potest minui in infinitum, sic per eius minorationem potest motus tardari in infinitum. Vel sic, arguitur quod infinita latitudo motus correspondet precise proportioni duplae vel eiusdem latitudini. Detur quod Socrates moveat AB motu qualitercumque intenso et ex quantacumque magna proportione voluerimus, et sit iste motus C. Totus motus Socratis correspondet precise proportioni duplae. Quod arguitur sic: sit Plato duplus in potentia ad Socratem, et moveat ista proportione, tunc motus Platonis est duplus ad motum Socratis, et Plato solum adquirit proportionem duplam supra proportionem quam iam habet Socrates; igitur tota latitudo adquisita a Platone supra B correspondebit proportioni duplae, et sic patet propositum. Vel sic, tota proportio Platonis ad A componitur ex proportione Platoni ad Socratem et Socratis ad A, et tota latitudo motus per quam motus Platonis excedit C motum, correspondet proportioni duplae quae correspondet proportioni inter Socratem et Platonem, quae est precise dupla quia Plato est precise duplus ad Socratem, et sic patet intentum.

37 Item, aliter arguitur quod infinita latitudo motus correspondet proportioni duplae. Quia si non, detur quod A sit tanta latitudo motus quae proportioni duplae correspondere non potest, et moveatur Socrates A gradu in B medio, duplata potentia Socratis B manente in eadem dispositione potentiae duplabitur motus, igitur tanta latitudo motus quantam continet A correspondebit
53v precise proportioni duplae adquisitae | supra illam proportionem quam nunc habet Socrates ad B. Illud patet assignato Socratem per 4 et B quantumcumque modicae resitentiae volueris existere et assignetur Plato sicut 8 et moveantur in B medio. <...>

<Capitulum 7>

38 Sequitur dicere qualiter motus sequitur proportionem, et primo modo exemplari exprimere ut qui in geometria non sunt exercitati exemplis grossis et sensibilibus veritatem ingrediantur et eius causam videant.

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11 Quod…sic] *marg.* 19 duplae…proportioni] *marg.*

39 Et primo supponendum est quod omnes motus eiusdem praedicamenti facti precise secundum eandem proportionem sunt aeque veloces. Istud patet ab omni opinione.

40 Secunda suppositio est quod omnis motus factus secundum maiorem proportionem est intensior motu facto secundum minorem proportionem, ita tamen quod uterque motus fiat in eodem praedicamento.

41 Tertia quod sive potentia maioretur sive minoretur ipsa autem movente secundum eandem proportionem continue motum aeque velocem producit. Ista patet immediate ex prima.

42 Quarta suppositio, quod latitudo motus et proportionis inter se aequaliter adquiruntur et deperduntur, sicut si spatium uniformiter adquiratur aliquo motu in die, quanta pars de motu adquiritur, tantum de spatio precise adquiritur et de tempore labetur. Ita, si latitudo proportionis uniformi motu intensionis adquiritur, quanta pars de latitudine proportionis adquiritur, quae est quasi spatium motus intensionis, tantum precise de latitudine motus sibi correspondentis adquiritur. Ista patent per ultimum argumentum factum contra primam positionem per hoc quod in omni intentione motus aequali latitudini proportionis adquisitae aequalis latitudo motus correspondet, quia aliter sequitur quod infinita latitudo cuiuscumque proportioni correspondet. Consequens falsum et impossibile.

43 Item, hoc sic patet. Si ex proportione dupla proveniat A motus, precise sequitur quod duplata causa, scilicet proportione, dupletur effectus, scilicet motus A. Et ex isto sequitur, quod motus inaeque veloces in proportione tali sunt inaequales in qua una proportio excedit aliam secundum quas illi motus fiant, ut si motus sit factus secundum proportionem duplam, quaecumque proportio continens illam bis precise duplum motum faciet, et quaecumque proportio illam ter continet, triplum motum faciet, et sic in infinitum; ita quod sicut duplabitur latitudo proportionis sic duplabitur latitudo motus. Hoc sic patet: si omnis motus verus sit secundum hoc quod motor dominatur supra resistentiam, sed cum motus non sequitur excessum potentiae motoris super motum, nec proportionem motorum adinvicem,

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13 aliquo motu] *marg.*

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10 patet…prima] *Cf.* § 39 *supra.* | 18 patent…19 positionem] *Cf.* § 1 *supra.*

igitur diversitas motuum sequitur proportionum diversitatem. Consequentia patet et antecedens per argumenta facta quoad opiniones improbatas.

44 Item, hoc patet posita hanc suppositione quod omnes motus facti secundum eandem proportionem sunt aeque veloces. Nam si ex proportione dupla solum proveniret B motus localis, et ex tripla proportione proveniat A motus, igitur permutatim sic se habet B motus ad A motum, sicut tripla proportio ad proportionem duplam. Et sic diversitas inter A et B est secundum diversitatem proportionum.

45 Item, si B motus fiat secundum proportionem duplam et quartam partem duplae et A secundum proportionem duplam, B est velocior A per quartam partem latitudinis motus contenti in A motu, quia per tantum proportio ex qua fit B est maior proportione ex qua fit A. Illud patet sensibiliter in quantitatibus, ut datur linea ABCDE latitudo motus et linea HFGIK latitudo proportionis, tunc ex proportione 16 in unum provenit A motus, qui continet tantam distantiam a quiete quanta est ab A usque ad E. Similiter proportio 16 ad unum continet tantam distantiam proportionis ab A usque ad proportionem aequalitatis quae est terminus latitudinis proportionis maioris inaequalitatis. Et si 16 minuantur ad 8 deperdent proportionem duplam, puta proportionem inter 16 ad 8 quae est dupla proportio. Et quia dupla proportio est quasi quarta pars totius proportionis 16 ad
54r unum, sicut AB est quarta pars AE lineae, ideo A | motus deperdet quartam suae latitudinis puta AB. Et si partibiliter minoretur proportio usque ad proportionem aequalitatis, partibiliter deperditur motus, ac si ad corruptionem AE lineae corrumperetur HK linea et econtra. Si igitur addatur proportio dupla ad proportionem quae est AE, fiet motus intensior per quartam sicut proportio est maior per quartam. Et si per medietatem duplae, tunc per medietatem quartae AC motus A augmentabitur, et sic in infinitum. Ut sicut 20 sunt plura 16 per quartam 16, scilicet per 4, sicut 32 addunt motui AE solum quartam, scilicet tantum quantum est inter AB, nam 20 continent 16 et eius quartam partem, sicut proportio 32 ad unum continet proportionem 16 ad unum et eius quartam partem proportionis,

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6 B] *corr. ad sensum, ms.*: gradus

scilicet proportionem duplam ultra. Et quia numerus ad numerum est sesquiquarta sic proportio ad proportionem est sesquiquarta, proportio 32 ad unum est sesquiquarta ad proportionem 16 ad unum. Ista patent per Campanum V Euclidis. Istum intellectum habet Aristoteles IV et VII *Physicorum* et cum Commentatore super eundem textum.

46 Item, ponamus lineam OTQM et intelligamus QM partem continere totam latitudinem proportionum inter 9 et 4 et G linea. Sit motus factus ex ista proportione quae est 9 ad 4, tunc tantum distat G motus a quiete sicut proportio 9 ad 4 a proportione aequalitatis. Si ergo 9 augmentetur ad 16, adquiret respectu 4 duas proportiones aequales, scilicet bis sesquiatertiam, quia proportio 12 ad 9 est proportio sesquiatertia, et proportio 16 ad 12 est proportio sesquiatertia. Ideo tota proportio 16 ad 9 est composita ex duabus proportionibus sesquiatertiis, sicut linea G componitur ex linea I X medietate illius partis et G X alia medietate. Et quia proportio 12 ad 9 est media proportio inter proportionem 16 ad 12 et 12 ad 9, ideo proportio 12 ad 9 continem medietatem totalis proportionis 16 ad 9. Et per consequens agens, cum fuerit sub 12, adquiret et habebit medietatem totius latitudinis motus et proportionis inter G et I et inter 9 et 16. Non tamen intelligendum est quod tota latitudo proportionum inter 12 et 9 se habet ad totam proportionem, quae est 9 ad 4, sicut proportio quae est inter 16 et 4 se habet ad proportionem quae est inter 12 et 4 quia uniformiter debet attendi penes terminum a quo incipit potentia maiorari vel minorari resitentia, ut si Socrates petranseat uniformiter AB spatium in hora, in secunda medietate pertransibit CD et DB partes, scilicet quartas, non tamen ita se habet DA ad CA sicut BA ad DA, sed utraque est quod pertransitur in tertia quarta sicut in ultima quarta. Consimiliter considerandum est de latitudine proportionum uniformiter adquisita.

47 Ex hiis patet quod in resistentia difformi omnia agentia quae per totam eandem horam sufficiunt movere aequales latitudines motus, deperdent similiter, sive fuerunt aequalia sive inaequalia in potentia. Quia si non, detur resistentia AB difformis respectu

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10 G...sicut] *marg.* 15 composita] et fit *add. sed exp.* 24 9…est[2] *marg.*

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4 Campanum…5 Euclidis] *Cf.* Euclides, *Elementa*, V, prop. 11.

Socratis et Platonis qui sunt inaequalis potentiae, gratia exempli, et sit A extremum asignatum ut 4 et B extremum ut duo, capiatur proportio inter Platonem et B extremum, et sit 6 ad 4, et sit potentia Platonis maior 4 quanta est linea DB maior linea AB, et sit potentia Socratis 8 et proportio Socratis ad A extremum sicut linea CB ad lineam AB, vel sicut 8 ad 4, capiatur gradus quem agit Socrates ad B resistentiam, et sit C gradus quem agit Plato ad B. Tunc sic, proportio Socratis ad B componitur ex proportione Socratis ad A, et A ad B. Item proportio Platonis ad B componitur ex proportione Platonis ad A et ex proportione A ad B. Igitur proportio A ad B, quae dupla est, est pars communis quasi quantitativa respectu proportionis Socratis ad B et proportionis Platonis ad B. Sed cum aequali latitudini proportionis correspondet aequalis latitudo motus, sequitur quod tantam latitudinem motus habebit Socrates in AB medio sicut Plato, quod est probandum.

48 Vel sic: si 6 agant ad B extremum, quod est ut duo, et 8 ad idem utrumque movendo usque ad A punctum, 6 tunc deperdent solum proportionem duplam, scilicet proportionem inter 4 et
54v duo. Sed cum tantam | proportionem deperdent 8 in AB, et aequali latitudini proportionis aequalis latitudo motus correspondet, igitur aequalem latitudinem motus utrumque, scilicet tam 6 quam 8, deperdent.

49 Vel sic: omne quod movebit per AB medium stante potentia eius sub eodem gradu et similiter resistentia AB, solum proportionem duplam deperdet per deperditionem motus vel duplam proportionem per intensionem motus adquiret. Igitur omnia mota in AB resistentia aequalem latitudinem motus deperdent.

50 Notandum est quod augmentatio proportionis semper est attendenda penes terminum a quo incipit augeri potentia, ut si Socrates sit ut 4 et moveatur in duobus, latitudo proportionis adquirenda in Socrate attendenda est respectu termini sub quo iam est Socrates, ut pro quolibet instanti tanta latitudo proportionis erit adquisita, quanta erit proportio Socratis pro eodem instanti ad 4. Et ideo pro quolibet instanti Socrates incipiet adquirere latitudinem proportionis a proportione

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26 deperditionem] remissionem *sed corr. marg.*

aequalitatis, scilicet respectu termini potentiae sub quo ipse est pro illo instanti, et illa sit adquisita et addita proportioni duplae primo habite inter Socratem et (...) duo facit maiorem latitudinem proportionis primo habita sicut quantitas augetur per additionem quantitatis incipientis a non quanto.

51 Praeterea ex hiis patet quod quocumque motu difformi accepto terminato circa quantitate, sive uniformiter secundum latitudinem, sive [maioratur] aliter adquisito, tantum precise pertransietur illo motu difformi taliter intenso, quantum precise pertransietur a duobus mobilibus motis in eodem tempore, quorum unum movetur motu gradu a quo incipit motus difformis intendi per idem tempus et aliud intendit motum suum a quiete consimiliter per tantam latitudinem in tempore aequali.

52 Ponamus Socratem intendere motum ab A gradu ad B duplum ad A in hora difformiter sive uniformiter, et moveatur Plato A gradu uniformi in eadem hora, et intendat Cicero a quiete ad A consimiliter adquirendo latitudinem motus sicut Socrates, et proveniat A motus ex proportione dupla precise. Tunc Socrates non pertransibit plus in hora quam Plato nisi per latitudinem adquisitam supra A, quae correspondet precise proportioni duplae, ut patet intellecto casu priori. Sed cum Cicero tantum pertransibit precise secundum illam latitudinem quantum Socrates habebit supra A, sequitur quod Plato et Cicero precise tantum pertransibunt istis duobus motibus quantum Socrates qui continue habebit A motum et adquiret latitudinem supra A correspondentem proportioni duplae, quod est probandum.

53 Istud patet per hoc quod omnis latitudo motus, sive fuerit addita intensiori sive remissiori gradui, semper aequaliter ei de spatio correspondet dummodo eodem modo adquiretur illa latitudo motus. Et causa est quia aequali latitudini proportionum et aequali proportioni correspondet, ut si 4 moveant 2 proportione dupla et intendatur potentia ad duplum, et adquirat unam proportionem aliam duplam ad aliam, non maius spatium correspondet illi latitudini proportionum adquisitae, hoc per se, quia si duo intenderentur supra quattuor et moveantur in quattuor dum tamen eodem modo intenditur motus. Et causa est quia semper gradus potentiae a quo potentia incipit augmentari

9 taliter] *marg.* 12 intendi] *marg.* 32 et^{1}…aliam^{2}] *marg.* 35 supra…intenditur] *marg.*

habetur terminus cui terminatur latitudo proportionis, sicut ad proportionem aequalitatis.

54 Item, sequitur quod omnis latitudo teminata ad quietem utriusque medietatis aeque intensa est precise, loquendo de medietate intensa et non de medietate in tempore. Nam proportio quadrupla componitur precise ex duabus duplis quarum utraque est tanta proportio sicut alia. Cum igitur latitudo motus se habet ad suas medietates intensive sicut proportio ad suas medietates, sequitur quod utraque medietas latitudinis motus est aeque intensa, sicut utraque medietas proportionis est alteri aequalis.

55 Item, non maior latitudo motus correspondet uni proportioni duplae quam alteri, et hoc precise, igitur motus proveniens ex proportione quadrupla componitur ex duobus motibus aequalibus intensive, sicut quadrupla proportio componitur precise ex duobus proportionibus duplis, quia aliter maior intensive motus correspondet uni medietati proportionis quadruplae quam alteri, quod est impossibile. Et per hoc
55r concluditur quod motus | et proportio et omnes tales qualitates, sive verae, sive imaginariae, componuntur ex partibus qualitativis in actu, vel saltem illo modo quo sunt qualitates et quod quaelibet pars est divisibilis in infinitum, et intensior qualitas non sit nisi per additionem talium partium qualitativarum, sicut proportio non maioratur nisi per proportiones additas incipientes a proportione aequalitatis, ut patet consideranti compositionem proportionum et eius naturam.

<Capitulum 8>

56 Contra istud sic arguitur. Nam ex hoc sequitur quod nulla latitudo motus terminatur ad gradum uniformem motus, ut ponatur quod Socrates moveatur iam a gradu proveniente ex proportione dupla, et adquirat Socrates duplum gradum, scilicet B, per augmentationem potentiae, et sit C latitudo precise adquisita a Socrate in hora data. Tunc per datum terminatur ad quietem. Quod sic arguitur, B componitur precise ex talibus duobus motibus sicut est A, eo quod B est duplus ad A; igitur sicut A terminatur ad quietem, ita et C latitudo adquisita. Et per consequens nec C latitudo terminatur ad A nec ad aliquem gradum uniformem, sicut nec quantitas terminatur ad quantitatem eiusdem speciei. Et ita per positionem sequitur nullam latitudinem terminari ad aliquem gradum uniformem. Et per hoc potest argui, quod nullus gradus uniformis caliditatis est terminus latitudinis caliditatis, cuius oppositum ut convenienter dicitur.

57 Pro istis est distinguendum primo, eo quod aliquid terminatur proprie, sive improprie, <improprie> ut secunda medietas pedalis terminatur ad aliam, id est post eam immediate in generatione quantitatis sequitur et in computatione sequitur. Et illo modo potest dici quod ternarius est terminus quaternarii et quintarius est terminus quaternarii, quia immediate succedunt. Similiter infinitum illo modo est terminus infiniti, quia in specie conveniunt. Et illo modo terminatur latitudo motus ad uniformem gradum et improprie sicut prima pedalis alicuius magnitudinis est terminus totius subsequentis. Proprie dicitur terminus qui cum terminato non convenit, ut punctus respectu lineae, et linea respectu superficiei, et superficies respectu corporis. Et proprie corpus non terminat corpus, quia omne corpus cum alio in toto vel in parte convenit in quantitate et <in aequalitate similiter. Sed nulla superficies cum corpore aliquo convenit nec convenire> potest eo quod omne corpus omnem superficiem imaginabilem incomparabiliter excedit.

58 Per hoc quando arguitur 'C latitudo terminatur ad quietem, igitur non terminatur ad A gradum a quo incipit', negatur consequentia quia C terminatur ad A improprie, et ad quietem proprie. Nam sicut pedalis quantitas adquirat aliam pedalem, haec secunda terminatur improprie ad primam pro eo quod sibi immediate additur. Ex qua cum quantitate adquisita fit additio alia maior, ita cum Socrates movetur iam A gradu, ille motus adquirendus addetur gradui A manifeste continue in Socrate. Ex quo motu adquisito partibus manentibus imaginare similiter rem velociorem sic motus intentio sicut proportio dupla respectu alicuius mobilis manet inter Socratem et suum medium, licet Socrates adquirat aliam duplam.

59 Similiter est imaginandum quod latitudo caliditatis non terminatur proprie ad gradum summum, sicut nec pedalis terminatur ad pedalem, nisi aliquis velit ponere quod gradus

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2 aliquem] *marg.* 4 caliditatis] *marg.* 26 sicut] si *add. sed exp.*

summus se habet ad latitudinem caliditatis sicut punctus ad lineam, quod non est ponendum. Nec secunda medietas latitudinis caliditatis terminatur ad aliam medietatem nisi improprie, id est ut eam sequitur in genere qualitatis summae. Et ideo sicut omnis latitudo finita terminatur ad duo puncta, ita si latitudo caliditatis in duplo remissior caliditatis vel distantia qualitativa alicuius qualitatis proprium haberet terminum, quaelibet pars eius distantiae et qualibet latitudo talis caliditatis ad eundem terminum posset terminari. Et quia ponatur, ut communiter, quod gradus summus terminat totam latitudinem caliditatis et quod una medietas terminat aliam medietatem ideo incidunt inconvenientes dicentes unam medietatem esse intensiorem alia. Nam proprie dicitur lineam terminari ad puncta, quia in nullo cum puncto continet linea, ita dicendum est de
55v termino caliditatis difformis, si terminum proprium | habeat, nec aliter proprie debet dici latitudo terminari ad aliud.

60 Secundo contra dicta sic obicitur. Si latitudo proportionum et motus aequaliter adquirantur et deperdantur, sequitur quod latitudo proportionis inaequalitatis aequaliter posset deperdi cum motu. Consequens falsum et consequentia tenet, ut videtur. Nam motor movet secundum proportionem maioris inaequalitatis et resistentia resistit secundum proportionem minoris inaequalitatis, eo quod motor ut ipsam movet resitentiam dominatur super eam. Et falsitas consequentis sic arguitur. Agant 4 in 2, et fiant 2 per augmentationem potentiae in hora aequalia illi 4 per uniformem adquisitionem proportionis. Tunc immediate ante finem horae infinite magna latitudo proportionis minoris inaequalitatis adquiretur, nam continue maiorabitur proportio minoris inaequalitatis in infinitum, quia quanto proportio maioris inaequalitatis minor fuerit, in tanto maior erit proportio minoris inaequalitatis, ut patet per suppositionem unam V Euclidis. Sed cum in infinitum maiorabitur proportio maioris inaequalitatis,

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6 caliditatis[1]] *marg.* 26 Tunc] *marg.*

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31 suppositionem…Euclidis] *Cf.* Euclides, *Elementa*, V, Theor. 8(?): Inequalium magnitudinum major ad eandem majorem rationem habet, quam minor: Et eidem ad minorem, maiorem rationem habet, quam ad maiorem.

sequitur quod in infinitum proportio minoris inaequalitatis maiorabitur.

61 Item, non posset esse maior proportio minoris inaequalitatis quam erit ante finem horae, igitur infinitam latitudinem proportionis adquiret, et solum finitam latitudinem motus deperdet. Sequitur quod non aequaliter adquirentur et deperdentur istae duae latitudines.

62 Item, sequitur quod non posset aliquid aequale alteri fieri nisi infinitam latitudinem proportionis adquiret.

63 Item, eadem est latitudo proportionis sive proportio 4 ad 2 et 2 ad 4, igitur si una latitudo sit infinita respectu termini quae est proportio aequalitatis, igitur utraque, tam proportio maioris inaequalitatis quam minoris inaequalitatis; vel sic, proportio 2 ad 4 est dupla ad proportionem 1 ad 4, igitur dupla est latitudo sive distantia proportionis.

64 Item, captis tribus gradibus latitudinis A B C, et distet A a B per certam distantiam et B a C et econverso, sic quod dupla sit latitudo sive distantia inter B et C quam inter B et A, et sequitur quod econtra B in duplo plus distat a C quam A a C. Sic fiat argumentum de proportione maioris inaequalitatis et minoris. Et sit A proportio quadrupla, et B dupla, et C medietas duplae proportionis, et fiat econtra comparatio secundum proportionem minoris inaequalitatis, et patet intuenti quod C in duplo plus distat a B quam B ab A, quoniam ad proportionem minoris inaequalitatis.

65 Pro istis, ut dictum est prius, omnis latitudo sive distantia potest dupliciter considerari, positive et privative, ut dicimus qualitatem intensam et ipsammet remissam, et eandem quantitatem magnam et parvam, ita latitudo proportionum dupliciter intelligitur: positive pro eo quod est proportio maioris inaequalitatis, privative pro eo quod est proportio minoris inaequalitatis. Nam captis tribus gradibus in caliditate A B C et sit C in duplo remissior B, et B in duplo remissior A, ex hoc non sequitur quod in duplo maior latitudo est inter C et B quam inter B et A, sed oppositum sequitur, ut patet. Sic intelligendum est in proportione minoris inaequalitatis, quia captis tribus proportionibus minoris inaequalitatis A B C, et sit C medietas

28 eandem] *marg.* 35 intelligendum est] *marg.*

duplae proportionis, et B dupla proportio, et A quadrupla proportio; C proportio est in duplo maior proportione minoris inaequalitatis quam B, et B quam A, non tamen in duplo maior est latitudo proportionum inter C et B, quam inter B et A, immo in duplo minor. Quia idem est dicere 'in duplo maior proportio minoris inaequalitatis est inter B et C quam inter B et A', et dicere quod est in duplo minor proportio maioris inaequalitatis. Vel sic: ponatur quod A B C sint tria puncta proportionaliter distantia respectu D termini, tunc C est in duplo propinquius D quam B, et B quam A; igitur in duplo maior est distantia inter C et B, quam inter B et A. Hoc non sequitur: sic est in proportione minoris inaequalitatis, quia secundum quod illa latitudo proportionis maioretur latitudo proportionis maioretur similiter, ut secundum quod aliquid sit magis parvum distantia quantitativa minuitur.

56r **66** Contra | hoc sic arguitur. Latitudo proportionis minoris inaequalitatis potest maiorari in infinitum sed non per maiorationem proportionis maioris inaequalitatis, igitur per maiorationem proportionis minoris inaequalitatis. Et per consequens infinita latitudo adquiretur et hoc proportionis minoris inaequalitatis, si minor fiat aequalis maiori.

67 Pro isto est intelligendum quod latitudo proportionis minoris inaequalitatis nulla est prout sic, quia latitudo, prout latitudo est, positivam importat ut intensum in qualitate et magnum in quantitate. Et ideo nulla est latitudo in quantitate secundum quam illa est remissa. Et sic arguitur: secundum quod habet distantiam in qualitate intensa est; sed nihil habet de latitudine. Desinit <enim> qualitatem ipsam, <ideo> est remissa. Quod sic arguitur: secundum quod aliquid desinit in qualitate illa est remissa et secundum quod habet distantiam in qualitate, intensius est, sed nullam habet latitudinem secundum quod aliquid <...> Sequitur quod qualitas nullam latitudinem habet secundum quam remissa est. Idem intellige de magno et parvo, et idem patet, quia in proportione nulla latitudo est maior alia secundum quod est proportio minoris inaequalitatis.

68 Ad argumentum. Latitudo proportionis minoris inaequalitatis potest maiorari in infinitum, et causa sophisticationis potest

7 quod est] *marg.* 11 Hoc] *lin.* 20 si…22 inaequalitatis] *marg.* 27 Quod…29 remissa] *marg.*

concedi et hoc solum propter minorationem proportionis maioris inaequalitatis, sicut latitudo remissionis.

69 Si admittatur modus loquendi 'maioratur solum per deperditionem latitudinis intensionis' ista patent alio exemplo. Si aliquis motus remittatur ad quietem secundum quod minoratur proportio maioris inaequalitatis, ita minoratur motus ille; igitur effectus maiorabitur, scilicet motus, secundum quod proportio maioris inaequalitatis maiorabitur, et secundum quod proportio minoris inaequalitatis maiorabitur, minoratur effectus, quia motus tardatur iuxta deperditionem proportionis maioris inaequalitatis; igitur latitudo proportionis non attenditur nec accipi potest secundum proportionem minoris inaequalitatis. Et per consequens in duplo maior proportio minoris inaequalitatis non facit duplam latitudinem proportionis, quae est conclusio probanda, sicut in duplo magis parvum non facit duplam distantiam quantitativam.

<Capitulum 9>

70 Sequitur solvere argumentum quod est fundamentum alterius positionis, cum supponatur quod tria agant in aliquod passum ut duo et agant alia tria post in eadem duo aequaliter resistentia, sequitur quod illa duo agentia agunt duplum quam unum per se, cum neutrum impedit aliud. Igitur, si prima tria agerent unitatem in alia duo et hoc latitudinis alicuius vel alicuius qualitatis in 2, sequitur quod 6 agerent duplum ad idem quod prius egerunt tria.

71 Item, 3 producunt aliquem certum motum localem, sit A ille motus, et fiat ille motus in B medio signato per 2, igitur sequitur quod 6 producunt duplum motum ad A motum, cum neutrum impedit aliud.

72 Pro istis et consimilibus supponendum est quod in omni actione tota actio agentis est totius agentis actio et eius cuiuscumque partis respectu illius actionis, ut sive agant 2 sive 4 in unum medium, tota actio 4 est actio unitatis sicut actio agentis unitatem per se, est actio unitatis. Cuiuscumque partis agentis eadem est actio, ut si A motus producatur a Socrate et a Platone in B, quorum neuter motum ita velocem sufficit facere in B, A motus est actio tam Socratis quam Platonis, et sic utriusque

22 in^1…hoc] *marg.* | vel alicuius] *lin.*

agentis est A motus. Quia aliter sequitur quod Socrates faceret unum motum per se, et Plato alium.

73 Item, si 6 agent in 2, sequitur quod esset una actio correspondens tribus illorum 6, et alia actio disparata correspondens aliis tribus. Et per consequens non esset dupla velocitas sed essent duae velocitates distinctae et duae qualitates distinctae ab illis, separatim inductae. Consequens falsum, quia eadem ratione esset una actio correspondens tribus et medietati unitatis, et sic infinitae actiones distinctae, et sic non esset ibi maxima velocitas danda. Consequens falsum et impossibile.

74 Per hoc ad argumenta quando concluditur quod duorum agentium quorum utrumque est ut tria aplicatum ad unitatem illa simul applicata duplum agunt ad illud quod unum per se, eo quod
56v | neutrum impedit aliud, igitur utrumque agit tantum sicut prius egit per se - conceditur consequentia et negatur antecedens. Et ulterius quando arguitur 'igitur duo agunt duplum ad prius actum', negatur consequentia. Quia tota actio est actio utriusque et utriusque idem agit et quia plus est actum quam prius, ideo utrumque plus agit quam prius, quia utrumque per iuvamentum alterius velocius agit, in quantum proportio totius ad passum secundum quem agens agit est maior quam prius erat illa proportio secundum quam utrumque per se agens alterebat vel movebat motu locali.

75 Pro isto est Commentator VIII *Physicorum*, commento 23, ubi ponit quod ita est de guttis quarum congregatio operatur in lapide, sicut est de multis hominibus qui movent navem, quoniam unusquisque non movet, id est per se. Ita non sequitur quod quaelibet gutta habeat operationem suam, sed intelligendum est quod quaelibet habet eandem, ut Socrates et Plato moveant A mobile B gradu, quo neuter sufficit movere A; B gradus est motus utriusque et cuiuscumque potentiae moventis in utroque illorum, non tamen per se, sed B motus est actio communis Socratis et Platonis.

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8 una] *lin.* 28 operationem…habet] *marg.*

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24 Commentator…27 se] *Cf.* Aver., *In Phys.*, VIII, com. 23 (f. 359raB): Sicut de multis hominibus qui movent navem quoniam si quemadmodum unusquisque eorum solus non movet navem, & omnes movent, ita si congregatio illarum guttarum operatur in lapide, non sequitur ut quaelibet gutta operetur per se in lapide.

76 Item, si quaelibet pars agentis haberet actionem per se, sequitur quod quilibet sufficeret agere in quodlibet. Consequentia tenet et arguitur: agant 4 in 2, et capiatur maximus gradus resistentiae quo non resistit pars illorum duorum, ut pars inexistit toti, et sit ille gradus B. Si igitur quaelibet pars illorum quattuor haberet actionem per se, et in illis duobus non est minor gradus resistentiae quam est B gradus; igitur quaelibet pars illorum 4 ageret per se in tantam resistentiam quanta est una sub B gradu, et sic fieret actio et motus secundum proportionem minoris inaequalitatis. Consequens falsum, quia tunc quodlibet ageret in quodlibet, cum non esset ratio assignanda quare secundum unam proportionem minoris inaequalitatis fieret quam secundum aliam, ideo etc.

<Capitulum 10>

77 Consequenter probatur latitudinem suo medio gradui correspondere.

78 Primo tamen monstratur quod si aliqua latitudo motus terminata ad quietem, intensiore gradu suo medio valet, medietatem eius remissiorem ad quietem terminatam remissiori medio eiusdem medietatis correspondere probatur.

79 Improbatur hoc sic: ponamus AF latitudinem motus, B medium gradum eiusdem, D gradum medium medietatis ad quietem terminatae, et C gradum correspondentem toti AF latitudini uniformiter adquisitae, et ponamus R gradum correspondere BF medietati, et sit F quies. Dico quod si C est intensior B, igitur R est intensior D. Nam pono Socratem adquirere uniformiter AF latitudinem in hora, patet quod Socrates tantum pertransibit in prima medietate ac si moveretur R gradu. Sed cum in secunda medietate horae habebit Socrates BA latitudinem citra quietem terminatam, igitur Socrates <tantum> pertransibit in secunda medietate quantum duo mota quorum unum movetur B gradu solum in eadem medietate horae et aliud movetur R gradu, eo quod latitudo BF <est terminata latitudo> sicut est BA. Consequentia tenet per secundam suppositionem capituli septimi illius partis. Et per consequens non tantum pertransietur precise in hora AF latitiudine sicut a duobus, quorum unum movebitur per horam R gradu solum et aliud

33 secundam…34 partis] *Cf. supra*

solum per medietatem horae B gradu, ut patet intuenti. Sed tamen B tantum valet per medietatem horae quantum ad pertransitionem spatii sicut D suus subduplus valet per horam. Sequitur quod AF latitudo uniformiter adquisita per horam valet R D gradus per eandem horam. Sed cum D R gradus per horam tantum valent precise sicut unus gradus compositus ex illis precise, sequitur quod G gradus, qui tantum precise distat a D gradu quantum R ab F, aequevalebit AF latitudinini uniformiter adquisitae per horam. Consequentia tenet per hoc quod G in uno moto per horam valet RD in diversibus motibus per horam. Sed 57r cum minor est distantia inter R et F quam inter D et B, et G | non addit supra D nisi latitudinem RF, sequitur quod G est remissior B, igitur gradus valens totam latitudinem AF uniformiter adquisitam per B, et per consequens C, est remissior B, quod est probandum.

80 Per idem probatur quod sic gradus correspondens BF medietati uniformiter adquisitae sit intensior D quam gradus correspondens totali AF est intensior B. Pro conclusione praecedente et aliis probandis in ista materia ponenda est haec suppositio, quod si sint duo gradus uniformes motus, ut R D inaequales sive aequales, quod tantum valet precise D per horam quantum unus, qui tantum addit de latitudine summa quantum R distat a quiete. Haec suppositio patet per conclusionem primam.

<Capitulum 11>

81 Sequitur probatio istius 'quod omnis latitudo etc.' Nam si aliqua latitudo incipiens a quiete uniformiter adquisita in transitu spatii suo medio gradui non correspondet, detur AQ latitudo talis, et sit Q quies, et A gradus eandem terminans, et sit B medius totalis latitudinis eiusdem, et D medius BQ medietatis eiusdem; si igitur AQ latitudo uniformiter adquisita suo medio in transitii spatii non aequepollet, detur igitur primo quod C gradus intensior B aequevalet AQ latitudini uniformiter adquisitae, et sit E gradus correspondens medietati BQ, quia igitur C est intensior B patet per praecedentem conclusionem quod E est intensior D. Dico igitur ex ista positione sequi quod latitudo uniformiter adquisita in hora gradui ipsam latitudinem terminanti aequevalet, quod est impossibile.

5 horam[1]] *marg.* | gradus[2]] *marg.* 8 gradu] *marg.* 25 non] *marg.*

82 Istud sic arguitur: ponamus Socratem adquirere AQ latitudinem uniformiter in hora. Patet per prius probata quod Socrates in prima medietate horae tantum pertransibit ac si per primam medietatem horae movisset E gradu, et in secunda medietate tantum sicut duo mota, quorum <unum> per totam horam movebitur E gradu et aliud E gradu per eandem medietatem. Igitur Socrates pertransibit AQ latitudinem sicut duo mota quorum unum per totam horam movebitur E gradu solum et aliud per medietatem solum B gradu. Et per consequens Socrates tantum pertransibit illa AQ latitudine uniformiter adquisita quantum pertransietur in hora E gradu et D qui est subduplus ad B. Consequentiae tenent intuenti praecedentem conclusionem. Cum igitur D et E gradus per horam facti valent AQ latitudinem per eandem, sequitur quod gradus qui valet AQ latitudinem in hora tantum addit de latitudine supra D quantum distat D a Q, qui est quies. Sed cum C per positum aequevalet AQ latitudini in hora, sequitur quod C tantum distat a B quantum distat D a Q. Istae consequentiae tenent per illam suppositionem. Tunc sic: aequalis est distantia inter D et Q sicut inter E et C, sed cum aequaliter distat D a B sicut D distat a Q, igitur aequaliter distat E a C sicut B a D. Consequentia tenet per illud medium 'quaecumque sunt aequalia sunt alicui tertio etc.' Si ergo aequalis est latitudo DB sicut EC, dempto igitur communi, scilicet BE, illa quae relinquntur, scilicet ED et BC latitudines, erunt aequales. Et per consequens tantum distat gradus correspondens AQ a B medio gradu eiusdem sicut distat E a medio BQ, quae est mediaetas AQ. Et per idem probatur quod per tantum distat gradus correspondens quartae AQ a medio gradu eiusdem quartae sicut distat E gradus a D gradu, et sic in infinitum, et per octavas et 16. Capiamus igitur GQ partem aliquotam AQ latitudinis, quae sit G, cuius medius gradus sit N. Tunc tantum distat N a G quantum distat E a D, vel sequitur quod minor sit distantia inter gradum eius medium et gradum sibi correspondentem quam sit distantia inter C et B. Et quod talis sit danda patet per argumentum factum, quia AQ latitudo est divisibilis in infinitum in partes aliquotas terminatas ad Q gradum, et cuiuscumque partis aliquotae dandae tanta est distantia inter gradum sibi

6 aliud] per medietatem solum *add. marg.* 17 a B] ab *sed exp. et corr. in*: a B E(*exp.*) 31 Tunc] *marg.* 32 sequitur] *marg.* 34 Et] *lin.*

correspondentem et eius gradum medium sicut inter C et B, nunc patet per argumentum factum. Et capiatur gradus correspondens GQ latitudini uniformiter adquisitae per tempus et sit M per 57v argumentum factum. Sequitur quod tanta est distantia | inter M et N gradus sicut inter D et E gradum. Sed cum tanta est distantia inter G et N sicut inter D et E, igitur tantum distat M ab N sicut G ab N gradu. Et per consequens idem est M gradus et G cui GQ latitudo uniformiter adquisita in hora correspondet, quod est probandum ex dato. Si igitur detur quod nulla est pars aliquota AQ cuius medius gradus precise tantum distat ab extremo quantum C a B, capiatur una latitudo cuius medius gradus minus distat ab extremo quam C distat a B, et sit illa GQ. Ex demonstratione facta patet quod GQ correspondet intensiori gradui quam extremali, quod est impossibile. Si ergo detur pars quantitativa quod latitudo uniformiter adquisita valet remissiori gradui medio eiusdem, sequitur per conclusionem praecedentem quod quaelibet latitudo minor illa terminata ad quietem remissiori medio suo gradui correspondet.

83 Intelligamus pro ista probatione C esse medium AQ et B remissiorem C gradu correspondentem AQ latitudini uniformiter adquisitae in hora, et D correspondentem EQ latitudini. Ex ista positione sequitur per aliquam latitudinem uniformiter adquisitam nihil adquiri, iuxta prius probata in illa conclusione patet quod AQ latitudo uniformiter adquisita in hora valebit D gradui per horam et C per medietatem horae. Et per consequens valebit AQ latitudo uniformiter aquisita in hora D gradui per horam et E gradui per eandem horam, qui E est subduplus ad C. Sed cum B per datum valet AQ latitudinem in hora uniformiter adquisitam, igitur B gradus tantum continet ultra D quantum distat E a Q. Consequentia tenet per suppositionem. Sed cum aequalis est distantia inter E et Q sicut inter C et E sequitur, quod aequaliter distat A a D gradu sicut E a C. Dempto igitur communi BE, relictae latitudines, scilicet ED et CB, erunt aequales. Capiatur igitur aliqua pars CQ medietatis, cuius medius gradus precise tantum distat a quiete sicut distat D ab E medio medietatis CQ, et sit latitudo illa GQ cuius latitudinis gradus medius sit N, et gradus eidem correspondens sit F, qui est remissior medio, ut patet per

5 gradum] *lin.* 11 latitudo] *marg.* 18 gradui] *marg.* 23 in illa] *marg.*

conclusionem praecedentem. Cum igitur tanta est latitudo inter D et E sicut inter B et C, non obstante quod CQ sit dupla latitudo ad EQ, et latitudo GQ vel est quarta pars vel octava latitudinis EQ, ita quod non in infinitum exceditur ab eadem, sequitur quod tantum distat F gradus correspondens GQ latitudini ab N medio latitudinis GQ quantum distat D gradus correspondens latitudini CQ ab E medio gradu CQ latitudnis. Sed cum quies tantum distat ab N medio GQ latitudinis quantum D distat ab E, igitur F gradus, qui correspondet GQ latitudini, est quies. Et sic sequitur quod per GQ latitudinem uniformiter adquisitam nihil pertransiri potest, quod est probandum ex dicto.

<Capitulum 12>

84 Aliter probatur 'quod omnis latitudo motus terminata ad quietem et uniformiter adquisita etc.' Hoc patet posita hac suppositione praedictis probatis, quod omnium graduum terminantium aliquas latitudines incipientes a quiete ad gradus correspondentes eis in latitudines incipientes a quiete ad gradus correspondentes eis in pertransitione spatii eadem est proportio, ut si AB AC sint duae latitudines ad quietem terminantae quarum AB sit maior, et D E sint gradus uniformes quibus tantum pertransietur in hora sicut AB AC latitudinibus uniformiter adquisitis, <sequitur> quod eadem est proportio B gradus terminantis AB latitudinem ad E gradum sicut AC ad D. Ex ista suppositione sic arguitur, et ponamus AF latitudinem uniformiter adquisitam in hora a Socrate. Dico quod tantum pertransietur AF latitudine in hora sicut AC eius medio gradu, quia si non, detur quod tantum pertransitur AF latitudine sicut AB gradu intensiori gradu medio eiusdem latitudinis. Et sit pedale maximum pertransitum a Socrate in hora et intelligamus F esse quietem AF | 58r
latitudinis. Capio tunc gradum subduplum ad B, et sit E, qui intensior est D gradu medio medietatis remissioris AF latitudinis, ut patet. Tunc sic: eadem est proportio A ad C sicut B ad E, quia dupla. Permutatim igitur, qualis est proportio A ad B gradum eadem est proportio C ad E gradum. Consequentia tenet per

1 D] *lin.* 16 latitudines…in] *iter.* 22 B] *lin.*

suppositionem V Euclidis. Tunc sic: eadem est proportio A ad B gradum sicut C ad E gradum, igitur per suppositionem datam sicut B correspondet AF latitudini uniformiter adquisitae, sic E gradus correspondet medietati remissiori eiusdem latitudinis. Consequentia tenet, quia unus est gradus in AF latitudine, qui se habet ad C sicut B ad A. Sed quia E est subduplus ad B gradum, et Socrates movebitur in prima medietate istius horae CF latitudine correspondente E pertransibit quartam totalis spatii pertranseundi a B gradu correspondente toti AF latitudini. Consequentia patet intuenti, et per consequens Socrates pertransibit in prima medietate horae quartam pedis. Sed cum Socrates in secunda medietate horae habebit uniformiter CA latitudinem, igitur tantum Socrates pertransibit in secunda medietate horae sicut duo mota in eodem tempore, quorum unum intendet uniformiter a quiete ad C gradum motu facto, et aliud movebitur continue C gradu. Consequentia patet per praecedentem conclusionem capituli septimi. Sed quia B gradu pertransiretur in secunda medietate horae, quia sic precise semipedale, quia sibi tota AF latitudo correspondet, et C est gradus remissior B per datum; igitur Socrates minus pertransibit in secunda medietate horae per C gradum et CF latitudinem quam tres quartas pedis. Igitur <sequitur> quod Socrates non in toto tempore pertransibit pedale, quod est contra hipotesim. Et consequentia tenet per hoc quod si Socrates movebitur B gradu et illa latitudine, tunc tres quartas pertransiet, sed iam minus quam tunc, igitur etc.

85 Si igitur gradus correspondens latitudini sit remissior eiusdem medio, ponamus igitur AF talem latitudinem ut prius et intelligamus B esse medium gradum eiusdem quartae correspondentem toti AF latitudini, qui C sit remissior medio gradu AF, ut datur, et sit E medius gradus BF latitudinis et sit D

6 E] *lin.* 18 F] *lin.* 28 quartae] et C gradum esse *add. sed. exp.*

16 Consequentia…17 septimi] *Cf. supra*

1 suppositionem…Euclidis] *Cf.* Euclides, *Elementa*, V, def. XII: Quantitates, que sunt in proportione una, antecedens ad consequentem et antecedens ad consequentem; dicetur econtrario sicut consequens ad antecedentem, sic consequens ad antecedentem. Itemque permutatim, sicut antecedens ad antecedentem, sic etiam consequens ad consequentem [H.L.L. Busard, *op.cit.*, 171]

subduplus ad C. Per prius argumentum factum patet quod eadem est proportio A ad C qualis est proportio B ad D, capta suppositione V Euclidis. Ponamus quod pedale sit maximum quod pertransietur a Socrate in hora uniformiter adquirendo AF latitudinem, quia igitur C est duplus ad D, et per C gradum precise pertransietur pedale in hora, quia latitudinem AF aequevalet, sequitur iuxta formam priorem, quod Socrates pertransibit in prima mediaetate horae quartam pedis. Et quia in secunda medietate horae Socrates tantum pertransibit precise quantum duo mota in eadem medietate, quorum unum movebitur B gradu totius AF latitudinis et aliud movebitur uniformiter adquirendo latitudinem BF, ut patet per conclusionem praecedentem, igitur Socrates pertransibit in secunda medietate horae quartam pedis et plus quam duplum ad quartam pedis. Consequentia tenet per hoc quod iuxta ultimam suppositionem B est magis quam in duplo intensior D gradu. Et sic patet quod plus quam pedalis quantitas a Socrate in hora erit pertransita AF latitudine, quod est contra dictum. Consequentiae patent pro illa parte intuenti argumenta prius facta.

86 Hic tamen nota quod haec demonstratio fundatur super hoc, quod si latitudo motus incipientis a quiete et uniformiter sit adquisita in aliquo tempore necessario in prima medietate eiusdem temporis quarta totalis spatii pertransietur et hoc medietate remissiori eiusdem latitudinis. Et ideo neque intensiori medio nec remissiori eiusdem illa latitudo in pertransitione spatii correspondet.

87 Ex istis duabus conclusionibus sequitur tertia, quod omnis latitudo finita circa quietem terminata suo medio gradui uniformiter adquisita correspondet. Detur latitudo AB citra quietem terminata cuius remissius extremum sit A. Dico igitur eam uniformiter adquisitam suo medio gradui correspondere, nam tantum pertransitur AB latitudine uniformiter adquisita in hora sicut A gradu continuato per horam uniformiter et latitudine
aequali AB incipiente | a quiete uniformiter adquisita, ut patet per 58v
unam conclusionem prius probatam. Sed cum omnis latitudo incipiens a quiete et uniformiter adquisita suo medio gradui

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27 conclusionibus] *marg.* 28 suo…adquisita] *infra col.* 32 AB] *marg.*

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3 suppositione…Euclidis] *Cf.* Euclides, *ibid.*

correspondet, ut patet per praecedentem conclusionem, igitur AB latitudine tantum pertransiretur in hora sicut a duobus motis per horam quorum unum movebitur A gradu solum per eadem, et aliud gradu medio tantae latitudinis incipientis a quiete. Consequentia patet per prius probata. Sed cum gradus medius AB latitudinis sit compositus ex A gradu uniformi et gradu medio tantae latitudinis incipientis a quiete, igitur tantum pertransietur AB latitudine uniformiter adquirenda sicut gradu medio eiusdem, quod fuit probandum.

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Index of Names

Summary

The main goal of our book is to answer the question raised first at the beginning of the 20th century about the continuation or of the discontinuation of the part of physics, namely the “science of motion” between the Later Middle Ages and Early Modern times. This aim was planned to be achieved through detailed analyses of the heritage of the English fourteenth-century philosophers of nature, who constituted the School of Oxford Calculators presented in both the secondary literature, and the medieval sources, that were, until now, to be found only in the Latin manuscripts. Therefore the present book is divided into two parts, the first of which consists of four chapters and the second part offers critical editions of these Latin manuscript texts. Given that contemporary researchers still formulate their opinions about the later medieval philosophy of nature on the basis of fragmentary and abbreviated presentations of the Oxford Calculators’ works, their incomplete knowledge frequently leads to mutually incoherent or even contradictory statements. Therefore, there was an urgent need to fill the blank spot within the history of the Oxford Calculators tradition in “mechanics” with the critical editions that are included in Part II of this work. We offer the critical editions from Latin manuscripts not only of the most famous Calculators’ works, such as William Heytesbury’s *De tribus praedicamentis*: *de motu locali* or John Dumbleton’s Part III of the *Summa logicae et philosophiae naturalis*, but also of a hitherto unknown work by Richard Kilvington, i.e., his question on local motion and the question on local motion written by the anonymous author of the treatise *De sex inconvenientibus*.

Studies on the Oxford Calculators’s thought begun with Pierre Duhem’s research published at the beginning of the 20th century. The

discovery of later medieval mathematical physics, which, in accordance to common opinion of historians of medieval science, was "introduced" by Thomas Bradwardine, initiated intensive research in the field. Konstanty Michalski, Marshall Clagett, Anneliese Maier, Henry Lamar Crosby, Curtis Wilson, John Murdoch, Ernest Moody, George Molland, John Longeway, Stephen Read, Fabienne Pironet, Sabine Rommevaux, and Edith Sylla, to mention only a few names, devoted their studies either to preparing critical editions of the Oxford Calculators' texts or to presenting the main ideas of the Calculators themselves.

The predominant belief, expressed by Edith Sylla, and commonly accepted, is that: "The Calculators carried their analyses and calculations a bit too far for it to be plausible that their main goal was discoveries in natural philosophy". In her opinion the works of such personalities of fourteenth-century Oxford philosophy as Richard Kilvington, Thomas Bradwardine, William Heytesbury, John Dumbleton and Richard Swineshead, albeit full of discussion on natural philosophical problems, were intended from the outset to be first of all, more or less advanced, logical exercises, meant primarily for advanced undergraduates. In the present book, however, we made an effort to prove that the Oxford Calculators works were aimed not at formulating increasingly complicated logical riddles, but rather at developing the natural science, with a special attention on "science of motion" within the typically Aristotelian scheme of theoretical science.

Taking into account how much has been discovered, edited and written on the Oxford Calculators up to now, we decided to revise and compare the results of our and other historians' studies on the intellectual heritage of these fourteenth-century English thinkers in order to provide those interested with an updated and well supplemented account on the Oxford Calculators' natural philosophy in perhaps its most fundamental aspect – at least from the point of view of Aristotelian philosophy – namely on the "science of local motion". In order to recount the history of the development of the theory of local motion, we have thoroughly examined the texts of all the Calculators from the beginning of the School, i.e., from Richard Kilvington's questions (1326) till the very conclusion with Richard Swineshead's treatise *De motu locali* from his "Book of calculations" (*ca.* 1350). We have also compared our own conclusions resulting from these studies with those formulated by other historians of medieval science. We have mostly focused our attention on topics that were important to medieval thinkers and not those that could be most interesting from a modern physicist's point of view. Thus we have directed our research on the Oxford Calculators' tradition in science towards prospecting the innovative character of their learning, and here first of all against the background of Aristotelian theories, and then the subsequent search for possible innovations which could have inspired early modern scientists. Although each of the Calculators dealt with four types of changes that Aristotle defined generally as motion, that is: generation, alteration, augmentation, and local motion, we decided to focus on their concepts of local motion, because some historians of science have claimed that Galileo took advantage of their solutions in this very respect. It is beyond any doubt that the local motion, firstly described by Aristotle in his Book IV and VII of the *Physics*, was the core interest for physicists until the twentieth century. Thus far historians of science had been focusing on the most famous achievements of the Oxford Calculators, such as "the new rule of motion" or "Bradwardine's rule", as it is commonly known, and "the mean speed theorem". Our goal was rather to answer the main question of the evolutionary or revolutionary character of science on the basis of many more sources derived from the School itself.

In Chapter I of this book brief biographies and descriptions of the works of the most influential Oxford Calculators, i.e., Richard Kilvington, Thomas Bradwardine, William Heytesbury, John Dumbleton, and Richard Swineshead, as well as the important anonymous treatise *De sex inconvenientibus*, are presented to illustrate the scope of their major philosophical interests. Accurate information on the availability of their works, i.e. critical editions, old prints, and manuscripts, was intended to indicate which of the Calculators' works have been most studied since their editing, and which have simply been forgotten in the general history of medieval science as they still remain in form the manuscript.

In Chapter II the scientific background and sources of inspiration of the theories of motion as proposed by the Oxford Calculators is presented. Most of their works were composed in order to meet the requirements of the curriculum of the University of Oxford, that, in the fourteenth century, obliged bachelors and masters at the Arts Faculty to comment on Aristotle's *On generation and corruption* and *Physics* as well as to teach logic. That is why Chapter II begins with Aristotle's theories and Averroes's commentaries. The latter introduced within his comments some new material presented in the context of discussing the ideas of his Arabic predecessors and those of his contemporaries. In fact, Averroes's interpretation of Aristotle's texts on natural philosophy gave the impulse to formulate new theories on motion. Latin philosophers in the fourteenth century interpreted Aristotle through Averroes's expositions being absolutely sure that this commentary mirrors and stays in accord with the theory of the Stagirite. The far-reaching moment in the history of "mathematical physics", as developed by the Oxford Calculators, was also the broad use of mathematics, which from the very beginnings of Oxford University was recognized as a demonstrative science and the proper tool of analyses within the philosophy of nature. It was the first chancellor of Oxford University, Robert Grosseteste, who was to introduce mathematics into his philosophical considerations. This attitude was adopted and enthusiastically propagated by Roger Bacon, John Peckham and Robert Kilwardby, among other English philosophers. The teaching of logic and mathematical disciplines such as geometry, arithmetic, optics, music, static and astronomy was far more developed in Oxford than in other medieval universities. This legacy was most obviously also inherited by the Oxford Calculators. In Chapter II of the book we have summarized the most significant theories of thirteenth- and early fourteenth-century English thinkers. The most influential, however, was – in our opinion – the original, innovative philosophy of William of Ockham. Ockham was only a bit older than the first Oxford Calculators, and his ideas – as we are convinced – gave them the first impulse to reinterpret Aristotelian theories in natural philosophy.

In Chapter III the detailed analyses of the theories of local motion offered by the above-mentioned Calculators are given. The analyses indicate clearly the continuous development of the theory of local motion while revealing the relationships of a varied kind (inspirations, borrowings, controversies, etc.) between the specific opinions of Calculators.

Finally, Chapter IV is focused on answering the question as to whether the achievements of the Oxford Calculators really gave the impulse for the development of the seventeenth century mechanics, or rather if they only provided a new interpretation of Aristotelian philosophy of nature.

Our final conclusions derived from these analyses are as follows. Historians of medieval science when analyzing Oxford Calculators' theories have described them from two different points of view: either from the perspective of a physicist, or that of a mathematician. So, like

Marshall Clagett, for example, they used modern mathematical language or "translated" medieval vocabulary into modern physical terms, thus suggesting that the 14th century theory of motion was not too far distant from 17th century mechanics. The second perspective has been adopted by researches like Edith Sylla, who from the very beginning of her research, i.e., since her Ph. D. dissertation (1970, published in 1991): "The Oxford Calculators and the Mathematics of Motion 1320-1350", up to the very last paper "Leibniz and the Calculatores", to be published in a volume dedicated to the history of the Oxford Calculators, consequently uses terminology suggesting that Calculators' "calculus of ratios", expresses the mathematical function. She is convinced that Bradwardine employed pre-Theonine's version of Euclid's "Elements" as well as Archimedes' and Apollonius' calculus, and he purposedly applied their theories to his theory of motion. As Jung proved, however, Bradwardine employed Klivington's arguments and theory of motion with a new calculus of ratios. It is also clear that Kilvington used the pre-Thoenine's version. Nevertheless Kivington was convinced that in order to describe properly continuous motion using such terms like speed, power, space and time, which are the continuous quantities the continuous proportion must be used, and this one is defined by Aristotle in his *Ethics*. It is also likely that Bradwradine and Kilvington were taught this theory of proportionality by the same math teacher at Baliol College, Oxford. They both maintained that the new theory of local motion is only a new interpretation of Aristotle's and Averroes' statements. Even if they introduced the new "calculus" to describe local motion, they followed, in a sense, Aristotle's "science of proportionality". Kilvington and Bradwardine, criticizing the rules of motion of Aristotle's and Averroes' presented in Books IV and VII of *Physics*, introduced arguments based on common experience, such as pushing a stone, dragging a barge across a river, or rolling a clock face due to an unevenness of suspended weights, while also following Aristotle's methods of observing events.

It is now widely accepted that in his *De proportionibus*, dated for 1328, Thomas Bradwardine advocated a new conception of the relations between ratios of motive powers to resistances and the resulting speeds, a conception that continued to be supported by Aristotelians until the early sixteenth century. What has not been recognized until recent times is that the theory called "Bradwardine's rule" ("the new rule of motion") was based on the mathematical theories of compounding ratios familiar to Oxford scholars in the earlier 1320s, i.e., well before 1328. So here it seems to be a case of "Stigler's law of eponymy", published by Stephen Stigler in 1980, which states that no scientific discovery is named after its original discoverer. In studying Kilvington's work, then, we find information about what was going on in Oxford natural philosophy before Bradwardine's *De proportionibus*, which previously had been recognized as the founding document of the Oxford Calculators' natural science. In attempting to trace the impact, spread, and decline of quantifying Aristotle, we should now realize that the activity of quantifying motion had a prehistory prior to 1328. In the opinion of Sylla and Murdoch, however, the tendency to remain close to Aristotelian "rules" of motion seems to be characteristic for all thirteenth- and early fourteenth-century commentators on the *Physics*. "The situation changed rather dramatically in 1328", when Thomas Bradwardine wrote his *Treatise on the Proportions of Velocities in Motion*. He removed the whole problem of relating velocities, forces and resistances from the context of an exposition of Aristotle's words, and investigated it in its own right. In the present book we have proved that this assertion is not well supported.

Nevertheless, even if, until now, the historians of medieval physics have been convinced and stated that that "Bradwardine's rule" concerns, and properly describes, relations in natural phenomena, namely in actually occurring local motions, yet they claim that fourteenth-century natural philosophers were not interested in description of changes taking place in the real world but in the world of the imagination. Bradwardine provided a proper rule describing motion that was recognized by his contemporaries as no more than a speculative tool in the description of the natural world; and thus, for constructing more or less complicated imaginable cases. Consequently, the resulting "science of local motion" became a substantial basis only for logical exercises. At first glance, it seems that the contents of the chapter *De motu locali* from William Heytesbury's "Rules for solving sophisms" affirms perfectly this conclusion.

It seems that the book of the last Calculator, namely Richard Swineshead's "Book of calculations" represents the most sophisticated stage in the development of natural philosophy within the circle of the Oxford Calculators, however, at the same time it is a perfect exemplary of how strongly attached, or even deeply affected by the Aristotelian worldview these otherwise ingenious thinkers were. At least in the context of his "science of local motion" Swineshead never crossed the boundaries of Aristotle's physics, even if he were to reach them in due course. In

Chapter XIV of the *Liber calculationum*: *De motu locali*, dedicated from the outset to establishing the "rules" of local motion, he simply adopts the new, Kilvingtonian/Bradwardinian rule of motion and exploits it to its limits, dictated by logical and mathematical applicability and consistency. As was presented in detail earlier, the consecutive cases he discussed there were formulated *a priori* by a consequent permutation of the imaginable changes in the factors of local motion, and the resulting changes of speed(s) were determined in a "geometrical" manner, on the basis of the already accepted or proven statements. The whole of his "science of local motion" is developed speculatively, Swineshead never referred to natural phenomena, either when formulating the "cases" or establishing the "rules". Swineshead was perfectly aware of the limitations and doubts formulated by his predecessors from the Oxford Calculators' circle, and tried hard to solve and overcome these. Therefore, Richard Swineshead's account on local motion should be appreciated with respect to the range and complexity of the cases he considered and "solved". Yet, it must be stressed here, that Richard Swineshead's aim was not to formulate any new, not to mention revolutionary, theory of local motion. He strove rather to supplement and complete the "science of local motion" formulated by his predecessors within the theoretical boundaries of Aristotelian natural philosophy.

In summarizing and reviewing what in the opinion of some historians of medieval science appears to be the most important departures of fourteenth century mechanics from Aristotle's physics we claim as follows. First of all, in the case of Oxford Calcualtors, there is a blend of the Aristotelian dynamics tradition and Archimedean statics and mathematical tradition. Secondly, there is a refutation of Aristotle's prohibition of *metabasis* and the use of mathematics as the proper, next to logic, method in natural philosophy. As we have emphasized, it was for the first time in the medieval period that mathematical strictness forced natural philosophers to invent a "new", i.e. consistent rule of motion. Thirdly, it was the differentiation between dynamics and kinematics, that led to the formulation of "the mean speed theorem" enabling one to compare the speed of a uniformly accelerated/decelerated motion with the speed of a uniform motion. Fourthly, there is the promotion of mental experiment as a method of confirming the established "rules".

Deeper insight into medieval mechanics, however, reveals the constant presence of the Aristotelian background. Even though Kilvington and Bradwardine had broken the Aristotelian prohibition of *metabasis*, they still remained within the framework of his physics, in which motion occurs because of the constant action of two necessary factors: moving power and resistance – acting as its direct causes. The speed of motion is determined by the ratio of moving power to resistance and "the new rule of motion" does not break this principle. Like Aristotle, Kilvington, Bradwardine, and their followers, maintained that constant motive power (and resistance likewise) causes a constant speed and not constant acceleration, something which was later only properly recognized by Galileo and formulated as the second law of motion by Newton in the seventeenth century.

Secondly, the notions 'uniform', 'uniformly difform' and 'difformly difform' motion were used not only to describe the distribution of changes in uniform, accelerated and decelerated motions. For when medieval natural philosophers considered the difformly difform speed, they had in mind not only non-uniform changes of speed, but also uniform changes of acceleration, i.e., a motion with equal increments/decrements of acceleration. Such motions do not occur as natural phenomena. Furthermore, such terms as 'uniformly difform' motion and 'uniform increasement of speed' were used in both contexts – of the motion of a free fall, i.e., downward motion, and of imaginable, uniformly accelerated upward motion. This is a part of medieval mechanics to which we do not pay enough attention, since we look only for properly recognized problems.

Thirdly, common as they were in the Middle Ages, mental experiments were rationalistic, only thought out, and not empirically rooted experiments, and these did not stimulate the development of an experimental science of motion.

Still, we agree with John Murdoch and Edith Sylla, who have pointed out that: "It would be an error to regard these new and distinctive 14th century efforts as moving very directly toward early modern science". Galileo's familiarity with late medieval physics' departures from Aristotle, which even made him repeat some of their erroneous solutions, did not affect his general idea, since he used fragments of medieval mechanics for completely different purposes. Galileo, whom we want to make responsible for the beginnings of Newtonian dynamics, rejected or rather reformulated "the new rule of motion" while going back to the theory expressed by Avempace. Likewise, he read Archimedes' works in a different way and context than did the medievals, which allowed him to create mathematical physics while recognizing the distinction between statics and dynamics. It also lead him to consider mechanics as a contemplative and mathematical science under geometry that could provide the mechanical arts with their principles and causes. With the two major achievements of Galileo's mechanics, namely the conception whereby the horizontal uniform motion of an unanimated body is held to be a state in which it remains until some external force causes it to change and the identification of free fall as a uniformly accelerated motion with the exposition of its role in nature, the new concepts in mechanics began a career that culminated in Newton's systematic exposition. In spite of this, Galileo was able to profit from the *secundum imaginationem* and *ceteris paribus* procedures, making broad use of mental experiments to convince his readers to accept Copernicus's heliocentric theory. Galileo's approach to the problem of a possibility of applying mathematical principles to physical phenomena was to view these principles not as pure mathematical abstractions but as laws that governed an experimentally rooted science of motion.

We would like to stress, however, that each step taken by new generations of fourteenth-century natural philosophers was a step forward, even though it was a step taken on the dead-end road of the Aristotelian science of motion. In our opinion, medieval mathematical physics was doomed, since even if it had succeeded in refuting the restrictive prohibition of *metabasis* associated with Aristotelian philosophy and accepted mathematics as its method, it did not develop empirical mathematics and experimental physics. This was because, ironically, the liberation of mathematics from the limitations of actual experience created a tool of theoretical analysis that would make it impossible to cross over the threshold of an exact science. Even though a tradition in "mathematical physics" was to continually develop in England from Grosseteste till the middle of the fourteenth century and then was to be continued by French, Italian, and Spanish thinkers until the end of the sixteenth century, it never made the final step forward to abandon Aristotle. Paradoxically, Aristotelian physics appeared to be perfectly prone to accommodate all medieval attempts at providing it with mathematical precision. The fourteenth-century revolution in mechanics was a revolutionary movement against the background of previous medieval theories, but not in relation to the seventeenth-century ones. The revolution was in the details. In its history medieval science, while taking an Aristotelian course, thoroughly explored that framework exposing its paradoxes and weakness yet reached the point where it was unable to overcome the lingering doubts. The big, decisive break was left to the successors of the medieval philosophers of nature.

After a deliberated study of the medieval science of motion and secondary literature we are forced to formulate the final conclusion: the fourteenth-century revolution in science should not be regarded as the first step towards the Scientific Revolution. In our opinion later medieval mathematical natural science should be treated only as a specific and fascinating phenomenon of medieval thought culture and evidence of the ingeniousness of the scholars that created it.

Our research confirms our belief that scientific truths in general, and even historical facts in particular, are never established once and forever, thus, through the present book we intend to revise the story of the Oxford Calculators' school.